How does causality protection constrain the form of physical law?

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Abstract. It's well known that a naïve approach to quantising gravity leads to causality violation and or infinite energies. Both of which would allow one to surpass the limit of what is Turing-computable. I suggest that we take this seriously and assume that quantum mechanics and general relativity are different facets of an underlying non-computable reality. Taking this view allows us to see an interesting duality between quantum mechanics and general relativity. I suggest that the forms of each are contrained by the specific rules we must enforce to prevent the observation of non-computablity and hence causality violation. Analysing the constraints implicit in these causality protection rules provides a possible explanation for the observed three dimensions of space and quantum non-locality and presents a possible rationale for the known symmetries of the Standard Model. It also suggests a new approach to the issue of state vector reduction. However it does imply that there can be no Unified Field Theory. I identify a number of key open problems that will need to be resolved before this approach can succeed.

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1. INTRODUCTION

This paper contains no new results. The purpose is to look at old results from a slightly different perspective and so gain insight into some hard problems. From this we identify areas requiring deeper investigation which can lead to new results. Rather than end up with a single or multiplicity of answered questions we end up with a multiplicity of new questions requiring answers. The hope is that if those questions can be answered then we will substantially advance our understanding of the operation of Nature.

It is well known that early attempts to unite quantum mechanics and general relativity were beset with problems involving infinite energies and causality violation. This led to many programs which held out the promise of finding a finite theory of quantum gravity, none of which have produced a generally accepted final theory of everything. The slightly different perspective I choose to adopt is to assume that in uniting quantum mechanics and general relativity infinite energy and causality violation are the right answers. This does mean that there can be no finite theory of quantum gravity and hence no unified field theory, but surprisingly it does allow us to take steps toward understanding some of the more tricky problems in present theories. The first part of the paper collects together known results which at first sight do not appear to have a connection. The second part draws lines of connection between these known results and hence identifies key questions that need to be answered. I take the view that we can learn the most by examining the most paradoxical aspects of our current theories.

First of all the notion that causality violation and infinite energy might be the correct result in quantum gravity is not so unusual. Both of these things have a common ground in that they would allow the computation of what would otherwise be non-computable. Penrose has already suggested that a theory of quantum gravity must include an element of non-computability [20, 21] and Hawking [15] has gone as far as suggesting that there may be an element of self-referentiality in the operation of physical law, and in consequence no theory of everything is possible. Such self-referentiality in physical law would put it beyond the limit of what it calculable by finite means, and is the same as saying that there is an element of non-computability in the functioning of Nature.

But we never experience causality violation or infinite energy, but then also we never experience both the classical, i.e the general relativistic, view of the world at the same time as the quantum mechanical view of the world. Our experience of these views of the world is separated by the mysterious phenomenon of state vector reduction and so we have two highly successful finite theories which seemingly cannot be reconciled. The key idea I propose is that each of these theories derives from complementary facets of a system which protects us from observing non-computable action. We can identify two types of "causality protection systems" and that these causality protection systems place severe constraints on the form of physical law which emerges. The constraints are so severe that the the physical laws which we experience are the only ones possible. The causality protection systems are distinct yet complementary and so in consequence we have distinct yet complementary theories of Nature’s functioning.

2. GATHERING KEY LESSONS FROM KNOWN RESULTS

2.1. The lessons from quantum mechanics

The most curious feature of quantum mechanics is that we could explain quantum phenomena if information could travel backwards in time. Recent advances in quantum computation have given us a deeper insight into the nature of quantum phenomena and such results as teleportation and counter-factual computation indicate that quantum computation cannot be regarded simply as a large set of classical computations running in parallel. These subtler process rely on the fact that quantum information can appear as if it could be transmitted backwards in time [22]. This feature is the basis of Hadley’s proposal that closed time like loops [14], which are allowed in general relativity, are the basis of quantum phenomena and we can derive the essential logic of quantum mechanics from general relativity.

The idea of information travelling backwards in time has a long history in our attempts to understand quantum mechanics; for example Aharonv et al [1], Crammer’s Transactional interpretation [10] and Deutch’s use of closed time like loops in [11]. Most useful for our purposes is Castagnoli and Finklestien’s [8] version of the same essential idea; *quantum computation can be understood as solving a problem in which the answer is part of the question.* When phrased like it abstracts the essential idea away from the notion of one thing happening after another in a causal sequence, i.e. time. In this respect their result should be regarded as more fundamental than other approaches which involve time and some means to transport information backwards in time. It is a simple statement of the essential idea of quantum mechanics, both the initial preparation and the final measurement contribute information which defines the question to be solved. *"The answer is part of the question"* is a statement of the essential feature of quantum mechanics stripped of the mathematical formalism. That formalism has to do with how that essential idea is implemented in the physical world. In fact it is relatively easy to construct logical statements with the feature that the answer is part of the question and they have many on the features we associate with quantum logic (see Small [25]).

Such a curious feature of quantum mechanics would, if it could be realised, allow us to create machines that could solve the halting problem. This is the origin of Kieu’s [17] and Calude’s [7] claim that a quantum computation could in fact compute that which is formally non-computable according to classical computation as defined by Turing. However Chaitin has pointed out that to use these systems one would need to make an infinite number of measurements. Since each measurement requires a finite amount of energy we would need an infinite amount of energy to use these quantum systems to solve the halting problem. Margolus [19] has made use of studies in to the energetics of computation to show that energy can be understood in computational terms as information being transported through time, therefore each measurement represents the addition of information to the system. Since a solution to the halting problem would encode all of mathematics, with an infinite number of axioms, then we would have an infinite amount of information, i.e an infinite amount of energy available to perform such computation. However, the universe is composed of a finite number of parts each of which can only be given a finite amount of energy. The universe is constructed to disallow the operation of such a machine. This is an important lesson; quantum mechanics would allow the operation of a machine would could compute that which is non-computable if we had an infinite amount of energy, but we don’t. Quantum mechanics comes with a built in feature, Planck’s constant h which places a limit on the amount of energy one can gather together in one place.

The feature of quantum mechanics which deceives us into thinking that a quantum computation could solve the halting problem is that super-position of eventualities provides a physical state which can represent the result of a computation involving a solution to the halting problem. Deutsch [11] has shown that the new logical states which quantum superposition introduces into the language of logical systems allows the quantum formalism to represent states one would encounter in the operation of a time machine, which would also allow one to solved the halting problem. Turing's proof relies on two valued logic and one form of Turing's proof of the non-existence of a classical machine to solve the halting problem is to argue that if such a machine could exist then one could construct machine which could examine itself and determine whether its computation will halt or not. One could then construct a machine programmed such that it would halt if it doesn't halt and not halt if it does. Such a state of halted and not halted at the same time is disallowed in classical mechanics and so constitutes a proof of the non-existence of a finite solution to the halting problem. However quantum mechanics would allow a machine to be in the state $|Halted\rangle + |Not\ Halted\rangle$ so the argument is not valid in the case of a quantum computer prior to measurement. But to gain knowledge of the state of a quantum computation we have to make a measurement, and to determine completely the value of Ψ we would have to make an infinite number of measurements, which we can't.

These seem to be the lessons from quantum theory;-

(i) The logic of quantum mechanics can be understood as a problem in which the answer is part of the definition of the problem to be solved.
(ii) The quantum formalism is sufficiently rich as to include physical states which can represent solutions to self-referential algorithms.
(iii) We can only gain complete knowledge of such states if we could make an infinite number of measurements, which requires an infinite amount of energy.
(iv) We can't have an infinite amount of energy.

2.2. The lesson from general relativity

The most curious feature of general relativity is that it allows the existence of closed time like loops and that these can be used to explain or to model quantum phenomena. This is the basis of Hadley's proposal [14] that one could derive the logic of quantum mechanics from general relativity and also of Deutch's observation that quantum super-position can represent the states encountered in the operation of such causality violating regions [11]. But we know that general relativity is a computable theory. This presents an problem, for there is no way that any finite procedure starting from a finite set of axioms can lead us being able to compute that which is non-computable. The usual procedure is to dismiss such solutions to Einstein's equations as being "unphysical" but it is more interesting to investigate how such solutions have crept in and hence to understand what being "physical" actually means. The point is that since general relativity proceeds by computable means from a certain set of assumptions to arrive at a result which would allow us to compute that which is non-computable then there must be something of a non-computable nature buried in those assumptions.

The lesson we've extracted from quantum theory is that one possible signature of non-computability is that the answer is part of the inputs that are required to fully define the problem to be solved. General relativity is a theory of space and time and so we must look for evidence that the development of the notion of space and time requires that the answer be part of the question. This evidence has already been recognised by Christian in [6, chap. 14] where he argues that general covariance, carries with it the implication that points in space-time cannot possess any sense of individuality a priori. As he puts it; -

> "Thus, strictly speaking the bare space manifold does not even become 'space-time' with physical meaning until both the global and local spatio-temporal structures are dynamically determined along with a metric. Further since spacetime points acquire their individuality in no other way but as a byproduct of a solution of Einstein's field equations, in general relativity 'here' and 'now' cannot be part of a physical question, but can only be part of the answer to a question. As Stachel so aptly puts it [16], the concepts 'here' and 'now' - and hence the entire notion of local causality - acquire ontological meaning only *a postereriori*, as a part of the answer to a question. "

However general covariance is just a mathematical technique not a physical principle and Barbour [3, 5] has argued that this argument from general covariance is not relevant because other theories can be cast in a generally covariant form. However while we have the option of casting other theories in a generally covariant form no such option exists in general relativity, it's an essential requirement. This discussion obscures the central idea, in general relativity "the concepts 'here' and 'now' - and hence the entire notion of local causality - acquire ontological meaning only *a postereriori*, as a part of the answer to a question. " Stated thus, shorn of the mathematical formalism which forms the details of how that idea is implemented in the context of space and time, we can immediately see the same central idea that we've extracted from quantum mechanics. In the case of quantum mechanics that idea is expressed in the sense that we prepare a quantum system in a state and then at a future point in time, as determined by the reference frame of the observer, we make a measurement on that system and determine a particular result, and that result in part determines the state of the system being measured. In general relativity the metric is determined by the arrangement of energy and momentum in spacetime, but we can't determine the arrangement of energy and momentum in spacetime without a metric. The answer is part of the question and this leads to the results that a space-time metric is only defined locally and that we are forced to use a generally covariant formalism to model the situation.

By stripping away the mathematical formalism to see the central ideas of both quantum mechanics and general relativity we can now recognise what appears to be an interesting duality of representations. In quantum mechanics the spacetime framework is purely deterministic and is conceived to exist *a priori* and yet the actions of material bodies in that framework follow rules which allow the answer to a problem be part of the definition of the problem to be solved. The material particles have well defined states only *a posteriori*. In general relativity it's the other way round, the material bodies behave in a purely deterministic manner but the definition of the spacetime framework in which they move involves the answer being part of the question. We summarise this in table(1)

TABLE 1. Is there a duality between general relativity and quantum mechanics?

	Quantum Mechanics	*General Relativity*
Spacetime framework	Strictly deterministic	Answer is part of the question
Particle interaction in the spacetime framework	Answer is part of the question	Strictly deterministic

Presented like this we can immediately see the what the core problem in quantum gravity is. Quantum mechanics is a representation of a particular idea in the context of particle dynamics and general relativity that same idea in the context of spacetime dynamics. We can't quantise gravity because it already represents the core idea of quantum mechanics but in a complementary way.

Presented like this the complementarity prompts an intriguing question. Can we find ways in which the computational possiblities afforded by one view of nature are exactly repesented in the other view? Deutsch [11] as already led the way by showing that the logical states encountered by the existence of closed time like loops find a representation in the quantum formalism as a superposition of possible outcomes. Similarly Hadley's [14] argument that the logic of quantum mechanics can be deduced from general relativity suggest that it ought to be possible to find ways to implement quantum algorithms in the framework of general relativity. A good test of this is as follows;- The idea that quantum computation might be able to exceed the speed of classical computation was first noted by Feynman [13] who observed that as simulating a quantum system on a classical computer requires an exponentially increasing amount of classical computation, then implementing that computation in a quantum system must be equivalent to having an exponentially increasing amount of classical computation available to use. We know that because "spacetime tells matter how to move and matter tells spacetime how to curve" then computation of results in general relativity quickly become quite complex. Is the degree of complexity the same as in quantum computation? If it is then we can represent calculations in general relativity on a quantum computer and the other way round. A quantum computer ought to be able to simulate a general relativitic system without the excessive computational load. If this is the can then we should be able to find analogues in general relativity of, for example, superdense coding or the Shor algorithm [24] for factoring large integers.

2.3. Lessons from other sources

Apart from the pleasure of being able to find computational analogues of quantum computation within general relativity such a study could yield a more important result. If we can map concepts from one view of the world into the other then we can gain a deeper understanding of the transition from the quantum representation of reality to the general relativistic, i.e the classical view. The notion that state vector reduction is in some way connected with gravity has been suggested by Penrose [22, 20, 21] but perhaps the most interesting idea comes again from Deutsch's paper[11]. In analysing the quantum representation of closed time like loops he notes that we can have an observer A of another observer B who is interacting with a closed time like loop in such a way as to put themselves in a quantum superposition of making one copy of themselves and not existing at all. In this case A sees B in a quantum superposition. But if A leaves their framework of time and joins B in their's then the superposition of possiblities vanishes and only the a classical certainty remains. This is an intriguing idea, one can implement state vector reduction by changing the framework of time.

We now collect a seemingly unconnected result from studies of quantum teleportation. Quantum teleportation requires observations on a single quantum system that has become extended in space. But to make the observations the parties have to exchange information via a classical route in order to agree on the framework of space and time in which the observations are to be made. If the parties do not exchange sufficient information then this lack of knowledge of the precise framework in which to make observations means that spatially extended quantum phenomena cannot be observed, (see [23] for a review). Effectively decoherence has occured, not being able to agree on a framework of spacetime in which to make observations has the same effect as decohering a quantum state. We put this together with Deutch's observation that a change in one's framework of time can effect state vector reduction and make a big jump, state vector reduction actually is consequent on changing the framework of spacetime. This idea comes in useful later on.

3. CONNECTING THE IDEAS

The central reason for the problems that arise in finding a theory for quantum gravity is that in general relativity the spacetime framework is dynamic and exists *a postereriori* and in quantum mechanics it is fixed and exists *a priori*. This is a consequence of different implementations of the same central idea. The central idea being that of a self-referencing, or non-computable system. If we attempt to unite the different implementations into a single view of reality without abandoning the key lessons of each then we end up with a theory which involves causality violation and/or infinite energies. The usual view is that to find a single theory we will have to abandon the key lessons from one or other. Either find a purely deterministic explanation of quantum phenomena or to abandon the notion of a dynamical spacetime, as in String/M-theories. But a better approach could be to assume that each of the theories is a limit representation of an underlying reality that has no finite representation and that each view introduces some measure which prevents the computation of what would be formally non-computable. Hence each allows a finite, though incomplete, representation of such an underlying reality. If we look at the methods we could use, but can't, to compute the non-computable then we find that there are exactly two methods. We are prevented from using these methods by rules which are built into physical law. One come from general relativity and separately one comes from quantum mechanics. We'll now examine those rules.

The first and most obvious system we could use to compute the non-computable is to send information backwards in time. So we'll create a rule C which states "we can't send information backwards in time" and then look at the constraints on our view of an underlying non-computable reality which then emerge. The second rule is less obvious. The specific statement of the non-existence of a method to solve the halting problem is that there exists no finite computation which could determine whether any given collection of data and code will result in the computer halting or not halting. If we have an infinite computation then we can do it. So our second rule is H "we can't observe an infinite amount of computation in a finite amount of time as experienced by the observer". Each of these rules has consequences as we'll now explore.

3.1. The consequences of rule C

Rule C "we can't send information backwards in time" carries with it the hidden implication that if we could evade the rule then we could send information backwards in time. So in applying this rule over an assumed underlying non-computable reality in order to model that reality as if it were strictly causal we have to find a place in our model for that notion of non-computability. From Goedel's Theorem's which are equivalent to Turing's we know that from a finite set of axioms only a limited set of Theorems can be proved in a finite number of logical steps. Gregory Chaitin [9] provides a useful insight from Algorithmic Information Theory.

> "This is because algorithmic information theory sometimes enables one to measure the information content of a set of axioms and of a theorem and to deduce that the theorem cannot be obtained from the axioms because it contains too much information. "

So there exist true theorems which cannot be derived from our starting axioms, or starting information. This means that we cannot progress in purely computable steps from an axiom set A to a theorem B if B does not exist inside the set of allowable configuration changes proceeding from A. Sending information backwards in time is not an allowable configuration change though if we could do that then we could create extra information as if from nowhere which would allow us to connect A with B. If we have another theorem C which we can deduce from A in a finite number of computable steps then the physical configuration representing C is causally connected to A. Under this rule we represent the position of B with respect to A by *space-like* separation from A that is we represent a formally undecidable proposition as an event outside the set of events which can be arrived at from A by a causal sequence of computations, but which could be connected if we constructed a causal loop, see figure (1).

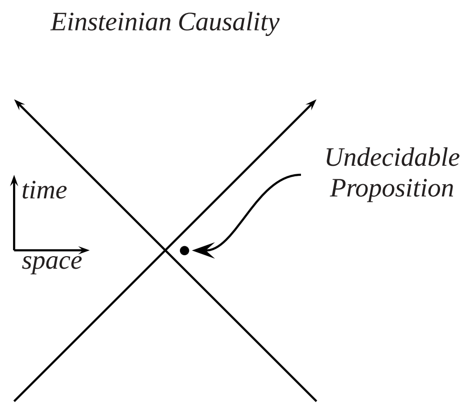

FIGURE 1. In special and general relativity a undecidable proposition is represented as a space-like separated event

The fact that the notion of space we've been forced to introduce carries with it the hidden implication that if we could send information between space like separated events then we could construct loops of causal sequences has an important consequence. We diagram the consequence here (2).

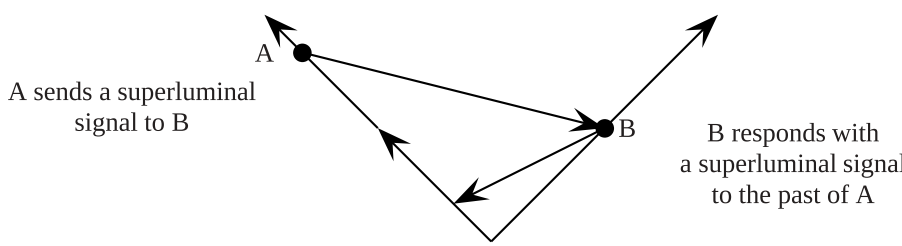

FIGURE 2. Superluminal signaling can be used to create a causal loop

The significant feature is that constructing a causal loop requires setting the arrows of causal propagation so that they are head to tail all the way around the loop. To do this we must make use of a particular feature of the circle known as parallelisablity. We can then see that to construct a causal loop we must make use of this feature of parallelisability. This feature of the circle, known as the sphere S^1, is shared by only two other continuous objects; the sphere S^3 and the sphere S^7. Each of these spheres has associated with it a complex division algebra. S^1 is associated with the complex numbers S^3 with the quaternions and S^7 with the octonions (see [2] for a review of the division algebras). A rotation on S^1 can be represented by complex multiplication and similarly rotations of S^3 and S^7 by quaternion and octonion multiplication. Any quaternion can be obtained by multiplication of two base quaternions, any octonion by multiplication from three base octonions. This means that any causal loop we want to construct using the largest parallelisable sphere, S^7, can be encoded in just three linearly independent numbers. Furthermore each of these three numbers must be complex with respect to the time dimension. This means we are forced to use a metric with signature $+---$.

Obviously this doesn't count as a new result because it's insufficiently rigorous, but by looking at old results with a slightly different perspective we have gained an insight into the nature of space which does lead to a possible solution to one of the hard problems of current physics. In this view the dimensionality of space is not determined by this year's fashion in particle theories by is completely constrained by what space-like separation actually means. We can develop theories with more than the observed number of space dimensions, but in those theories space-like separation cannot have the hidden meaning that we can create causal loops through super-luminal signalling since the largest number of independent measures of information required to do this is just three. We also note that the operation of rule C results in a barrier between what is computable and what is not computable, which we identify as the light cone. Therefore we must have a quantity which defines the maximum speed of propagation of information i.e c.

If we add extra space dimensions to a theory then because they are extra degrees of freedom which cannot carry with them the implication of causal separation they will appear to have no spatial extent in the framework of normal 3+1 spacetime. Though these extra degrees of freedom do have an interesting connection with the parallelisable spheres. But first I'll identify some important open problems in this approach;- in attempting to map the space of computations from a given starting set of axioms we've arrived that the conclusion that we can measure the degree of non-computablity of an undecidable proposition by using a complex number of computational steps. We need to show that we are justified in using complex numbers to measure the amount of non-computable computation required to connect A to B. The fact that it's possible to understand energy and momentum in computational terms and that Einstein's equations set up an equivalence relationship between the spacetime metric and energy and momentum is a good clue that mapping sets of computations to physical space would be instructive. But if we do this then we have an extraordinary possibility, that the information content of axioms added to an axiom system can be measured in at most three linearly independent ways. Understanding this strange observation is another open problem.

3.2. The consequences of Rule H

A computation that goes through an infinite number of steps in a finite amount of time is called a supertask. So we rephrase rule H as "supertasks are not allowed". In the same way as above we attempt to find a way to represent an underlying reality, which is non-computable, within a computable framework with this rule applied. In this case evading our rule would allow us to create a supertask so we have to find a representation for such an implied evasion. An easy way to visualise this though at thought experiment "Thomson's Lamp" [26], we look at what could happen if we could implement an infinite number of configuration changes in a finite amount of observers time. If we set up a light switch that is on for 30 seconds, off for 15, on for 7.5 and so on then at the end of one minute we will have had an infinite number of configuration changes. Is the light on or off at the end of one minute? It can't be on because that would imply only a finite number of steps, and it can't be off for the same reason. There is no classical representation for the state of the light a the end of an infinite number of on/off transitions. But in quantum mechanics we do have a way to represent such a state;- $|ON\rangle + |OFF\rangle$. Just as the quantum formalism can be used to represent the logical states encountered in a causal loop so it can be used to represent the logical state of a system at the end of a supertask. If we introduce the notion of quantum superposition then the light can be both on and off at the same time. See figure (3) An undecideable proposition then has a representation in the quantum formalism as a superposition of states. See figure 4

Now, if we put figure (1) alongside figure (4) it is clear that the notion of spacelike separation, which is the place for undecidable propositions under rule C is not needed under rule H and so we would expect that spatial separation in quantum mechanics does not carry with it the same meaning of causal separation that is does under rule C. This is the reason for the non-locality of quantum phenomena. Since the operation of a supertask is computationally equivalent to the creation of a causal loop then the logic of quantum mechanics will appear to be explained by the introduction of causal loops. But since we're operating under rule H then we can't actually observe superposition, just as we never

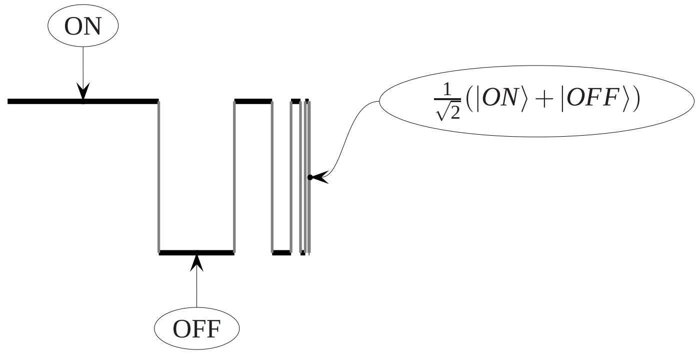

FIGURE 3. The end result of a super-task is a superposition of states

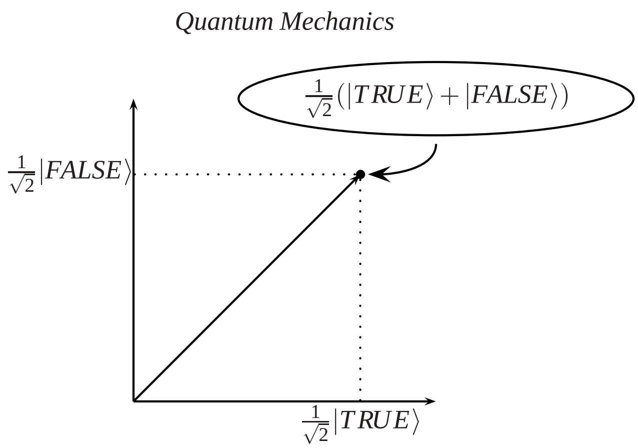

FIGURE 4. Quantum mechanics represents an undecidable proposition as a super-position of alternative outcomes

observe an event which is space-like separated from us. Space-like separation and quantum superposition are notions we have to introduce in order to explain observations made within a causal framework but the causality protection rules G and H prevent direct experience of non-computability.

Since a super-task goes through an infinite number of steps in a finite amount of time measured by reference to an external observer, then we prevent a super-task from happening by enforcing a rule that there is a finite maximum number of configurations that a system can go through in a finite period of time measured according to the external reference frame. Following Margolus [19] we define $\Omega(t)$ the number of configuration changes in a unit of time. This is a dimensionless number, but if we call this number action we can then create another concept energy which is the maximum rate at which configurations can change. We then have a constant of proportionality, let's call it K which puts a boundary on the number of configuration changes in a unit time via

$$Et = K\Omega(t) \tag{1}$$

So that K has the dimension of $energy \times time$ since $\Omega(t)$ is dimensionless. The actual equation in our world is

$$Et = \frac{h}{2}\Omega(t) \tag{2}$$

So that $K = h/2$, hence we can see that the operation of rule H demands a fundamental constant which has the units of action. But it also requires us to have an external reference framework of time.

Rule H brings with it the strange implication that to prevent the observation of an infinite amount of computation in a finite amount of time there has to be a finite limit to the amount of computation that can be observed. It doesn't matter what the limit is only that there is a finite limit and this has odd consequences which we'll come to later on. In the meantime we note that it is precisely this quantity h that prevents infinite energies. Our causality protection rule H "there will be no supertasks" is implemented by Planck's constant h. The rule is that it is not possible to observe an infinite number of configuration changes in a finite amount of external time. This makes it clear that it is with reference to the extrinsic framework of time as experienced by the observer that we have to limit number of configuration changes. This is why quantum mechanics has to be formulated with extrinsic time. There can be no formulation of quantum mechanics with an intrinsic time because there would then be no h.

Evading rule H to create a supertask is equivalent to evading rule G and creating a causal loop. Since the representation of a supertask under rule H is as a superposition of possibilities then quantum superposition can represent causal loops [11]. But rule H does not bring with it the notion of space like separation as a place-holder for undecideable propositions so the possible degrees of freedom are not limited to three dimensions of space. If we try to find a place for these extra degrees of freedom in the framework of space-time generated by rule G then it will appear as if quantum systems have additional spatial degrees of freedom more then three spatial dimensions but these extra dimensions have no spatial extent. But because these extra degrees of freedom that comes from superposition are equivalent to constructing causal loops the action of quantum systems follows logic constrained by the algebra of the parallelisable spheres. An important result by Dixon [12] shows that the $U(1)\times SU(2)\times SU(3)$ symmetry of the Standard Model can be uniquely derived from the properties of the division algebras. But this result does not fit well with current efforts to find a unified field theory or even a grand unified theory and has therefore been ignored. However if we abandon the search for a unified field theory and instead seek out explanation for the structure of our more limited theories then the significance of this result becomes apparent. It would appear that the symmetry of the Standard Model is completely determined by the requirement that we represent a non-computable reality as if it were a causal sequence. But such a derivation allows the observed symmetries of the Standard Model to pop fully formed out of the higher order division algebras, not via symmetry breaking from some higher order symmetry group. This would imply that not only is there no unified field theory but there's no grand unified theory either. Which though controversial does at least accord with current experimental evidence.

We'll now deal with the odd consequences of rule H. The fact that it places a finite limit on the amount of computation that can be observed from the point of view of any particular external framework of time is odd because being finite we could certainly add more energy to the system to get it over the limit, whatever that limit is. A suggestion by Magueijo [18] that the total energy of a *coherent system* is the important measure leads to a useful understanding of the mechanism of decoherence. If we take the total energy of a coherent quantum system over the limit this would allow us to evade the rule and create a supertask which is equivalent to having infinite energy. We then have a very strange counting system. If our limit was say 100 configuration changes in 1 unit of extrinsic time and the system has enough energy to go through 90 units per unit time and we add enough energy to allow an extra 20 configuration changes per unit time then that would be equivalent to going up to infinite energy. The only way round this problem is to say that the system can no longer be observed in a single framework of external time. Deutsch's work [11] and research on long distance entanglement which shows how important it is to maintain a common framework of space and time to prevent decoherence (see [23] for a review) now becomes useful. The idea that (a) changing one's notion of causal ordering, i.e. time makes what was experienced as a superposition of possible outcomes to become a single certainty and (b) communicating information about the framework of spacetime between two observers is essential to be able to observe the operation of a spatially extended quantum system. Together these ideas bring a new understanding of state vector reduction. To observe the operation of a single spatially extended quantum system observers have to agree on the causal ordering of events. That means they have to share enough information to be able to make measurements on the system within a single framework of time. If they don't or can't share that information then they cannot observer the consequences of entanglements and the system has effectively become decoherent.

The result of this is that there is another way to exceed this limit imposed by rule H. If two observers are not able to experience the joys of quantum teleportation or whatever because they haven't exchanged enough information to be able to agree on a single spacetime framework in which to interpret their observations subsequently do exchange enough information, then what was previously decoherent would become coherent. The formalism of quantum mechanics assumes a single external framework and hence it allows entanglement to propagate up to the largest bodies, there is no place in the formalism where decoherence happens. The implication is that if our observers could exchange enough information about the spacetime framework then they could perceive entanglement effects where previously they couldn't. By exchanging information they could build a single coherent framework within which they could observe otherwise decoherent systems as a single coherent quantum system with energy that exceeds the limit imposed by rule H. To enforce rule H we have to prevent observers exchanging enough information to allow them to build a single framework in which to observe a coherent quantum system.Therefore rule H which places a limit on the amount of information that can move through time can only work if there is a corresponding limit on the amount of information that can move through space. Since in 3d space communication between observers is through 2d surfaces the amount of information that can cross any surface has to be limited. If the amount of information that has to be exchanged between two observers in order to perceive coherent quantum effects exceeds that limit then it will not be possible to have a single framework of time and the system appears decoherent. We'll call that limit H_L the Holographic Limit.

We'll now put these ideas together with Chaitin's insight from Algorithmic Information Theory as quoted above. The fact that some theorems cannot be deduced from a certain set of axioms because they contain more information than those axioms carries with it the implication that if we add information to an axiom system then theorems which could otherwise not be deduced now can be deduced. That means that by adding information we've increased the number of causal connections we can create between things. Looking at figure (1) which says that we can create a space of computations where some things can be placed into causal order and some things can't then it's clear that adding information will make some things which could not be put in causal order, i.e. they are space like separated, now become causally ordered, i.e time like separated. So there must be a measure of the number of new causal connections that can be made with a certain amount of information. Since energy and momentum can be understood in computational terms (Margolus [19]) then we should expect that there will be a relationship between the amount of energy/momentum that is required to alter causal relationships between things. In a sense G is a count of the number of connections that change from being non-causal to causal in response to a certain quantity of energy. In Algorithmic Information Theoretic terms G measures the number of new theorems that can be deduced from an additional quantity of information.

But adding information to make undecidable theorems become decidable is equivalent to performing a non-computable task, i.e a supertask. So there must be a relationship between this measure G and the measures of h and H_L. These limits arise from the need to prevent the observation of a supertask and have to be defined with an implied external framework of time. G is a measure of the degree to which the framework of external time is altered by energy/momentum. So it's also a measure of how much energy/momentum is needed to go over the limit imposed by h and H_L. The larger these quantities are the smaller the value of G.

If we were to go over the limit imposed by h and H_L and a non-computable task is computed then within the view imposed by rule C the light cone has been tipped. Two observers of the system now have to agree on which events are space-like separated and which aren't, if they can't agree then then they can't find a single framework of time and for them the system becomes decoherent. This is similar to Penrose's suggestion that the collapse of the wave function is caused by the creation of a graviton. But in this case we can only say that tipping the light cone and collapsing the wave function are different facets of the same phenomenon. By exceeding the finite limit of computation that is permissible within single framework of time we have achieved a supertask, in doing this we have altered our perception of the causal ordering of events. In the view imposed by rule C events which were spacelike separated become timelike separated. In the view imposed by rule H what was a superposition of states becomes, in a new system of causal ordering, a single state.

Since the degree to which spacetime bends in response to energy i.e G is therefore inversely proportional to both these quantities then we can write;-

$$G \propto \frac{1}{H_l \hbar} \tag{3}$$

Which is very close to

$$S_{BH} = \frac{kc^3 A}{4G\hbar} \tag{4}$$

which we rearrange as

$$G = \frac{c^3}{\frac{S_{BH}}{A}\hbar} = \frac{kc^3}{4H_l\hbar} \tag{5}$$

We need to find an explanation for the remaining $\frac{kc^3}{4}$, we could just add the term to make the equation dimensionally consistent but that doesn't give us any useful insight. What we do notice is that we've come close to the black hole entropy equation by examining a possible mechanism for state vector reduction. This would appear to vindicate Penrose's view [20] that the two are closely related. It's possible that deeper analysis along these lines will allow us to fill in the gap and make a start on understanding state vector reduction.

4. CONCLUSIONS

I suggest that the relationships between quantum mechanics, general relativity and a fundamental non-computable reality can be summarised in figure (5)

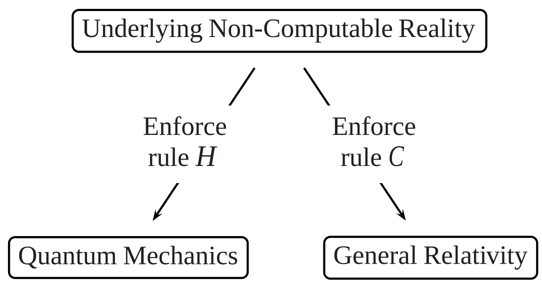

FIGURE 5. Conjectured relationship between quantum mechanics general relativity and an underlying reality

By assuming that a finite theory of everything, or even a theory of quantum gravity, is not possible and then tracing through the consequences we've been able to find vaguely plausible arguments for three dimensions of space, quantum non-locality, the symmetry groups of the Standard Model, and even state-vector reduction. This implies that either we're on the right track or have descended into pseudo-science. To find out which we first have to answer some key open questions. These are ;-

1. I've suggested that the computational complexity of quantum systems and general relativistic systems might be the same. We have circumstantial evidence that this might be the case because quantum superposition can be used to represent the states encountered in closed time-like loops and quantum logic can be deduced from such CTL's [11],[14]. We need to establish that this is actually the case. If it is then this would immediately open up a whole new field of study;- finding general relativistic analogues of known quantum quantum algorithms. For example in superdense coding [4] by using entanglement parties A and B can encode two classical bits of information into a single particle. To do this A and B have to agree on the measurement framework through the classical channel and then two classical bits of information are sent via the quantum channel once they've both measured the particles. We now switch the concepts around, parties A and B both agree on the state of the particle, i.e. the particle is classical, but A and B communicate information about the reference framework of time. This means that at the end of the communication, if any information has been transferred, the reference framework will be different to that at the start of the communication. Which in turn means that some events which would be interpreted as space like separated are now interpreted to be time like separated. The only way this can be is if the light cones for A and B are altered by sending the classical particle. The general relativistic analogue of the quantum computational concept of superdense coding looks like the gravitational mass associated with a particle.

   Such a notion of mass would imply a deep link to entanglement. It has already been suggested by Vedral [27] that entanglement with virtual particles may be responsible for particle mass. However he's hedged his bets by suggesting it's due to entanglement with virtual Higgs bosons, but such an idea would work with any virtual particle and Higgs bosons would not be required.

2. I've suggested that a supertask and a causal loop are computationally equivalent, and that the end state of a supertask such as Thompson's lamp can be represented by quantum super-position. This needs to be rigorously proved.

3. I've assumed that if we count steps of causal computation separating two states A and B using integers, then we must count the steps of "non-computable" computations using complex numbers. We need to show that this is the case. Also the implication is that if we measure the information content of the axiom system in terms of Algorithmic Information Theory then the information content of new axioms added has to be measured in at most three distinct dimensions. This seems very odd and needs to be proved.

4. I've suggested that the symmetry group of the Standard Model which springs fully formed out of the division algebras does not require symmetry breaking from a higher order symmetry and therefore there's no grand unified theory. This suggestion will be tested when the LHC comes online in the next few years, but in the meantime we need to find out if this really is a prediction that can be made from these ideas.

5. An analysis of state vector reduction takes us part of the way to the Beckenstein-Hawking black hole entropy formula. We're missing an explanation for the remaining $\frac{kc^3}{4}$ factor. This is an open problem.

As promised at the start I've ended with a host of new questions, but ones which if resolved would deepen our understanding into the operation of physical law.

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