Curvature of the trajectory traced out by a monochromatic plane electromagnetic wave in a medium with rotation

V. O. Gladyshev
N. É. Bauman Moscow State Technical University, 107005 Moscow, Russia

(Submitted 30 April 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 8, 593–597 (25 October 1993)

A new relativistic effect has been observed in a medium with a permittivity ϵ and a magnetic permeability μ which has a rotation Ω. This effect is a curvature of the trajectory traced out by a monochromatic plane electromagnetic wave. An interference method is proposed for measuring this curvature. The method makes use of a rotating, symmetric, nonconfocal resonator filled with a medium with ϵ and μ.

A curvature of the trajectory of an electromagnetic wave arises in the Sagnac experiment[1] when the refractive index of the medium between mirrors satisfies the condition $n>1$, upon removal from the moving frame of reference of the radiation source and detector. If the latter condition does not hold, this system is independent of the refractive index in the nonrelativistic approximation, as was correctly pointed out in Refs. 2 and 3. If the velocity of the medium has no tangential component ($U_{2t}=0$, $U_{2n}\neq 0$), there is a longitudinal entrainment in the classic Fizeau experiment.[4]

The solution of the dispersion relation for the propagation of an electromagnetic wave in a medium is valid for an atomic layer with a thickness on the order of a few wavelengths of the electromagnetic radiation.[5] For calculations on each layer of the medium, the only properties available are the frequency ω_0 and the angle of incidence on the interface between the two media, ϑ_0. The motion of a given layer of the medium affects the coordinates of the point at which the wavefront intersects the next layer. In general, for a region of the medium in which the velocity is not constant, it is necessary to solve a wave equation for each neighboring local region of the medium. The complete solution is the set of local solutions for the regions in which the velocity of the medium is constant at the physically necessary accuracy.

We consider a medium in the half-space $Z<0$ which has ϵ_1 and μ_1 in its rest frame. There is also a medium at $Z>0$, with ϵ_2 and μ_2 in its rest frame. We choose a frame of reference in which the medium at $Z<0$ is at rest, while the other medium is moving at $U_2=U_{2x}\mathbf{e}_x+U_{2y}\mathbf{e}_y+U_{2z}\mathbf{e}_z$, where $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ are unit vectors. A monochromatic plane electromagnetic wave of frequency ω_0 is incident from the first medium on the surface of the tangential discontinuity. The wave vector of the incident wave, $\mathbf{k}_0$, is in the X, Z plane and makes an angle ϑ_0 with the Z axis. According to the requirement that the phases of the incident, refracted, and reflected waves be equal at the interface, the tangential invariant corresponds to $I_t=k_{0x}=k_{1x}=k_{2x}$. The invariant $I_1=-\omega_0=-\omega_1=-\omega_2$ corresponds to equality of the frequencies, due to the zero normal component of the velocity of the interface. For this system, the coordinate solution of the dispersion relation[6] for the refracted wave is, if we ignore absorption and dispersion of the moving medium,

$$k_{2z}=\frac{\omega_0}{c}\left[-\kappa_2\gamma_2^2\beta_{2z}\xi_2\eta_2+(\eta_2\cos^2\vartheta_0+\kappa_2\gamma_2^2\xi_2^2\eta_2^2)^{1/2}\right], \tag{1}$$

where

$$\xi_2=1-\beta_{2x}\sin\vartheta_0,\quad \eta_2^{-1}=1-\kappa_2\gamma_2^2\beta_{2z}^2,$$

$$\kappa_2=\epsilon_2\mu_2-1,\quad \beta_{2z}=\frac{U_{2z}(x)}{c},\quad \beta_{2x}=\frac{U_{2x}(z)}{c},$$

$$\gamma_2^{-2}=1-\beta_2^2,\quad \beta_2^2=\beta_{2z}^2+\beta_{2x}^2.$$

For a given law of rotation centered at the point $x=0$, $z=a_0$, the tangential and normal components of U_2 correspond to

$$U_{2x}=\Omega(a_0-z),\quad U_{2z}=\Omega x. \tag{2}$$

The angle through which the electromagnetic wave is refracted, ϑ_2, is found from the relation $\tan\vartheta_2(x=0,\ z=0)=k_{2x}/k_{2z}$, where $k_{2x}=(\omega_0/c)\sin\vartheta_0$. We impose a boundary on the trajectory of the electromagnetic wave in the second medium: a surface of radius $R=a_0$. We require $R\gg\lambda_0$, where $k_0=2\pi/\lambda_0$.

The trajectory lies in the X, Z plane. The implicit equation

$$z=\int_0^{x_{\max}(x,z)}\frac{k_{2z}dx}{k_{2x}} \tag{3}$$

corresponds to this trajectory. Here

$$x_{\max}(x,z)=\frac{1}{2}\sin 2\vartheta_2\{a_0-k\tan\vartheta_2+[a_0(a_0-2k\tan\vartheta_2)-k^2]^{1/2}\}, \tag{4}$$

$$k=x-z\tan\vartheta_2$$

is the coordinate of the expected intersection of the trajectory of the electromagnetic wave with the cylindrical surface. This coordinate is drifting with x, z.

Since there is no explicit general solution of (3), it is preferable from the accuracy standpoint to use the expression $\tan\vartheta_2(x,z)$ for numerical estimates of the curvature of the trajectory. The geometric length of the trajectory of the electromagnetic wave in the rotating medium is then described by the equation

$$L_t\int_0^{x_{\max}(x,z)}\sqrt{1+\cot^2\vartheta_2(x,z)}\,dx. \tag{5}$$

Using the expression for the geometric length of a rectilinear trajectory to the point with the coordinate $z_{\max}$, i.e., $L_{0t}=\sqrt{2a_0 z_{\max}}$, we find the equivalent difference in path lengths for waves which have traversed the path from the point (0, 0) to the point $(x_{\max}, z_{\max})$ with $\Omega=0$ and $\Omega\neq 0$:

$$dL_{cr}=n_2(L_t-L_{0t}). \tag{6}$$

Since n_2 is not a function of the velocity of the medium, the path difference due to the longitudinal Fizeau effect clearly does not enter dL_{cr}. Working from the relation for the propagation velocity of an electromagnetic wave in the medium, $c' = -I_1 \cos \vartheta_2 / k_{2z}$, we can write an equation for the equivalent length of the trajectory:

$$L_e = 2c \int_0^{x_{\max}(x,z)} \frac{k_{2z}\, dx}{(-I_1) \sin 2\vartheta_2(x,z)} . \tag{7}$$

Experimentally, one might measure the accumulated difference between the path lengths traversed by two electromagnetic waves which are incident on the interface between two media at an angle ϑ_0. One of these waves would be propagating in a medium with $\Omega = 0$, and the other in a medium with $\Omega \neq 0$. This accumulated path difference corresponds to

$$dL_e = L_e - L_{0e} . \tag{8}$$

The accumulated path differences due to the transverse and longitudinal entrainment effects are, respectively,

$$dL_t = n_2 (L_t - L_0), \tag{9}$$

$$dL_l = L_e - n_2 L_t . \tag{10}$$

Equations (5)–(10) determine the physical and geometric characteristics of the transformation of an electromagnetic wave in a frame of reference with rotation.

We turn now to the results of some numerical calculations and some implications. The basic result of the calculations is to confirm that there are curvilinear propagation trajectories for electromagnetic waves in a medium with $\Omega \neq 0$, as follows from Eqs. (1) and (2). This effect has a clear physical explanation, based on the circumstance that only one component of the wave vector, k_2, changes in a moving medium. Since the electrodynamic equations are written in an inertial frame of reference, there is a change in $\vartheta_2 = \arctan[k_{2x}/k_{2z}(U_2)]$ in each local region of the trajectory. In other words, secondary electromagnetic waves change direction in each local region of the trajectory because of a change in the projection of the velocity of the atoms of the medium onto the wave vector of the excitation wave. As a result, there is a drift of the phase velocity, and there is curvature of the trajectory representing the superposition of all waves.

Interestingly, the wave trajectories with $\Omega = 0$ and $\Omega \neq 0$ intersect on the straight line $z = a_0$ for arbitrary ϑ_0. Numerical values for the transverse and longitudinal entrainment effects are shown for comparison in Fig. 1 in plots versus ϑ_0 for the following parameter values: $k_0 = 10^{-7}$ m^{-1}, $n_2 = 1.5$, $a_0 = 0.1$ m, and $\Omega = 10^4$ rad/s. From the shape of the curves for dL_t and dL_l we conclude that there is a competition between effects with increasing ϑ_0. In the integration, the size of the local region corresponded to $\sim 10^{-5}$ m; a reduction of this value had essentially no effect on the calculated results.

Some experiments[7,8] which have been carried out agree well with the solution of the electrodynamic equations for moving media. However, these experiments consti-

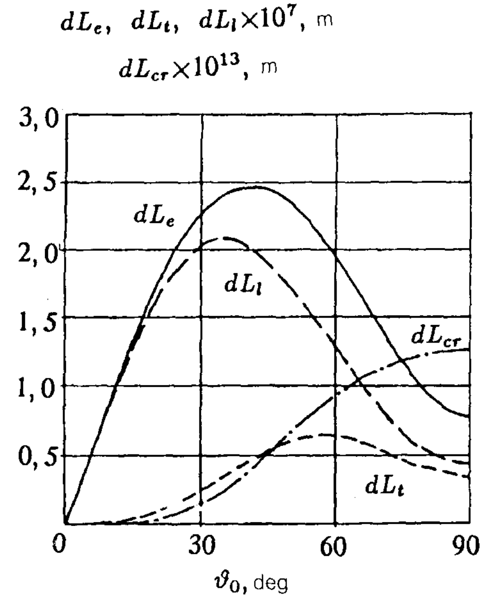

FIG. 1. Equivalent path differences of the electromagnetic waves, $dL_e(\vartheta_0)$, $dL_t(\vartheta_0)$, and $dL_l(\vartheta_0)$, for $\Omega=0$ and $\Omega=10^4$ rad/s, along with the geometric path difference due to the trajectory curvature, $dL_{cr}(\vartheta_0)$.

tuted an experimental test of only the part of the equations associated with the motion of the interface, not of the medium itself. The passage of an electromagnetic wave through a medium with rotation opens up the possibility of an experimental test of the part of the solution of the wave equation which contains terms with U_{2x} and U_{2z}. It follows from Fig. 1 that for the assumed value of Ω and for $\vartheta_0 \sim 45°$ the accumulated pass difference is on the order of λ_0 for a single passage through a medium. This quantity increases linearly upon multiple rereflection at a cylindrical surface of radius a_0, which forms a symmetric nonconfocal resonator. There is accordingly a large margin in terms of accuracy for studying the relativistic curvature of the propagation trajectory of a light beam in a laser interference experiment.

In summary, from the experimental standpoint a study of this physical effect—the curvature of the propagation trajectory of a monochromatic plane electromagnetic wave in a medium with $\Omega \neq 0$—respresents not only a determination of the curvature and the possibility of carrying out a new experimental test of the electrodynamics of moving media but also the construction of a new type of relativistic interferometer.

[1] M. G. Sagnac, J. Phys. (Paris) **4**, 177 (1914).
[2] A. A. Logunov and Yu. V. Chugreev, Usp. Fiz. Nauk **156**, 137 (1988) [Sov. Phys. Usp. **31**, 861 (1988)].
[3] C. V. Heer, Phys. Rev. A **134**, 799 (1964).
[4] d'H. Fizeau, Ann. Chim. Phys. **57**, 385 (1959).
[5] D. Censor, IEEE Trans. Microwave Theory Tech. **MTT-16**, 565 (1968).
[6] B. M. Bolotovskiĭ and S. N. Stolyarov, Usp. Fiz. Nauk **159**, 155 (1989) [Sov. Phys. Usp. **32**, 813 (1989)].
[7] J. S. Hey, J. T. Pinson, and P. Smith, Nature **179**, 1184 (1957).
[8] O. G. Zagorodnov, Ya. B. Faĭnberg, and A. M. Egorov, Zh. Eksp. Teor. Fiz. **38**, 7 (1960) [Sov. Phys. JETP **11**, 4 (1960)].

Translated by D. Parsons