Inertia, Mach's Principle and Expansion of Space

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Abstract

From the experimental observations of space together with Newton's second law of mechanics, *F=ma*, the expansion velocity of space is deduced on the basis of the Mach principle. It was found that the expansion rate of space is decreasing, not accelerating. An explanation is proposed as to how inertia can stay constant in the expanding space in spite of the decreasing density of space.

Key words
Inertia, Mach's principle, principle of equivalence, dark energy

1. Introduction

Isaac Newton defined /1/ inertia to be a primordial property of mass in the absolute immovable spatial co-ordinate system. This concept was criticised by George Berkeley /2/ 20 years after the Principia. He found it difficult to accept acceleration with no reference point and proposed that fixed stars could serve as the universal reference for motion. One hundred and fifty years later Ernst Mach continued /3/ this discussion and he introduced the idea that the total mass in space is the basic cause of inertia.

It was known from Newton's time that gravitational and inertial masses have a constant ratio within experimental accuracy. Mach never /4/ formulated a mechanism, physical or mathematical, as to how the mass of space will result in the locally observed inertia. However, Albert Einstein gave credit /5/ to Mach when he developed the General Theory Relativity. One key problem was the connection between inertia and gravity. Einstein solved this problem by postulating in the principle of equivalence stating that the ratio between inertial and gravitational masses is constant everywhere in space. This is an extrapolation from the experimental result that acceleration given by gravity to a mass is independent of its composition.

In the times of Newton, Mach and Einstein, knowledge of the universe was limited and space was rather a topic of philosophical discussion than of observational science. Our present knowledge that space is expanding will add a new problem to be solved. According to Mach, inertia depends - although in an unspecified manner- on all masses in space. In expansion, the mass density in space is diluted and the average distance between masses increases. Why, then, do we nevertheless find from astronomical observations that inertia appears to remain constant, despite the dilution of mass density?

2. The structure of space

We can summarise some of the observed characteristics of space as follows:

1. According to astronomical observations mass, on the average, is evenly distributed in space, and space seems to be both homogeneous and isotropic on large scale.

2. Space appears to be expanding uniformly relative to any reference point in space.

3. The cosmic microwave background radiation has the same spectral distribution and intensity from all directions, excluding the Doppler shift due to the velocity of the Earth in relation to the background radiation /6/, and very small temperature fluctuations in the background.

4. The inertial force is the same in all directions. The inertial force affects the accelerated object without any delay.

5. The laws of physics are independent of time and position. This can be concluded eg. from the fact that by taking into account the Doppler shift in the observations, the characteristic spectral lines of various chemical elements are identical everywhere in space.

All of the above observations suggest that space is symmetric and homogeneous. Its geometry is hardly very complicated. A simple and natural solution of the geometry of space is an expanding three -dimensional surface of a four-dimensional sphere. Einstein /7/ proposed this geometry already in 1917 (without expansion). Richard Feynman /8/ formulated this idea by stating "*...One intriguing suggestion is that the universe has a structure analogous to that of a spherical surface. If we move in any direction on such a surface, we never meet a boundary or end, yet the surface is bounded and finite. It might be that our three-dimensional space is such a thing, a three-dimensional surface of a four sphere. The arrangement and distribution of galaxies in the world that we see would then be something analogous to a distribution of spots on a spherical ball."*

3. Inertia and Mach's principle

3.1. Inertia in homogeneous space

The second law of Newton, $F = ma$, is so commonly accepted that its origin is not often thought to be worth enquiring.

One of the very mysterious properties of inertia is that it appears without any noticeable delay on the object accelerated. Distances to other masses in space are huge. How is it conceivable that masses far away in space can immediately have an effect on the locally accelerated objects, knowing that the velocity of light cannot be exceeded?

Mach assumed that inertia is caused by other masses in space. Such an dependence, however, is not evident in Newton's formula

$$F = ma \tag{1}$$

which, in the first place, is an expression of experimental observation and which then has been used as a postulate in classical mechanics. Let us assume that the correct form is

$$F = f \cdot ma \tag{2}$$

Here the multiplier function *f* contains space-dependent variables, namely the total mass in space, *M*, the distance *R* from the origin of expansion and its expansion velocity *dR*/*dt***.** To be strict, in fact, we do not have to specify what the true "meaning" of *R* is; it is sufficient to say that it is a length-like property of space. Following the cosmological principle, we assume that, any time, both *R* and *dR/dt* are same everywhere in space. By further including the gravitational constant *G*, the function *f* can be expressed as

$$f = F\left(G, M, R, dR/dt\right) \tag{3}$$

Because there are no astronomical observations showing that inertia should depend on time, it follows that the function *f* must be dimensionless and constant (= 1). Hence, the argument of the function *f* is also dimensionless and constant. The simplest argument for this function is then

$$q = G^i M^j R^k \left(dR/dt\right)^l \tag{4}$$

the exponents *i*, *j*, *k* and *l* have to be found so that *q* is dimensionless. The dimension of *q*, given according to (4), is

$$[q] = \left[\left(\mathrm{m}^3/\mathrm{kgs}^2\right)^i, \mathrm{kg}^j, \mathrm{m}^k, \left(\mathrm{m/s}\right)^l\right] \tag{5}$$

The trivial solution for (*i*, *j*, *k*, *l*) is (0, 0, 0, 0). This is a simple identity and does not tell us anything about the possible, albeit perhaps hidden, dependence on the argument variables introduced in (3) and (4).

The only non-trivial solution turns out to be (*i*, j, k, *l*) = (1, 1, -1, -2). This yields for the constant argument *q* the following result

$$q = \frac{GM}{R\left(dR/dt\right)^2} \tag{6}$$

From this it follows

$$\left(\frac{dR}{dt}\right)^2 = \frac{1}{q}\frac{GM}{R} \tag{7}$$

If we assume that the mass of space *M*, and the gravitational constant *G* are constant we can solve this simple differential equation and get

$$R(t) = \text{constant} \times t^{2/3} \tag{8}$$

If note present time by *T* and present *R* by R_H (the Hubble distance) we get

$$R(t) = R_H \left(\frac{t}{T}\right)^{2/3} \tag{9}$$

and

$$\frac{dR(t)}{dt} = \frac{2}{3}\frac{R(t)}{t} \tag{10}$$

According this equation (10) the expansion rate of space is decreasing. This result follows on the basis of Mach principle and Cosmological principle. The first of the principles says that inertia is caused by all masses in the universe via gravitational interaction. The second principle, the Cosmological principle tells that universe is symmetric and laws of physics appear same everywhere and at all time.

How can Mach's principle be interpreted from the previous? Equation (9) shows how space is expanding as a function of time. The expansion is determined by the total mass of space. Since we know that inertial force appears instantaneously this means that average information of the whole space must be present everywhere at all time. It is evident that potential of masses of universe is present everywhere in the universe. For this reason there is no need to assume infinite velocity of the propagation of gravity. The expanding mass of space manifests itself in the inertia of local mass. In fact there is no separate inertial and gravitational mass. Both properties follow from the gravitational interaction.

3.2. Zero energy principle

Multiplying by *M*, equation (7) can be written in the form

$$M\left(\frac{dR}{dt}\right)^2 = \text{constant} \times \frac{GM^2}{R} \tag{11}$$

If the expansion velocity of space is same as the velocity of light this formula is very close to the "zero energy principle" /9/

$$Mc^2 = \text{constant} \times \frac{GM^2}{R} \tag{12}$$

where c is the velocity of light. The right-hand side of the equation is the negative of the total gravitational energy of all masses in the space, and the left-hand side is the rest energy of all masses in the space, as given by the theory of relativity. This expression emerges independently in the results of many workers, most recently by Tuomo Suntola /10/ 2000. This formula proposes that the total energy of the universe is zero. Feynman /8/ pondered this mysterious equation in his Lectures on Gravitation in 1962. He expressed this in the following words:

"*If now we compare this number [total gravitational energy M^2G/R] to the total rest energy of the universe, Mc^2*, *lo and behold, we get the amazing result that $GM^2/R = Mc^2$, so that the total energy of the universe is zero — — Why this should be so is one of the great mysteries—and therefore one of the important questions of physics. After all, what would be the use of studying physics if the mysteries were not the most important things to investigate*".

The idea that the velocity of light is the same as the expansion rate of space is not presented here for the first time. Suntola /10/ has developed a novel cosmological model based on the zero energy principle and showing that for conserving the zero energy balance, the maximum velocity in space, which also appears as the velocity of light, is equal to the expansion rate of space. In this model frequency of atomic oscillators follow always exactly the velocity of light. For this reason the observed velocity of light appears constant at all time. Suntola calculated also a value for the constant in equation (12), which simply comes from the geometry of space. The huge numbers in this equation fit amazingly well and give direct connection between the velocity of light and the constant of gravitation.

4. Conclusions

Isaac Newton defined that inertia is a primordial property of matter. The discussion presented here shows that he was not necessarily right because inertia is more a property of local space than matter. However, it is primordial in the sense that the property of local space depends on the total mass of space which, probably, is not actually variable but very much primordial.

The Mach principle, if valid, is more general than what has been thought up until now. The total mass of space M together with distance R_H will also determine the actual velocity of light. From Mach's principle it also follows that the expansion rate of space is decreasing as a function of time. One may argue that this is against experimental results. However, it is not necessarily so because actual measurements are red shifts and magnitudes from distant objects. The conclusion that expansion is accelerating is based on the standard cosmological model. The expansion formula (9) fits well to red shift/magnitude data measured from supernova's /11/ by using Suntola's model /12/. (His model also predicts that space appears flat.) In his model no hypothesis or adjustable parameters of such a thing as negative gravitation or "dark energy" are needed. And, certainly, it is according to good scientific practice that one should avoid unnecessary hypotheses when those are not necessary.

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