MOVING CLOCKS, TWIN-CLOCK PROBLEM AND THE UNIVERSE: ANALYSIS, NEW INSIGHTS AND THE FINAL SOLUTION

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Introductory remarks

The theory of motion is also about the theory of space and time, and usually the 'background space-time' is defined and specified before one discusses kinematics and dynamics. In Newton's theory of motion, both absolute space and relative space were used, and time was considered universal and absolute. This notion changed with the study of Lorentz's theory of relativity when it was realized that both spatial and temporal intervals are affected by motion. However, it is in Einstein's special theory of relativity one finds full expression to the modification of spatial and temporal intervals as an integral part of the general theory of motion. Interestingly, it is with Einstein's theory the issue became debatable and relatively ambiguous at the level of ontology[1]. We examine the relevant points first before discussing the problem of comparison of moving clocks.

The Lorentz-Poincare theory was built on the notion of motion relative to a luminifeorus ether that provided an absolute reference frame for electromagnetic phenomena, including the propagation of light. Therefore, the physical effects like time dilation and length contraction were real physical effects happening to bodies that move through the ether. In particular, the time dilation in the theory happens to those clocks that move with respect to the ether, in proportion to the square of their 'absolute' speed with respect to the preferred rest frame of the ether.

When one comes to Einstein's theory, coordinates and their transformations take precedence and the Lorentz transformations used in the theory are based on relative velocities between frames of reference moving in empty space, and not based on a velocity relative to a preferred frame. Since ether is denied in special relativity (SR), there is no preferred frame, and all frames of references in inertial motion are equivalent. Since the theory deals with motion in empty space, space and its metric remain invariant under motional transformations.

This brings up an immediate ontological problem. If there are two frames that are moving relative to each other, each may claim a state of rest, and then the Lorentz transformation applies to clocks and measuring scales in the other frame. Therefore, the transformations are symmetrical. (Strictly speaking, the transformations are symmetrical even when noninertial motion is involved, since what matters is the instantaneous relative speeds, and at each instant one can replace the two frames by two locally inertial frames.) Naturally, the question arises whether the relativistic modifications of spatial and temporal intervals are physically real. Usually, the answer given is that the length contractions are symmetrical, and each observer may claim that the measuring scales in the other frames are shortened. Since the phenomenon happens only while the frames are in motion, a comparison after coming to relative rest will not reveal any residual effect. But this answer cannot be maintained when time dilation is considered because time is an accumulated quantity, like phase, and if the rate of a clock is affected during motion, then one expects a permanent imprint of that modification in the integrated recorded time. So, it makes empirical sense to ask whether the two clocks in relative motion will show equal times or different times at the end of the experiment if they are brought to the same spatial point and compared. If all effects depend only on relative speeds, then of course the time dilations are symmetrical, and the two clocks should show identical readings. But this does not fit in consistently with the calculations from either of the frames, because each observer will conclude that he was at rest and other clock was moving throughout the experiment, and hence should show less time (younger). The twin-clock problem in relativity theory has been a topic of discussion since the early days of Einstein's special theory of relativity. The point of contention was that a theory that predicted physical effects which depended only on relative velocity would have time dilations of clocks that are symmetrical in the relative velocity, and hence each inertial observer equipped with a clock would conclude that it was always the “other” clock that went slower. But each clock cannot be showing lesser reading than the other. Therefore, it is clear from the outset that some physical element that is beyond the physical description of motion in SR is involved in the analysis of the problem of time dilation of moving clocks. The reality of time dilation is linked to some issue that breaks the symmetry of relative motion, and it goes beyond the special relativistic description of relative inertial motion. While most text books and writings on SR denies this conclusion, by asserting that the issue can be resolved fully within SR, Einstein’s own conclusion was exactly the opposite, and he categorically stated that a theory beyond SR was required to satisfactorily handle the problem of comparison of clocks. It will become clear that the crux of the problem, and the solution, is to identify the exact physical reason for the asymmetry of time dilation, and it will turn out that all the conventional reasons based on the noninertial nature of the motion of one of the two clocks are entirely misplaced and irrelevant.

Basic results on time dilation from Special Relativity

The Lorentz time dilation formula applied to intervals of time contains the basic mathematical description of time dilation in special relativity. Time intervals in two frames of references are related by

$$dt' = dt\sqrt{1 - v^2/c^2} \tag{1}$$

In this formula, the interval dt' is read by a clock that is in a frame that is moving at speed v relative to a frame in which the clock reads the interval dt. Therefore, the clock in the moving frame records less time compared to the ‘proper’ time in the rest frame. Such time dilation effects have been unambiguously established in several clock comparison experiments. But, the transformation is completely symmetrical and one could write

$$dt = dt'\sqrt{1 - v^2/c^2} \tag{2}$$

as a description of clocks relative to the other frame, in which the ‘primed’ clock is at rest. This is the origin of the twin paradox. Which clock really goes slower? Of course this cannot be answered empirically unless the two clocks in relative motion are brought together to the same spatial point for a comparison, as Einstein stated the physical effect in his 1905 paper[2]; "If one of two synchronous clocks at A is moved in a closed curve with constant velocity until it returns to A, the journey lasting t seconds, then by the clock that has remained at rest the travelled clock on its arrival at A will be (1/2)tv^2/c^2 second slow". Note that Einstein does not insist that the clock is in inertial motion even though the theory was constructed to deal with only clocks in inertial motion. This stand is physically reasonable since the closed trajectory can be built from a set of ‘piecewise inertial’ trajectories and the formula suggests that only instantaneous speeds matter for the calculation and not whether the trajectory is inertial or not. Indeed, modern experiments have confirmed that the relativistic modification of he life time of unstable particles are identical for motion along straight inertial trajectories and for motion along curved trajectories with high acceleration, provided the speed are identical. Therefore, acceleration is irrelevant for estimating time dilations. We will come back to this point and establish it with high logical rigour, and also with sufficient empirical evidence. But before we proceed it is instructive to examine some experimental results.

Some relevant experimental results

The modification of the life time of unstable elementary particles, especially the muon, is quoted extensively as a precision test of the relativistic time dilation predicted by SR. The half-life of muon at ‘rest’ is 2.2 μs. Muons in an accelerator storage ring are observed[3] to have a longer half-life of 64 μs while moving at a speed of 0.9994 c. This agrees with the formula (1), relative to the clocks in the laboratory. The only point we specifically note here is that this measurement is consistent with the prediction from the Lorentz ether theory as well, since in this case of ultra-relativistic speed, there is no measurable difference between the effect based on the absolute speed with respect to a preferred frame and the effect based on the relative speed with respect to the laboratory frame. In fact, it is a fact of the history of the tests of relativity theories that not a single test of time dilation to date rule out a theory of relativity with a preferred frame, and in particular, the ether based theory.

Direct comparison of clocks transported along the lines suggested by Einstein in the 1905 paper became feasible with the development of atomic clocks in the fifties and sixties. Cesium clocks were used in many tests of relativity, though some of the pioneers, like L. Essen, became skeptical and critical later on[4]. One of the first clock comparison experiments in which atomic clocks were taken around in close round trip trajectories was performed by Hafele and Keating[5] of the Naval observatory in 1970-71, and the results support a theory of relativity with a preferred absolute frame, like the Lorentz-Poincare theory, and invalidates the SR prediction based on relative speeds between frames of references. In fact, if this experiment were performed around 1910, the results would have been immediately interpreted as going against Einstein's theory, and as empirical support for Lorentz's theory! Let us examine the results from this experiment.

The Hafele-Keating experiment compared three sets of atomic clocks. One set was stationary in the Earth laboratory, one set was flown westwards, and another set eastwards in commercial jet aircraft in a round trip. The total duration of these round trips were around 40-50 hours. The average ground speed of the airplanes was around 220 m/s. The round trips (Washington D.C.→Washington D.C.) covered landing points in the northern hemisphere with latitudes ranging from 42^o N (Rome) to 13^o N (Guam), including Bombay (19^o N) with an average latitude of about 30^o N. (This information is not directly available from the published papers, but it is contained in the information available directly from the US Naval observatory). Note that all clocks are in noninertial motion, and the clocks in the flights are indeed more noninertial than the clocks on the surface of the earth. But special relativistic Lorentz transformations are supposed to be applicable for noninertial motion as well once the effects of gravity are subtracted out – otherwise one has to concede that there is no real test of special relativity (muons in storage rings are very non-inertial, and so are elementary particles created and annihilated in particle interactions. Also, Dirac equation for electrons in atoms assumes that special relativity is applicable, even though the electron dynamics is thoroughly noninertial).

The figure below sketches the scheme of the experiment. T0 is the reference clock that is stationary in the laboratory and T1 and T2 and compared to T0 after a round trip.

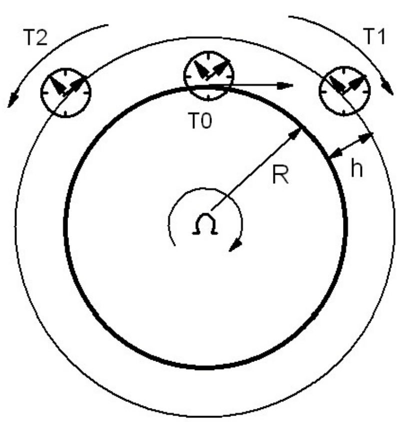

Figure 1: Round trip comparison of clocks T1 and T2 relative to a clock T0 that is stationary in the laboratory. The laboratory itself of moving through the cosmic frame at an instantaneous speed V, of which all contributions except the one due to the rotation of the earth are common to all the clocks. Thus, one expects the tangential speed of earth surface at the appropriate latitude to contribute to the relative time dilations in a preferred frame theory.

Since T1 and T2 are at a distance $R+h$ from the centre of the earth, compared to the distance R for the clock T0, the gravitational time dilation factor, gh/c^2, where g is the local gravity, causes the clocks T1 and T2 to go faster than T0. This factor has to be corrected for to get purely velocity dependent time dilation. The gravitational contribution has been calculated by the experimenters accurately from flight data containing altitudes to be +144±14 ns for the eastward clock and +179±18 ns for the westward clock, the 20% difference coming from the 20%

difference in flight durations. As expected from gravitational time dilation, these clocks in flights age faster compared to a ground clock, and this is indicated with the positive sign. Once this contribution is subtracted, all the clocks can be compared for purely velocity dependent effects. The observed data from the experiment and predictions from relativity theories with a preferred frame (RT-PF) and from special relativity (SR) are summarized in the table. All time comparisons are in nanoseconds, relative to the clock stationary in the earth laboratory, exactly as it was done in the experiment (it is important to note that in the experimental procedure, the clocks are initially synchronized and then finally compared after the round trip with no intermediate clocks or light signals used for comparisons or time transfers).

	Eastward clock T1 Flight time ~41 hrs.	Westward clock T2 Flight time ~49 hrs.
Total observed time delay T_O (relative to T0) ns	-59 ± 10	$+273 \pm 7$
Gravitation time delay T_G (relative to T0, from general relativity) ns	$+144 \pm 14$	$+179 \pm 18$
Velocity dependent time delay $T_O - T_G$, observed, ns	-203 ± 17	$+94 \pm 19$
Prediction from RT-PF	-210 ± 10	$+95 \pm 10$
Prediction from SR	-41 ± 10	-49 ± 10

The prediction from SR is contained in the Einstein formula, $\delta T = -Tv^2 / 2c^2$ where *T* is the duration of the experiment (the exact value is not important since $T \gg \delta T$). Clearly the results from the experiment do not agree with the prediction from SR. What is almost shocking is the fact that one of the transported clock has run faster than the clock that remained stationary in the reference frame.

The simplest explanation for the observed results is obtained by noting that the observed time dilations agree perfectly with the predictions from a theory that uses absolute velocity through a preferred frame, instead of the calculation using relative velocities. In fact, the experimenters did a calculation of this sort to explain the experimental results, by assuming hypothetical clocks distributed in space such that they do not move with the reference clock as the earth rotates. If the calculation done by assuming instantaneous Lorentz frames on the other hand, at each instant the laboratory observer is to be considered at rest, and the clocks in the airplanes are moving at some relative velocity. Adding up all such infinitesimal time dilations will end up giving a net time dilation of clocks T1 and T2 relative to T0. SR can never give the result that the transported clock registered more time than the reference clock in the rest frame.

The prediction from a theory in which there is a preferred frame defined by the matter distribution and radiation in the universe that is isotropic and homogeneous on the large scales agrees very well with the observed results. The orbital motion and the motion of the solar system are common to all the clocks, and the difference in absolute speeds come only from the rotational velocity of the earth. This is about 460 m/s at the equator, and reduces with increasing latitude. At the latitude λ , the tangential speed is $v = R\Omega \cos\lambda$. So, the clock T0 will have an absolute time dilation of its proper time that depends on this constant speed relative to the preferred frame. The other two clocks in the airplanes change their speeds and latitudes, but generally their ground speed is maintained at around 220 m/s on the average, at the average latitude of about 30 degrees. The actual latitudes of the three clocks are different and should be properly accounted for in a precision calculation. The reference clock at Washington D.C. is at rest in the laboratory at about 39 degrees latitude. The preferred frame speed of the clocks T0, T1, and T2 are then approximately 360 m/s, 621 m/s, and 181 m/s respectively. Using these absolute velocities, the time dilations of the various clocks can be calculated and then compared using the formula $\delta T = -TV^2 / 2c^2$ where V is the speed relative to the preferred frame. The results are shown in the table below. Negative sign indicates slower rate of the clock.

	Eastward clock T1	Westward clock T2	Reference clock T0
Velocity relative to the preferred frame	621 m/s	181 m/s	360 m/s
Time dilation factor per second	2.14×10^{-3} ns	1.82×10^{-4} ns	7.2×10^{-4} ns
Approximate time of flight (at average speed of 220 m/s)	1.476×10^{5} s	1.76×10^{5} s	
Predicted time delay relative to T0 (with preferred frame)	-210 ns	$+95$ ns	
Observed velocity dependent time delay (relative to T0)	-203 ± 17 ns	$+94\pm19$ ns	

The data clearly favour time dilation factors based on the absolute speed with respect to the preferred frame with overwhelming statistical confidence. The westward clock indeed registered shorter proper time than the reference clock that was stationary on earth's surface, and ran faster (aged more, rather than less, contrary to the expectation in SR).

It is important to note that the slightly noninertial nature of the reference clock is completely irrelevant for the analysis and for the results of the experiments. If the experiment is done in a configuration shown in figure 2, where the reference clock is always in inertial motion, the results will be identical. We have already argued, and also cited the experimental result from muon storage ring, that accelerations of the clocks T1 and T2 are also irrelevant.

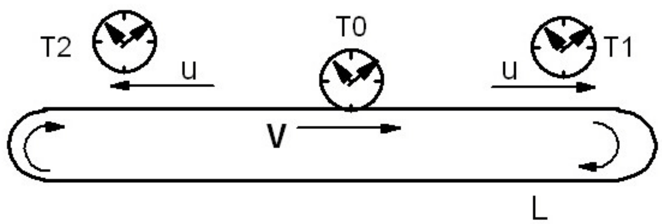

Figure 2: Round trip clock comparison experiment in which the reference clock T0 is inertial. The final results are identical to the clock comparison experiment depicted in figure 1, when the parameters of the experiments are identical. The total path length L in the 'race-track' equals the circumference of the circular track in figure 1.

There are other more recent experimental results that support this fact[6]. Also, it is well known in the context GPS time keeping that the time dilation factors based on the preferred frame speeds are put into the algorithms that calculate the time delays. Indeed, all our usage of time dilation factors are always based on the speeds with respect to the preferred frame, and not based on relative speeds between different frames. In the context of the GPS, the algorithms can ignore the common speed of all the satellites because it contributes equal time dilation to all clocks, and then the remaining contribution to the satellite's preferred frame speed is only from the tangential surface speed of the earth to which their speeds are referenced. An algorithm based on instantaneous relative speeds between GPS satellites will fail, because physical of clocks based on SR is incorrect. Though the GPS algorithm uses the prescription correctly, because it is empirically known, the physical reason and the fact that GPS data continuously validate cosmic relativity with the isotropic cosmic frame as the preferred frame have been completely missed.

Clocks and Acceleration

It is now time to analyze rigorously the role of acceleration in determining time dilation. Since the time dilation factors in SR depend only on relative speeds, it is clear that the theory by itself does not have any acceleration dependent effect on time registered by clocks. If the velocity happens to be time dependent, the infinitesimal Lorentz factors can be integrated over the path, and the correct time dilation is obtained, for example in the case of elementary particles in storage rings etc. In fact, Einstein's original prediction of time dilation itself is for a noninertial trajectory of the clock[7]. The issue becomes relevant in the context of the twin paradox. In the usual description of the twin paradox, a calculation done in the frame of the non-accelerated clock that is preferentially considered to be at rest in the laboratory frame gives the answer that the transported clock ages slower, and this result is independent of the accelerations in the trajectory of the transported clock, and only velocity needs to be taken into account for the calculation. But, the calculation done in the frame of the other clock, using special relativity, gives the symmetrical answer that the laboratory clock ages slower. Since this does not agree with experiments, and since the symmetrical calculation gives the paradoxical answer, usually it is claimed that somehow it is accelerated clock that finally ages slower. While it is true that the acceleration breaks the symmetry between the motion of the clocks in the ideal standard case, the reasoning given for how this asymmetry picks the second clock preferentially to age slower is not unique, signaling the general confusion on this issue. Some texts attributes the asymmetrical aging to different Doppler factors in the up and down trips, considering light pulses being send from one frame to the other and so on, though the clock comparison experiments typically do not keep sending pulses to each other – there is no need to do that. Once the clocks are synchronized, they can be compared at the end of the experiments. This 'resolution' then do not depend on the acceleration itself to obtain the time dilation, but projects the total time dilation as a residual of asymmetric Doppler shifts of light propagating between the two frames, despite an explicit warning in some writings, like that of Eddington in his book[8], against confusing Doppler shift with relativistic time dilation. The second category of resolutions claims that there is a jump of the relative times of the clocks at the point of acceleration because the line of simultaneity has changed while one of the clocks reversed the trajectory. Here also, the acceleration is not used as a physical agency to induce time dilation. Convincing arguments as to why these resolutions are not correct may be found in reference[9] (9). Also, Einstein himself did not think that these were sufficient to resolve the twin-paradox. The third category mentions that acceleration is important as a physical influence for determining the time dilation, but rarely completes the argument or calculates the expressions.

Interestingly, Einstein's own resolution of the twin paradox contradicts all the standard text book resolutions! Einstein's resolution makes use of the acceleration, and its equivalence to a homogenous gravitational field. Einstein asserted that general relativistic physics is essential to resolve the twin paradox in special relativity. This is consistent with his not writing anything that adequately addressed the twin paradox until 1918.

In 1918, almost three years after the completion of the General Theory of Relativity, Einstein published a paper[10] directly addressing the criticisms of the theory of relativity in the journal Die Naturwissenschaften , and it was entitled `Dialogue about objections to the Theory of Relativity'. This paper is a dialogue between a critic and an `adherent' of the physical theory of relativity (relativist), representing Einstein himself. This paper is hardly referred to in writing on the twin paradox. Curiously, the discussion starts with a complaint by the critic that none of the relativists had adequately responded to the criticisms of relativity by many in journals. In fact, the critic accuses relativists of `shirking' the issue. This certainly suggests that Einstein's considered that none of the earlier discussions adequately addressed the problem and that it was necessary to respond. As the physical cause of the asymmetry he uses the pseudo-gravitational field and the gravitational time dilation of general relativity, after admitting that special relativity is not suitable for resolving the issue due to the fact that one of the twins undergoes accelerations during their trip. However, he does explicitly states that the special relativistic effect can be calculated from the point of view of the intermittently accelerated twin for those portions of the journey that was inertial, and he concludes that during these portions of B's journey it is the twin A who ages less.

Einstein discussed the problem from the point of analysis of each of the frames -- the inertial frame K in which the twin A remains at rest and the twin B moves at velocity v, and the frame K' in which the twin B is at rest, but experiences accelerations. From the frame K, the analysis is identical to the one usually found in text books, leading to the $tv^2/2c^2$ time dilation of B, where t is the duration of the trip. Then he analyzes the situation from the frame K', and agrees that the situation is symmetric except at the point of acceleration. Then he splits the total time dilation into the part that comes from special relativistic kinematics and another part due to the gravitational time dilation that should be present according to the general theory of relativity and the equivalence principle. In the paper he explains to the critic that such effects are expected even in a `pseudo-gravitational field', and that one need not distinguish between such a field and a true gravitational field in this context. The special relativistic part is indeed a time dilation of $tv^2/2c^2$ seconds of the twin A relative to B. Then Einstein explains that since, 'according to the general theory of relativity, a clock works the faster the higher the gravitational potential at the place where it is situated', and since there are homogenous gravitational fields equivalent to the acceleration experienced in the frame K', one should add this contribution in the calculation. He asserts, "calculation shows that the consequent advancement amounts to exactly twice as much as the retardation during stages of inertial motion. This completely clears up the paradox...". In passing I note that Einstein admits to the existence of a paradox within special relativity, without resort to the pseudo-gravitational field of general relativity.

Einstein did not include the calculation itself in the paper, presumably because it is a simple calculation that involves only the time dilation formula for two clocks in a uniform gravitational field g, separated by a height h,

$$\delta t = tgh/c^2 \tag{3}$$

where t is the duration over which the comparison is made. The full calculation was reconstructed recently in reference (9). But, unfortunately, Einstein's resolution fails totally because of its dependence on the fact that a real acceleration that can be felt and measured exists in B's frame. The entire experiment can be done without the clocks undergoing any acceleration by adding auxiliary clocks to the experiment. Also, the simple device of freezing or switching of the clock in frame B during the short duration of acceleration eliminates any possibility that acceleration is important in determining time dilation[11]. Usually we do not think of this possibility because the typical example is discussed in terms of live twins!

The correct resolution of the twin paradox is that the clock that moved more, in the sense made precise later, relative to the preferred frame of the universe is the clock that runs slower. The actual physical effect is due to the gravitational potentials of the matter in the universe as we will see in a later section. As a prescription for calculation all one has to do is to calculate the time dilation using the absolute speed through the preferred frame, and the answer is always correct. The entire voluminous writings on the twin clock problem and the paradox can be replaced by this one-sentence resolution, and one is guaranteed to be right. Thus, cosmic relativity offers the unambiguous, correct and rigorous resolution of the twin-clock problem, which is based on a real physical interaction consistent with the presence of the matter around us in the universe, making all future debates on this irrelevant. This is the final solution.

Clocks moving in the matter-filled universe

The correct metric even in flat space, asymptotically far away from gravitating objects like the bodies of the solar system and the earth itself, is the Robertson-Walker metric of the matter filled universe that is homogeneous and isotropic when averaged over large scales. The correct metric in a frame that is moving through the preferred commoving frame can be obtained by transforming the R-W metric, and the transformed metric is not the Minkowski metric[12].

For the privileged comoving observer in this universe with its average matter density close to the critical density, the spatially flat R-W metric is

$$ds^2 = c^2dt^2 - R^2(t)(dx^2 + dy^2 + dz^2) \tag{4}$$

Since the rate of expansion is extremely small, only about $\dot{R}/R \simeq 2\times 10^{-18}\ m/s/m$, (this is just the Hubble expansion parameter written in metric units), we can consider that the expansion is negligible for any experiment done on human time scales. Thus $R(t) \simeq R_0$, nearly a constant. Therefore, the metric becomes

$$ds^2 = c^2dt^2 - R_0^2(dx^2 + dy^2 + dz^2) = c^2dt^2 - (dX^2 + dY^2 + dZ^2) \tag{5}$$

Thus $g_{00} = -1, g_{ii} = 1$ and all other metric components are zero.

For an observer who is not comoving the metric will obviously be different. In a reference frame that moves through the matter distribution of the universe, the metric has to be anisotropic, as obviously shown by the dipole anisotropy of the CMBR, since there is a large current matter in such a frame that defines an anisotropy vector. This means that the new metric will have anisotropy and hence off-diagonal elements. The metric that follows the anisotropy of the space is obtained by making a purely geometric coordinate transformation to such a frame.

Consider motion along the direction *X*. Then $X' = X - Vt$. We get,

$$g'_{00} = -(1 - V^2/c^2),\ g'_{ii} = 1,\ g'_{0X} = g'_{X0} = V/c \tag{6}$$

All other metric components remain zero. It is this changed gravitational metric that is responsible for physical changes in moving clocks and measuring scales – we can easily see how the gravity of the matter current, and the resulting modified gravitational potentials, cause gravitational time dilation.

A clock in such a frame will be gravitationally affected to read the modified proper time,

$$d\tau = \sqrt{-g'_{00}}\,dt' = (1 - V^2/c^2)^{1/2} dt' \tag{7}$$

Here the velocity is relative to the preferred frame in which the metric is isotropic, or equivalently, the velocity is always relative to the CMBR isotropic frame. We see that a clock in a frame that is moving relative to the CMBR is gravitationally slowed down with a Lorentz factor corresponding to the absolute speed. Therefore, if such a clock is compared with another that is in a frame at rest in the CMBR isotropic frame, or with a clock that is moving slower, a time dilation will be measured.

When the speed of the clock is small compared to the speed of light, we get

$$d\tau = (1 - V^2/c^2)^{1/2} dt' \simeq (1 - V^2/2c^2) dt' \tag{8}$$

Therefore, the relative time dilation is

$$\delta T = -V^2/2c^2 \tag{9}$$

The difference between this formula in cosmic relativity and a similar formula in special relativity is that in CR it is the velocity with respect to the preferred frame that is used whereas in SR it is the relative velocity between frames of reference that is used. The physical difference is enormous, as we have already seen in the analysis of transported clocks. It is this formula that we used earlier to estimate the time dilations in the Hafele-Keating experiments and the observed results agreed remarkably well with the predictions of the CR.

Instead of writing the formula in terms of the absolute velocities, one can also write it in the frame that is moving relative to the CMBR. In such a frame the metric, as we derived earlier, has the components $g'_{00} = -(1 - V^2/c^2),\ g'_{ii} = 1,\ g'_{0X} = g'_{X0} = V/c$.

The full expression for the time dilation of clocks T1 and T2 relative to the clock T0 is

$$\delta T_{1,2} = -\frac{Tv^2}{2c^2} \mp \frac{(Tv)R\Omega_E \cos\lambda_{1,2}}{c^2} - \frac{T\Omega_E^2 R^2(\cos^2\lambda_{1,2} - \cos^2\lambda_0)}{2c^2} \tag{10}$$

The last term goes to zero when the average latitudes of the flight clocks and the reference clock are the same. Then the time dilation can be written as

$$\delta T = -\frac{Tv^2}{2c^2} \mp \frac{2\pi R^2\Omega_E \cos^2\lambda}{c^2} = -\frac{Tv^2}{2c^2} + \frac{2\vec{S}_\lambda \cdot \vec{\Omega}_E}{c^2} \tag{11}$$

where we have written the (latitude dependent) area S of the round trip path and the angular velocity of the earth as vectors. This form is special for rotations, and the general expression involves the integral of the form

$$\delta T = -\frac{Tv^2}{2c^2} + \frac{1}{c}\oint A_i dx^i \tag{12}$$

where the vector product of the gravitomagnetic potential and the path length is integrated over the trajectory in the frame moving relative to the preferred cosmic frame. The gravitational vector potential is essentially the off-diagonal metric component. The expression is physically and mathematically analogous to the expression for the quantum phase of a charged particle in the electromagnetic vector potential, for obvious reasons. In this case, the coupling is gravitational, and it is universal. Therefore, the expression is very general, and applies to clocks as well as quantum and classical phases in frame moving through the cosmic frame. This is why the same expression holds for the Sagnac phase, the gravitational Aharonov-Bohm phase and the clock delays in cosmic relativity[13]. They are all due to the gravity of the matter in the universe. In any case, a clock is just a phase measuring instrument, once the frequency of the oscillator is fixed.

Physically, the gravitational time dilation due to the matter in the universe is because the Newtonian gravitational potential energy of the clock with all the matter in the universe is modified due to its motion. The gravitational potential due to the earth, in units of the quantity c^2, is $\phi_{Earth} = -GM/Rc^2 \simeq 10^{-9}$. Naively one might assume that the potential due to more distant matter is comparatively smaller or even negligible. This turns out to be grossly incorrect. The potential due to the Sun is about 10 times larger, and the potential due to the Milky Way galaxy is 1000 times larger. The potential due to the visible Universe can be calculated assuming uniform mass distribution around a point on earth, and then adding up the potentials all the way up to the Hubble radius of about 10^{28} cm. Since the amount of matter in a shell at radius *R* increases as the square of the radius, and since the potential decreases as 1/*R*, it is always the distant shells dominate in the contribution to the cosmic gravitational potential. Since the universe is isotropic and homogeneous over very large scales, we calculate the cosmic potential as

$$\Phi_U \simeq \frac{4\pi G}{c^2}\int_0^{R_H} \frac{\rho R^2}{R} dR = \frac{2\pi G \rho R_H^2}{c^2} \approx 1 \qquad (13)$$

R_H is the Hubble radius, and we have taken the critical density, as indicated by cosmological observations, for the calculation. This turns out to be almost billion times more than the gravitational potential due to the earth. Clearly, it is the distant matter that is more important for any effect that depends on the gravitational potentials.

If the effective gravitational potential at a clock that is at rest relative to the CMBR is written as ϕ_U, the potential at a clock moving at velocity V relative to the CMBR is

$$\phi_U' = \phi_U \Big/ \sqrt{1 - V^2/c^2} \simeq \phi_U (1 + V^2/2c^2) \qquad (14)$$

This is larger than the potential for the stationary clock and one expects that the two clocks will run at different rates purely because of the different gravitational potentials. The difference in rates is given by

$$\frac{\Delta t' - \Delta t}{\Delta t} = \frac{\phi_U' - \phi_U}{c^2} \simeq \frac{\phi_U}{c^2}\frac{V^2}{2c^2} \qquad (15)$$

Clearly this effect depends on the amount of matter in the Universe. In an empty Universe this gravitational time dilation would be zero. The quantity $-\phi_U/c^2 \simeq 1$, and therefore the clock that moves relative to the CMBR at velocity V will age slower than the clock that is stationary relative to the CMBR by an amount $-V^2/2c^2$.

Time dilation of the life-time of unstable particles

It is often stated that the modification of the life-time of unstable particles like the muon is related to the special relativistic kinematical time dilation, since and the Einstein's formula accounts for the observed time dilation very well. However, this is one case where it does not make any fundamental difference whether we take the absolute preferred frame speed or the relative speed with respect to us as observers because the speed of the particle is much larger than the velocity of the our frame with respect to the cosmic frame. What is known from experiments is that the time dilation factor is identical for muons in straight motion and for muons in a storage ring where the acceleration is enormous[14]. Therefore, acceleration is irrelevant. These experiments prove that the accelerating clock has exactly the same time dilation as a non-accelerated clock, for the same speeds, and that nothing special (other than the usual velocity dependent time dilation) happens to the rate of the clock when there is acceleration.

We have showed that due to the matter in the universe, all moving clocks will have a gravitational time dilation given by

$$\tau' = \tau\sqrt{1 - V^2/c^2} \qquad (16)$$

For muons in storage ring this dilation factor is about 30 in some of the early experiments. The velocity is about 0.9994 c, and the agreement with the formula is established to better than 0.1%. Since the galaxies and other matter exist in reality, as can be verified by direct observations, the gravitational time dilation predicted by cosmic relativity is inevitable. Even if one is willing to take the gravitational time dilation due to only visible matter in the universe, it will amount to about 5% of what is observed in storage rings. But the existence of vast amount of matter presently invisible is also established, and it is clear that there is no way to accommodate a discrepancy of even 1% within the experimental accuracy, There is no choice but to accept that there is in fact no kinematical time dilation and that all the observed time dilation in this case is due to the gravitational effect of the visible and invisible matter in the universe. Hence, the muon life-time measurement is to be seen as a precision test of certain cosmological parameters within cosmic relativity, which has other independent empirical support.

Concluding remarks

Moving relative to the preferred frame of the isotropic distribution of matter in the universe generates gravitational time dilation, and it is this gravitational time dilation that is observed in all the experiments performed so far, and not the special relativistic kinematical time dilation. Both cannot be accommodated in the context of precision experiments and since our physics has to be consistent with the fact that galaxies and other matter are visible and they have gravitational fields, one has to conclude that all time dilations are of gravitational origin. It is the speed relative to preferred cosmic frame that is to be used for calculating the time dilation, and not relative velocities between frames of reference, and this completely solves the often debated twin-paradox, and also clarifies the physical reason why a particular clock runs slower than another. It is not the acceleration, not the Doppler shift, and not the changes in the line of simultaneity that is responsible for one clock running slower than the other. The clock that has a higher (absolute) speed with respect to the cosmic frame during the experiment will suffer larger gravitational time dilation and will register less proper time. The correct theory of motion and relativity is obtained by replacing the Lorentz ether with universe and by considering the gravitational effect of the matter in the universe. This is the final solution to the questions on whether motion is relative or absolute, and this theory, called Cosmic Relativity, should replace the special theory relativity. A rigorous calculation of the effects on clock in motion has been presented within this theory starting from the Robertson-Walker metric of the universe, and the physical interpretation based on modified gravitational potentials in moving frames has been discussed. All problems concerning the rates and comparison of clocks are unambiguously resolved with this new paradigm. Cosmic Relativity and its solution to the problem of time registered by clocks in motion achieve a fundamental unification in that it replaces the presently dual view that time dilation can be caused either by relative motion, in a kinematical context, and also by an interaction, in the context of gravitation, by the unified view that all time dilations are gravitational. What has been thought to be pseudo-gravitational fields and pseudo-forces in the context of accelerations are actually real cosmic gravitational fields, and they are the only agents responsible for all observed time dilations of clocks in motion with respect to the preferred isotropic cosmic frame.

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[11] See reference 9 for a detailed discussion. Possibility of freezing the clock readings provides powerful counter arguments to standard resolutions of the twin paradox, and positively reiterates the solution based on cosmic gravitational effects. What is to be remembered is that the experiments compare clocks and their readings, and not ‘time’ in an abstract sense.

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[13] Reference 1, 12.

[14] Reference 3.