

'Algebraic Chemistry' Based on a 'PIRT'*

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Abstract

The present paper continues on from my 2004 PIRT paper entitled "Can Chemical Data Support a 'PIRT'?". The present paper first provides more detail on the variation of ionization potentials within the periods of the Periodic Table. The description developed refers to the standard quantum numbers n, l, s ($n=1$ to infinity, $l=0$ to $n-1$, $s=-1/2, +1/2$) for single-electron states that are incorporated into successively larger atoms. The empirical Madelung and Hund rules that are found in today's chemistry textbooks provide the nominal filling order for single-electron states. Using this filling order, a simple empirical formula is developed for the local slopes on a log plot of the ionization potentials. The formula is quite accurate for first ionization potentials, but becomes less accurate for higher-order ionization potentials. This fact could well be related to the fact that the filling order that actually occurs in Nature departs from the standard one for about 20% of the known elements. Accordingly, the filling order for single-electron states is itself investigated next. A more efficacious empirical rule is developed. Like the standard one, it involves sums of traditional quantum numbers, but unlike the standard one, judiciously chosen coefficients that are powers of 2 provide the needed improvements. The resulting formulation automatically directs attention to the elements for which the departures from the textbook rules actually do occur. The paper concludes with an indication of future work.

* Dedicated to the memory of Al Kelly, our fondly remembered and much missed colleague at these PIRT meetings.

1. Introduction

This paper is part of an on-going research program that aims ultimately to produce theoretical explanations for many puzzling empirical facts, a great many of them from chemistry. The standard quantum chemistry (QC) of today, based on the standard quantum mechanics (QM) of yesterday, does not fully explain the chemical behaviors addressed here. So a new approach seems needed. The new approach tried here is based on a new founding postulate for QM. Instead of taking Planck's constant as a fundamental constant of Nature to begin from, the new approach starts from a new founding postulate for special relativity theory (SRT), and then applies the resulting expanded SRT to atomic systems.

The new approach has been introduced to this audience in PIRT papers starting with year 2000. The approach is called "Two-Step Light". What does that name mean? Conventional models for light assume steady, linear propagation of wave fronts, or photon particles, across space, characterized by constant light speed c . Two-Step Light is not so steady; the wave or photon is contained within a boundary-defining 'envelope' function, and the 'propagation' involves that envelope first expanding from a source, then collapsing to a receiver. In contrast to the 'wave' idea based on Maxwell, the emphasis is not on the oscillation, the thing described by a differential equation, but rather on the envelope, the thing that imposes the boundary conditions. In contrast to the 'photon' idea developed by Einstein, the image is not a 'bullet', a thing whose size is small, constant, and not very physically relevant, but rather a 'balloon', a thing that is spatially extended, flexible, and able to receive or discharge contents over a period of time.

The idea of Two-Step Light is a starting point for a development of a covering theory for Special Relativity Theory (SRT), which includes the familiar relationships, along with many new relationships, and overall a context within which to understand everything more fully and eliminate the numerous paradoxes that occupy the existing literature of SRT. That was the point of the 2000 PIRT paper "How Can Paradox Happen?". The text emphasized a fact well appreciated in the engineering world: mismatch between mathematical model and physical process can wreak havoc like that which we see in the literature of SRT.

A restart on SRT offers the opportunity to have also a restart on atomic theory and quantum mechanics (QM). The PIRT 2002 paper "Do Atoms Really Have States?" developed this idea in regard to Hydrogen atoms. It argued that the ground state of Hydrogen is determined by a relativistic phenomenon: signal delay due to finite propagation speed. Finite signal speed causes radiation and energy loss (well-known), plus internal torquing and energy gain (not previously appreciated at all!) The balance of the two produces the ground state. The paper further argued that sin-

gle atoms do not have ‘excited states’ *per se*; that instead, systems of multiple atoms working together constitute ‘excitation’. This leads to a new understanding of its various ‘quantum states’. In particular, the $n > 1$ for an excited state of Hydrogen does *not* refer to an eigenvalue identifying one member from the complete ortho-normal basis set of possible solutions to a differential equation formulated by Schrödinger for the complex amplitude of a probability wave. The $n > 1$ just refers to how many Hydrogen atoms are involved in a multi-atom system.

A restart on QM offers in turn the opportunity to have a restart on quantum chemistry (QC) and chemistry overall. The PIRT 2004 paper “Can Chemical Data Support a PIRT” introduced a visual image representative of a new start. It put the information from the well-known Periodic Table (PT) into a different format, called the Periodic Arch (PA), displayed here as Fig. 1. The purpose was to shake up the viewer’s thinking by displaying the information in from a different point of view. Wherever there is an arch, there is a ‘foundation’. For the PA, the foundation is the algebraic pattern ‘ $2N^2$ for $N = 1, 2, 2, 3, 3, 4, \dots$ ’ which describes the number of elements in the rows of the Periodic Table (hidden) or the layers of the Periodic Arch (revealed).

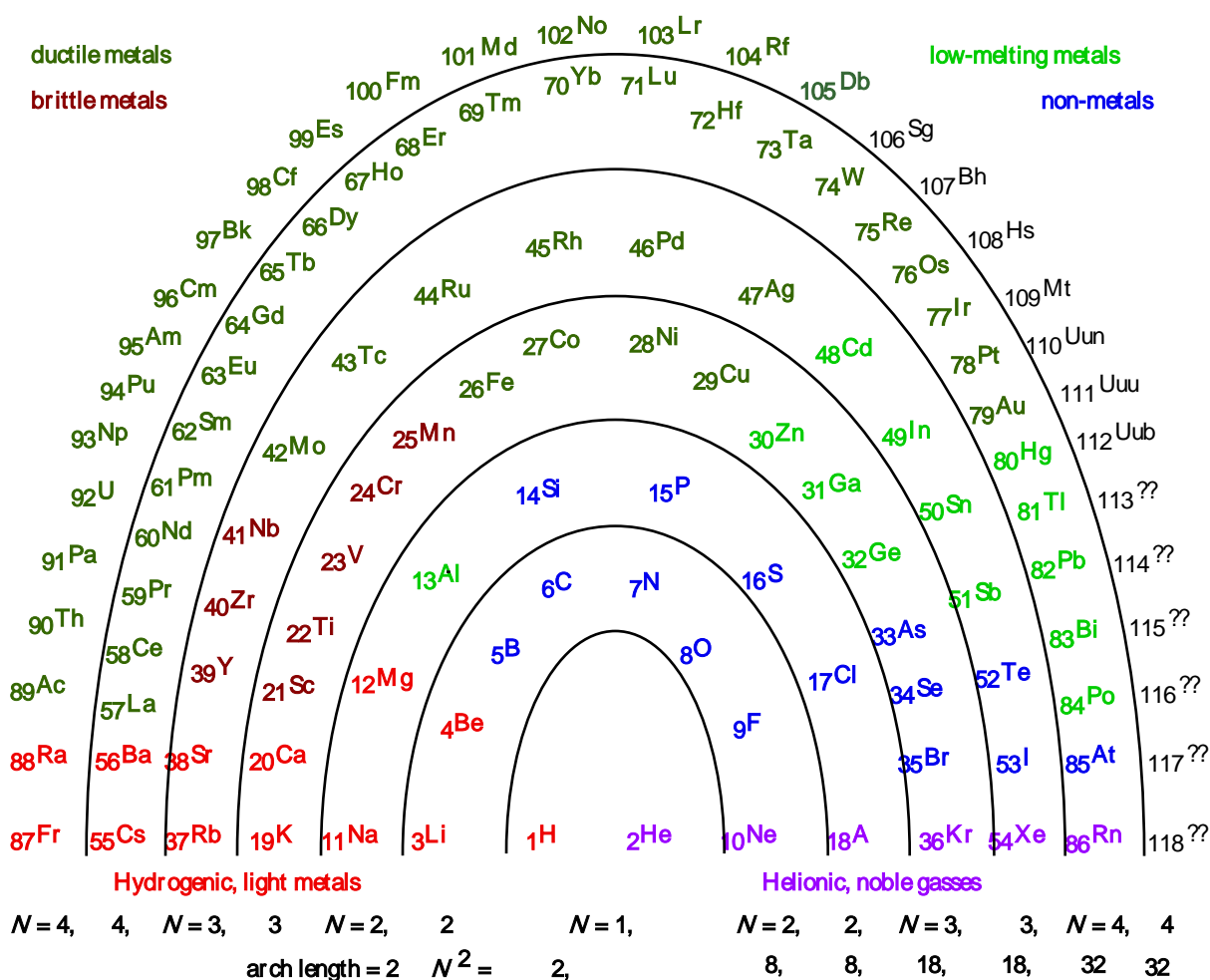


Figure 1. Nature’s Periodic Arch.

The payoff for enduring the shake up caused by this altered viewpoint is new insight into the complexities of real-world chemistry. There exists a huge body of rather chaotic-looking data about ionization potentials (IP ’s) for all the elements and all the orders of ionization they permit. The ideas developed in the PIRT 2004 paper “Can Chemical Data Support a PIRT” made considerable sense out of the data. There has since been further development, reported in “Algebraic Chemistry”, Hadronic Journal 29 (1) (whole issue) 2006. The present paper recaps that in-

formation, and adds a bit more information. Two-Step Light is indeed a ‘PIRT’ that the body of data from chemistry richly supports.

Section 2 reviews essential information about the Two-Step Light and about the mechanism of electromagnetic balance in the prototypical atomic system: the Hydrogen atom.

Section 3 goes more deeply into the detailed pattern followed by ionization potentials within a period on the PT/PA. The pattern has been characterized in terms of conventional quantum numbers n, l, s ($n = 1$ to infinity, $l = 0$ to $n - 1$, $s = -1/2, +1/2$) for single-electron states (SES’s) that are incorporated into successively larger atoms, at least according to the nominal rules of the game. Without at all presuming that the conventional interpretation of those quantum numbers is adequate, we can nevertheless use them to characterize empirical facts concerning IP ’s. The simple formula developed for the local slopes on a log plot of the IP ’s is quite accurate for first-order IP ’s, but becomes less accurate for higher-order IP ’s. This fact could be related to the fact that the filling order for SES’s that actually occurs in Nature departs from the standard one for about 20% of known elements.

Section 4 Investigates SES filling order. The nominal order follows the totally empirical Madelung and Hund rules that are found in today’s chemistry textbooks. These rules say: fill states in order of increasing $n + l$; within a given $n + l$, fill states in order of increasing n ; within a given l , fill all of one s value first, then all of the other s value. There is no known theoretical reason why sums of quantum numbers should be important in determining filling order. Even so, an investigation can help develop a more efficacious quantum-number sum rule. Section 3 develops a better one. Like the standard one, it involves sums of quantum numbers, but unlike the standard one, judiciously chosen coefficients that are powers of 2 provide the needed improvements. The resulting formulation automatically directs attention to the elements for which the departures from the textbook rules actually do occur.

Section 5 gives a brief commentary and preview into future research.

2. Review of the Hydrogen Atom Analysis

All of the present analytical work is all rooted in a revisionist re-development of Einstein’s Special Relativity Theory (SRT). That redevelopment produces a covering theory for SRT known as ‘Two-Step Light’ theory. Two-Step light is summarized visually in Fig. 2. The spots represent, for successive times, two atomic systems: the source and the receiver. The arrows represent the light at the successive times. The propagation process consists of two steps: expansion from the source, followed by collapse to the receiver. This vision is something quite different from the vision that underlies SRT.

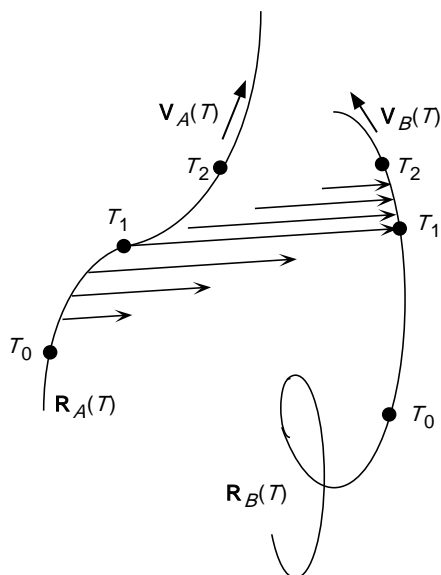


Figure 2. Illustration of light propagation in two steps.

Einstein’s point of departure for SRT was Maxwell’s nineteenth-century triumph: the theory of electromagnetism. That background put Einstein’s initial focus on the differential-equation aspect of the propagation problem, and on the waves that Maxwell’s differential equations describe. By contrast, Two-Step Light theory does not concentrate on the idea of light as a wave, or on any differential equation as the description of a propagation process. Instead, it focuses on the boundary-condition aspect of the propagation problem. Einstein later had to introduce something like this change of focus too, when he confronted the photoelectric effect. The idea of a ‘photon’ suggests a spatially confined, even point-like, light particle. Indeed, he quietly used this ‘confinement’ idea even in SRT, in conceptualizing the idea of ‘signal’. In short, even to say ‘signal’ is to say ‘tightly confining boundary’. He should have had a third postulate acknowledging that fact. Today we use the confinement idea to reconcile the phenomenon of dispersion, which can make a phase speed in excess of c , with the SRT requirement that nothing travel at a speed greater than c .

But Two-Step Light theory does *not* imagine a point-like light particle. Instead, it imagines a spatially-extended, elastic and time-varying, wave boundary within which the light waves (or whatever) are contained: a sort of ‘balloon’. The arrows on Fig. 2 represent the essential information about the balloon at any given time: its current length along the propagation direction.

Although Two-Step Light theory does emerge in response to the kind of phenomenology that drove the twentieth-century development of quantum mechanics (QM), the tie to QM is cautiously selective. Two-Step Light theory does not feature any kind of wave-particle ‘duality’; *i.e.*, the kind of enigma and mystery characteristic of the Copenhagen interpretation of QM. But it does accept apparent experimental facts, like non-local correlation, time reversal invariance, and wave function collapse. These are all now-familiar and sound ideas to enlist and build upon.

Two-Step Light theory replaces Einstein’s Second Postulate (*i.e.* constant c with no specific reference). The light speed c is relative to specific objects: first the source, and then the receiver. But there is much more to say about it. Observe that during expansion, the rear end of the light balloon stays with the source, while the front end moves away. The center of the balloon moves at c relative to the source; therefore, the front end moves at $2c$ relative to the source. Similarly, during collapse, the front end of the balloon is fixed to the receiver, while the rear end moves inward. The center of the balloon moves at c relative to the receiver; therefore, the rear end moves at speed $2c$ relative to the receiver. The occurrence of a $2c$ anywhere in the theory stops many readers cold. Nevertheless, that is the postulate for launching Two-Step Light theory.

Note that whatever the wavefronts (or whatever else one may imagine) may do, that behavior is not involved in the Two-Step Light Postulate. Neither is any assertion about how things evolve are in directions transverse to the propagation direction. Only the arrow matters. For all else, refer to Newton: *Hypotheses non fingo*.

Two-Step Light theory can be applied to the exchange of electromagnetic signals within a Hydrogen atom. This problem is of historic importance because it was among the problems that led to the development of QM. It was expected from Maxwell’s theory that the orbiting electron, being in acceleration, would give rise to radiation, and thereby to catastrophic energy loss from the system – which did not happen. The application of Two-Step Light theory leads to a second physical phenomenon: internal torquing. This produces an energy gain that can counter the radiative loss – and thereby account for the known facts without a separate and radical postulate to found QM..

For balance, the rate of energy loss due to radiation must equal the rate of energy gain due to signal propagation by Two-Step Light. For hydrogen, the balance is

$$2^5 \epsilon^6 / 3c^3 m_e^2 (r_e + r_p)^4 = \epsilon^4 / m_p \alpha (r_e + r_p)^3 \quad (1)$$

where e is the electron charge, m_e is the electron mass, m_p is the proton mass, r_e is the electron orbit radius, and r_p is proton orbit radius. One can solve for the equilibrium value of $r_e + r_p = 32m_p \epsilon^2 / 3c^2 m_e^2 = 5.5 \times 10^{-9}$ cm - close to the presently accepted value for r_e , which is 5.28×10^{-9} cm. Then one can evaluate the corresponding total orbital energy from $E_H = -\frac{1}{2} \epsilon^2 / (r_e + r_p) = -14.25$ eV - close to minus the accepted $IP = 13.5984$ eV.

Eq. (1) admits scaling for arbitrary atoms and ions. For an arbitrary atom, $r_e + r_p$ generalizes to $r_e + r_N$, where r_N is the radius of the nuclear orbit, and its evaluation generalizes to $r_e + r_N = 32M\epsilon^2 / 3c^2 m_e^2$, where M is the nuclear mass. Observe that $r_e + r_N$ is a constant times M . So in general, one may expect linear dimensions of atomic systems to scale with M ; *i.e.* be governed mainly by the nucleus for that kind of atom.

Total electron energy for an arbitrary atom or ion is $E = -\frac{1}{2} \times Q_- Z / r_e + r_N$, where Z is the atomic number, or nuclear charge, and Q_- is the negative charge present: for a neutral atom, $Q_- = Z$; for an ion, $Q_- \neq Z$. Thus total electron energy scales as $Q_- Z / M$.

Energy per electron is predicted to scale as Z / M . This prediction conflicts with the classical ‘Rydberg factor’, for which scaling goes as $1 / (1 + m_e / M)$, which has much weaker dependence on M . The $1 / (1 + m_e / M)$ is always close to 1, but the Z / M varies from 1 to about 0.4.

3. Ionization Potentials

The simplest way to test the prediction of Z / M scaling is to look at ionization potentials, IP 's. These too are predicted to scale as Z / M . This prediction is important for organizing IP data. To put all atoms on a common basis for comparison, we scale all empirical data on IP 's by M / Z . That reveals very simple patterns. They are displayed on Fig. 2, which is developed slightly beyond what they were two years ago.

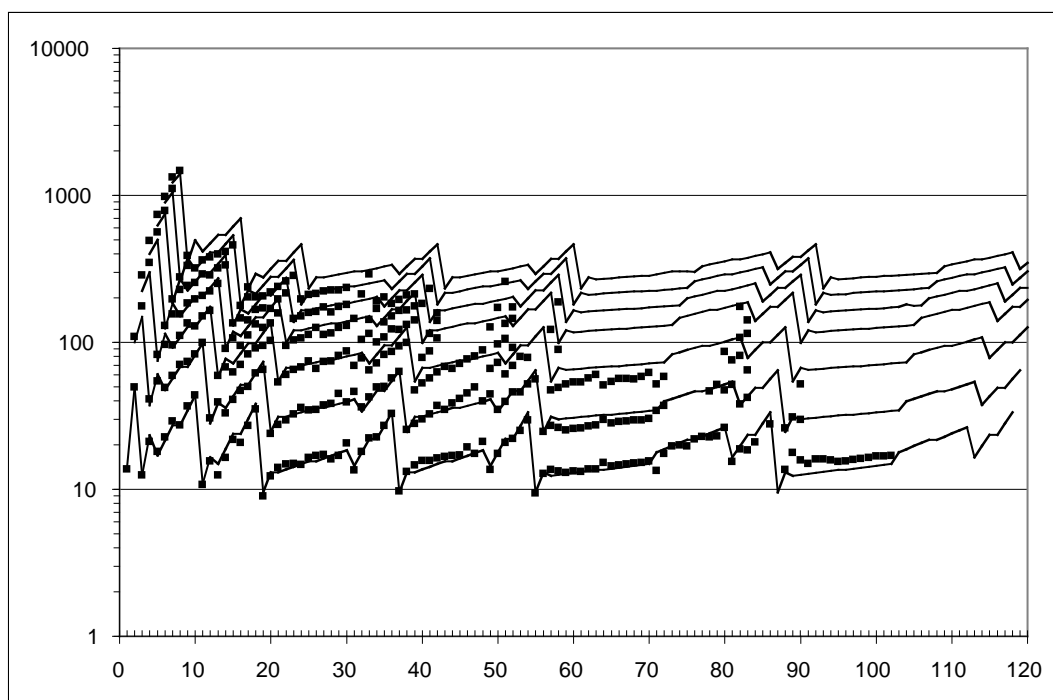


Figure 3. M / Z -scaled IP 's for $IO=1$ to 7 .

Observe that there certainly is a recurring similarity to Hydrogen: the M / Z -scaled first IP 's for elements around the mid points of periods on the PA are numerically close to that of Hydrogen. There is also a lot of structure. There are definite rises over periods and they follow a definite numerical pattern. Figure 4 lays out this pattern. The ratios actually used in constructing Fig. 4 are in bold font; the other, equally valid but redundant, ratios are in small print.

There is also a next level of detail within the periods. The representation of this detail has been refined over the last two years. There appear to be straight-line rises associated with the traditional angular momentum quantum number l . The intercepts are at the end of the period if the l run is at the end, and otherwise appear to be in the middle of the run. The rises appear to be a function of l and the parameter $N = 1, 2, 2, 3, 3, 4, 4$ that belongs to the period, The following function was developed:

$$\text{incremental rise} = \text{total rise} \times \text{fraction} \quad \text{where} \quad \text{fraction} = \frac{(2l+1)(N-l)}{N^2} \quad (2)$$

Since conventional QM does not even *use* the parameter N , it cannot come up with a function like this. The function is uniquely available from the present analysis approach. Figure 5 shows the evaluation of the fraction involved in Eq. (2) for all the circumstances that arise in the known PA.

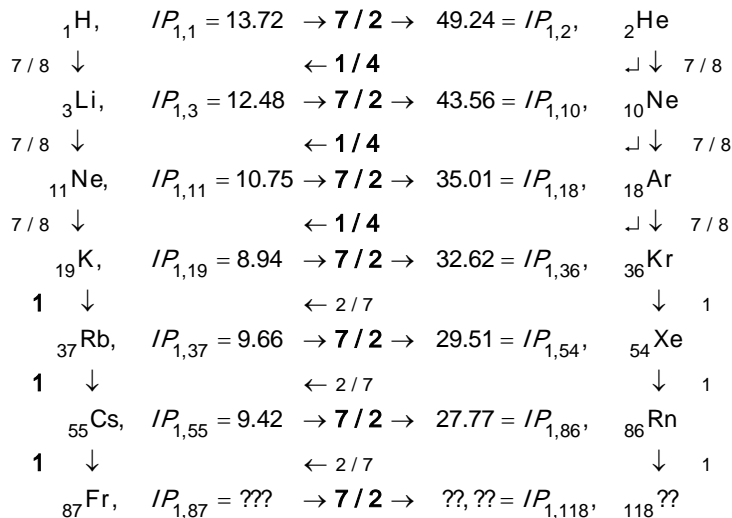


Figure 4. Map of main highways through the periods for $IO = 1$.

N	l	fraction	l	fraction	l	fraction	l	fraction
1	0	1						
2	0	1/2	1	3/4				
2	0	1/3	1	3/4				
3	0	1/4	2	5/18	1	2/3		
3	0	1/4	2	5/18	1	2/3		
4	0	1/4	3	7/48	2	5/16	1	9/16
4	0	1/4	3	7/48	2	5/16	1	9/16

Figure 5. Map of local roads through the periods.

The pattern described by Eq. (2) and evaluated on Fig. 3 is fairly reliable for first IP 's, but gradually becomes less reliable for higher-order IP 's. This behavior may occur because the higher-order IP 's involve more 'single-electron states', the filling order of which is not perfectly understood. So the more of them involved, the less certain one can be about them. There is a nominal state filling order, but it is violated about 20% of the time. So this work has included an effort to better describe SES filling order.

4. Single Electron State Filling Order

The nominal filling order according to Madelung and Hund rules uses the traditional radial quantum number $n = 1, 2, 3, 4, 5, 6, 7$ and angular momentum quantum number $l = 0$ to $n - 1$, along with the traditional spin quantum number $s = \pm 1/2$. Madelung's rule says: advance with the linear combination $n + l$, and within a given value of $n + l$, advance with n . Hund's rule says: for a given n and l , first fill all occurrences of one s value, then all occurrences of the other s value.

For historical reasons, l values are often represented, not by numbers 0, 1, 2, 3, but by letters s, p, d, f . That is slightly unfortunate, since it means the symbol s has two meanings; namely ' $l = 0$ ' and 'spin'. With that caution, one may lay out the nominal filling order as shown in Fig. 6.

The triangular layout of the two sets of information corresponds to the rows of the PT. The triangles look so economically narrow because every l value actually involves $2(l + 1)$ repetitions, and hence embraces $2(l + 1)$ elements.

The first triangle of information shows the standard SES naming scheme, and the second triangle of information shows the numerical value of the $n + l$ parameter that the Madelung rule applies.

n, l designation	$n + l$ parameter
1s	1
↓	↓
2s → 2p	2 → 3
↙	↙
3s → 3p	3 → 4
↙	↙
4s → 3d → 4p	4 → 5 → 5
↙	↙
5s → 4d → 5p	5 → 6 → 6
↙	↙
6s → 4f → 5d → 6p	6 → 7 → 7 → 7
↙	↙
7s → 5f → 6d → 7p	7 → 8 → 8 → 8

Figure 6. Nominal SES filling order according to the traditional nomenclature.

Obviously, the Madelung $n + l$ parameter doesn't really resolve the SES ordering very much, since the same numerical values recur multiple times. Perhaps worse than that, there exist many anomalies in the SES filling order, and the Madelung $n + l$ parameter offers no warning or explanation about that. The first departure from the almost-reliable rules comes with ${}_{24}\text{Cr}$. That element is expected to have electron configuration $3d^4 4s^2$, but it actually has $3d^5 4s^1$; that is, a $3d$ electron state ($n + l = 5$) is taken onboard instead of a $4s$ electron state ($n + l = 4$). That is, a $3d$ SES is filled instead of a $4s$ one. Indeed, the same thing happens with ${}_{29}\text{Cu}$, and the similar thing ($4d$ in place of $5s$), happens with ${}_{41}\text{Nb}$, ${}_{42}\text{Mo}$, ${}_{44}\text{Ru}$, ${}_{45}\text{Rh}$, ${}_{46}\text{Pd}$, and ${}_{49}\text{Ag}$, and the next similar thing ($5d$ in place of $6s$), happens with ${}_{78}\text{Pt}$ and ${}_{79}\text{Au}$. And that is only the beginning. In the Lanthanide series: there are instances of $5d$ SES's replacing $4f$ ones: in ${}_{57}\text{La}$, ${}_{58}\text{Ce}$, and ${}_{64}\text{Gd}$. And in the Actinide series, there are instances of $6d$ SES's replacing $5f$ ones: in ${}_{89}\text{Ac}$, ${}_{90}\text{Th}$, ${}_{91}\text{Pa}$, ${}_{92}\text{U}$, ${}_{93}\text{Np}$, and ${}_{96}\text{Cm}$. I see 19 anomalies so far, and I suspect that we do not completely know about elements much beyond ${}_{96}\text{Cm}$, so I estimate a 20% anomaly rate.

How can all those mysteries be understood? One possibility is that some different linear combination of quantum numbers can be more informative than $n + l$ is. This possibility has been investigated in detail, and the end result is as follows. Instead of looking at $n + l$, one can more profitably look at a new parameter

$$R = \frac{1}{2}[4n + 2(l + s) - 1] \quad (3)$$

This parameter still involves n and l , but with n weighted twice as heavily. It also involves s , which gives it the power to replace Hund's rule as well as Madelung's rule.

Figure 7 illustrates the performance of the R parameter. The Figure shows evolution of the R parameter, juxtaposed with the corresponding display of the nominal SES's. Clearly, the R parameter produces more resolution than $n+l$ can produce, since it varies from 1 to 15 instead of just 1 to 8 (as in Fig. 6). Also, it is not monotonic in its progress. The R values that have regressed from, or just returned to, the maximum so far in that period are set in boldface. They are all at what would nominally be d or f SES's. The SES's where the anomalies actually occur are also indicated by bold font. They too are all located at what would nominally be d or f SES's. There is a striking correlation between the bold R 's and the bold SES's. Evidently, the regressions in R allow, but don't always mandate, an anomalous SES filling.

$R = \frac{1}{2}[4n + 2(l + s) - 1]$	SES's
1 → 2	1s(-) → 1s(+)
┘	┘
3 → 4 → 4 → 5	2s(-) → 2s(+) → 2p(-) → 2p(+)
┘	┘
5 → 6 → 6 → 7	3s(-) → 3s(+) → 3p(-) → 3p(+)
┘	┘
7 → 8 → 7 → 8 → 8 → 9	4s(-) → 4s(+) → 3d(-) → 3d(+) → 4p(-) → 4p(+)
┘	┘
9 → 10 → 9 → 10 → 10 → 11	5s(-) → 5s(+) → 4d(-) → 4d(+) → 5p(-) → 5p(+)
┘	┘
11 → 12 → 10 → 11 → 11 → 12 → 12 → 13	6s(-) → 6s(+) → 4f(-) → 4f(+) → 5d(-) → 5d(+) → 6p(-) → 6p(+)
┘	┘
13 → 14 → 12 → 13 → 13 → 14 → 14 → 15	7s(-) → 7s(+) → 5f(-) → 5f(+) → 6d(-) → 6d(+) → 7p(-) → 7p(+)

Figure 7. Performance of the more sensitive SES filling parameter $R = \frac{1}{2}[4n + 2(l + s) - 1]$.

5. Concluding Comments

About IP 's: This fascinating problem now seems to show far more order than it did in the established chemical literature. My ambition now is to find more data, or to stimulate the taking of more data. Please help me if you can.

About SES filling: I do not know what the R parameter means physically. No one knows what the Madelung $n+l$ parameter means physically, or what the Hund rule means physically. People think they know what the traditional quantum numbers n , l , and s mean, but I am not at all convinced of that. Remember, in past PIRT papers I have questioned even the most basic quantum number, the 'radial' quantum number n used for distinguishing radial 'states' of a single H atom. I argued that n is not at all what it is claimed to be, but rather it is the number of H atoms involved in a complex system.

Our present situation in QM is as if the physical facts of QM were written in Russian, the QM explanation written in German, and our understanding of QM spoken in English. Too many translations garble the message; to understand better, we should read directly in Russian. Our 'English' words are interpretations, like 'radial' or 'angular momentum' or 'spin' for the quantum numbers involved in SES's. The 'German' words are the actual quantum numbers n, l, m, s , whose corresponding mathematical functions constitute a complete, ortho-normal basis set, which is sufficient to 'say' anything in German. The Russian words have to be the parameters of some other equally good ortho-normal set of functions. Certainly the N parameter is one of them, and the atomic number Z is another. And there must be at least one other. But what? I believe it is R . That is why I gave R its name: in remembrance of 'Russian'.

About applying Two-Step Light theory: There is much work yet to be done in atomic physics and micro-physics generally. The founding Hydrogen problem has recently been revisited to remove all small-angle approximations. In Eq. (1), on the left side, the radiative loss side there should be some cosines of angles for vector projections, and

on the right side, the torquing gain side, there should be some sines of angles for vector cross products. With the small-angle approximations, the cosines are set to unity and the sines are set to the angles themselves.

Removing those approximations and inserting the actual trigonometric functions makes the Hydrogen problem more complicated, so that algebraic solution is no longer possible. But graphical solution is perfectly feasible. The result of that effort is displayed as Fig. 8. The horizontal axis is system radius, on a log scale, going from 10^{-15} cm to 10^{-5} cm. The vertical axis is also on a log scale. Data Series 1 is the upper curve and represents rate of energy loss due to radiation, and Series 2 is the lower curve and represents the rate of energy gain due to torquing.

The original ground-state solution is where the curves cross on the right side of the plot. The removal of small-angle approximations possibly allows a solution slightly closer to the accepted value of $r_e = 5.28 \times 10^{-9}$ cm. The graphical solution is somewhere between $r_e + r_p = 5.41 \times 10^{-9}$ cm and $r_e + r_p = 5.84 \times 10^{-9}$ cm, as compared to $r_e + r_p = 5.5 \times 10^{-9}$ by algebraic solution with small-angle approximations.

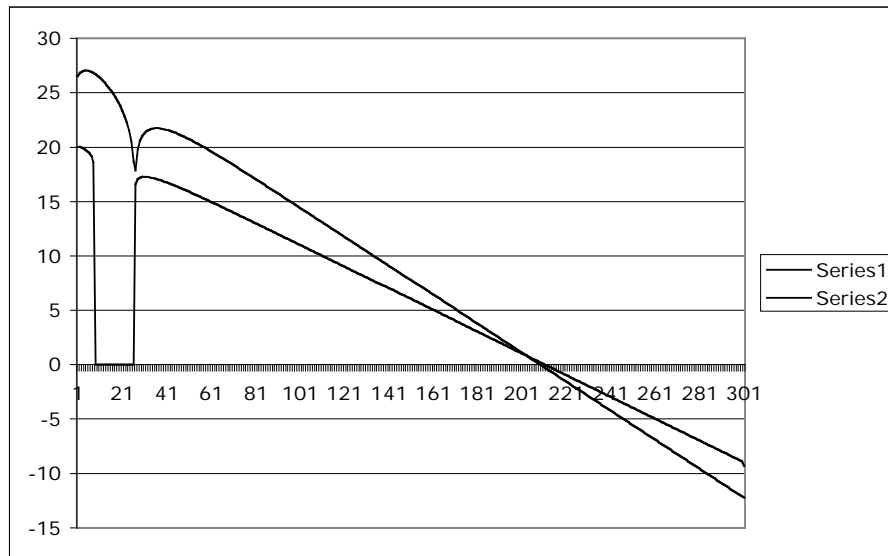


Figure 8. Result of Hydrogen stability analysis with small-angle approximations removed.

The really interesting development is to the left side of Fig. 8. Clearly a family of sub-ground solutions is emerging. In this region, the system radius is extremely small, and the electron orbit velocity is extremely superluminal. The first sub-ground solution is associated with $V_e = 2\pi c$. At that speed, the angles in the problem are such that both the radiative loss rate and the torquing gain rate actually rate drop to zero, but the radiative loss rate drops more quickly, and so crosses the torquing gain rate, making a new solution. (The crossing does not show explicitly, because it comes between data points.)

Beyond this new solution, at even smaller system radius and higher electron speed, the angles are such that the torquing turns negative, and so cannot be displayed on a log plot. Not until $V_e = 4\pi c$ does it turn positive again. And when it does, that creates the opportunity for another solution, just out of view on the left. That solution would be associated with $V_e = 6\pi c$.

The development of the new solution displayed, and the others further left and off the page, is a major subject for future investigation. Their energies can be characterized as a constant times the square of an odd integer. That behavior is like having a 'fractional' radial quantum number. There exists an extensive experimental literature about Hydrogen sub states that are characterized in that same way, reported by Randall L. Mills and associates. Dr. Mills

tells the story quite comprehensively in “The Nature of the Chemical Bond Revisited and an Alternative Maxwellian Approach”, *Physics Essays* **17** (3) 342-389 (2004).

Acknowledgment

This PIRT paper, like those that preceded it, was developed from an earlier and more preliminary presentation given before conferences of the Natural Philosophy Alliance in the United States. These conferences occur every year, at locations throughout the US, often associated with regional conferences of the AAAS. Proceedings volumes are produced for future reference.