Physical Interpretations of Relativity Theory X, Imperial College, London, Sept. 2006

AN EXPLANATION OF THE ANOMALOUS DOPPLER FREQUENCY SHIFT OF THE PIONEERS

Walter Petry

Mathematisches Institut der Heinrich-Heine-Universitaet Duesseldorf,
Universitaetsstrasse 1, D-40225 Duesseldorf
E-mail: petryw@uni-duesseldorf.de
wpetry@meduse.de

Abstract: The analysis of the Pioneer 10 and 11 data demonstrated the presence of an anomalous Doppler frequency blue-shift drift where post-Newtonian approximations in the pseudo-Euclidean geomety are used. To explain this shift the spacecraft in the solar system is studied in the universe. The time of the observer is introduced. The calculated frequencies of the arriving photons emitted from the spacecraft at different times in the universe agree with the corresponding measured frequencies, i.e. an anomalous frequency shift does not arise by considering the solar system with the spacecraft in the universe.

1. Introduction

There are many papers confirming an anomalous Doppler frequency blue-shift drift of the Pioneers (see e.g. Anderson at al.[1],Markwardt [2], Turyshev at al. [3], and many others). The frequency shift follows by considering the solar system with the spacecraft to post-Newtonian accuracy in the pseudo-Euclidean geometry, i.e. neglecting the universe. In general, the shift is interpreted as an anomalous acceleration of the spacecraft but it is difficult to explain this anomalous acceleration. In the last time several autors suggested that the universe cannot be neglected (see J.L.Rosales et al. [4], J.L.Rosales [5], A.F.Ranada et al. [6], Petry [7]). In this paper, the results of Petry [7] are reconsidered. The theory of gravitation in flat space-time is used which has been studied in several papers (see e.g. Petry [8]). A summary of this theory with applications can be found in the paper of Petry [9]. This theory of gravitation gives for gravitational redshift, light deflection, perihelion precession, radar time delay, post-Newtonian approximations, gravitational radiation and the precession of the spin axis of a gyroscope in the orbit of a rotationg body the same results as general relativity to the accuracy needed for experimental results. Birkhoff's theorem is not valid for this theory. Furthermore, the theory of gravitation in flat space-time gives non-singular, homogeneous, isotropic cosmological models and the space is flat for all models (see e.g. Petry [10-14]). In these models the arriving frequency of a photon emitted at a distant object in the universe is calculated (see Petry [10]). The post-Newtonian approximation of several mass points in the universe is given in the paper [14] . The time of the observer is introduced in addition to the time given by the pseudo-Euclidean geometry which is given in the paper [7]. These results are used to calculate the Doppler frequency shift of the spacecraft moving through the solar system in the universe following along the lines of the paper of Petry [7]. The Doppler frequency blue-shift drift does no longer arise and there is no anomalous acceleration. Hence, the solution to the stated anomalous frequency shift is of cosmological origin.

2. Auxiliary Results

A covariant theory of gravitation in flat space-time studied by Petry [8] is the starting point of all subsequent considerations. The theory of gravitation in flat space-time has a flat space-time background metric given by

$$(ds)^2 = -\eta_{ij} dx^i dx^j \tag{2.1}$$

The gravitational field is described by a symmetric tensor g_{ij} satisfying covariant (with respect to the flat space-time metric (2.1)) differential equations of order two where the source for the field equations is the total energy-momentum tensor inclusive the one of the gravitational field. The theory can e.g. be found in the papers of Petry [8, 10] and is omitted here. The proper time τ (atomic time) is defined by

$$c^2 (d\tau)^2 = -g_{ij} dx^i dx^j. \tag{2.2}$$

The application of this theory of gravitation to homogeneous isotropic cosmological models is studied in several papers (see e.g. Petry [10-14]). The used background metric is the pseudo-Euclidean geometry, i.e.

$$(\eta_{ij}) = diag(1,1,1,-1) \tag{2.3}$$

where x^1, x^2, x^3 are the Cartesian coordinates and $x^4 = ct$. The four-velocity of the universe is

$$u^i = 0 (i = 1,2,3),\ u^4 = c\frac{dt}{d\tau} \tag{2.4}$$

and the potentials have the form

$$(g_{ij}) = diag(a^2(t), a^2(t), a^2(t), -1/h(t)). \tag{2.5}$$

It follows from (2.2) by the use of (2.4) and (2.5)

$$d\tau = dt/\sqrt{h(t)}. \tag{2.6}$$

The energy-momentum tensor consists of matter (dust), radiation, an additional matter (for some models) with a stiff equation of state, i.e. $p_a = \rho_a$, cosmological constant and gravitation. Then, the field equations of gravitation give two coupled differential equations of order two for the two functions $a(t)$ and $h(t)$. The initial conditions at present time $t_0 = 0$ are

$$a(0) = h(0) = 1,\ \ \dot{a}(0) = H_0,\ \ \dot{h}(0) = \dot{h}_0 \tag{2.7}$$

where the dot denotes the time derivative, H_0 is the Hubble constant and $\dot{h}_0$ is a further constant being zero for Einstein´s theory. These differential equations can be found in several papers (see e.g. Petry [10,11]). It is shown that non-singular cosmological models exist under conditions on the density parameters of the universe similar to those of Einstein´s general theory of relativity for a flat space. The space of this theory of gravitation is flat and the functions $a(t)$ and $h(t)$ are defined for all $t \in]-\infty,+\infty[$. The functions $a(t)$ and $h(t)$ are not used subsequently and therefore, they are omitted.

Furthermore, we need the post-Newtonian approximation of a perfect fluid in the universe which can be found in the paper [14]. Let

$$\rho(x,t), \quad v(x,t) = (v^1(x,t), v^2(x,t), v^3(x,t)), \tag{2.8}$$

be the density and three-velocity of a perfect fluid. Then the density

$$\rho^* = \rho(x,t)\frac{dt}{d\tau} \tag{2.9}$$

implies the conserved mass

$$M = \int \rho^*(x',t) d^3x' \tag{2.10}$$

and the equations of motion to Newtonian approximation have the form $i = (1,2,3)$

$$\frac{\partial}{\partial t}\left(a^2\sqrt{h}v^i\right) + \sum_{j=1}^{3} v^j \frac{\partial}{\partial x^j}\left(a^2\sqrt{h}v^i\right) = -\frac{1}{a\sqrt{h}} k \int \rho^*(x',t) \frac{x^i - x^{i'}}{\|x - x'\|^3} d^3x'. \tag{2.11}$$

Here, $\| \ \|$ denotes the Euclidean norm and k is the gravitational constant.

In addition, we need the formula for the energy loss of a photon emitted at the distant moving object in the universe and arriving at the observer. This is studied in paper [10]. Let us assume that a distant atom in the universe is moving with velocity $(v,0,0)$ and emits at time t_e a photon moving to the observer. Put

$$\gamma = \left(1 - \left(\left(a\sqrt{h}\right)(t_e)\frac{v}{c}\right)^2\right)^{-1/2} \tag{2.12}$$

and let E_0 be the energy of the photon emitted from the same atom at rest and at present time in the universe. Then, the observer at $t_0 = 0$ receives from the moving atom the emitted photon with the energy

$$E(0,t_e) = \gamma^{-1}\left(1 + \left(a\sqrt{h}\right)(t_e)\frac{v}{c}\right)^{-1} a(t_e)E_0. \tag{2.13}$$

Finally, the time epoch dt given by the use of the pseudo-Euclidean geometry (2.1) with (2.3) at time t in the universe is measured by the observer at time $t_0 = 0$ and denoted by dt'. The relation of the two time epochs is derived in the paper of Petry [7] and it holds

$$dt' = dt / \left(\left(a\sqrt{h}\right)(t)\right). \tag{2.14}$$

The total time t' of the observer since the beginning of the universe till the time t of the pseudo-Euclidean geometry as background is given by

$$t' = \int_{-\infty}^{t} dt / \left(a\sqrt{h}\right)(t). \tag{2.15}$$

Equation (2.15) defines a unique relation between the times t and t'. It is worth mentioning that the age of the universe as measured by the observer is finite whereas it is infinite by the use of the time t. Furthermore, let us introduce the atomic time τ given by (2.6) then the equation

$$\tau = \int_{-\infty}^{t} dt / \sqrt{h(t)} \tag{2.16}$$

gives the relation between t and τ. It follows that the age of the universe measured with atomic time is also finite and it holds

$$\tau(0) < t'(0) < \infty. \tag{2.17}$$

The time of the observer (2.14) with (2.15) implies for the relation (2.2) with (2.5) the proper time

$$c^2 (d\tau)^2 = -a^2(t)\left(\sum_{i=1}^{3}\left(dx^i\right)^2 - (dct')^2\right) \tag{2.18}$$

and for (2.1) with (2.3) the background metric

$$(ds)^2 = -\left(\sum_{i=1}^{3}\left(dx^i\right)^2 - \left(a^2 h\right)(t)(dct')^2\right) \tag{2.19}$$

where t must be replaced by t' by the use of (2.15).

3. Explanation of the Doppler Frequency Shift

Let us follow along the lines of the paper of Petry [7]. Relation (2.18) implies for the observer that the light velocity in the universe is at any point and for all times equal to the vacuum light velocity c. The velocity $v(t) = \left(v^1(t), v^2(t), v^3(t)\right)$ of any object with respect to the time t is by the use of (2.14) transformed to the observer´s velocity

$$v^{i'}(t') = v^i(t)\frac{dt}{dt'} = \left(a\sqrt{h}\right)(t)v^i(t) \quad (i = 1,2,3) \tag{3.1}$$

where t must be replaced by t' using (2.15). Then, the equations of motion (2.11) in the universe have the form

$$\frac{\partial}{\partial t'}\left(av^{i'}\right) + \sum_{j=1}^{3} v^{j'}\frac{\partial}{\partial x^j}\left(av^{i'}\right) = -k\int \rho^*(x', t(t'))\frac{x^i - x^{i'}}{\|x - x'\|^3}d^3x' \ (i = 1,2,3). \tag{3.2}$$

These equations give by applying to several mass points $x_j = \left(x^1{}_j, x^2{}_j, x^3{}_j\right)$ with mass M_j and velocity $v'_j = \left(v^{1'}{}_j, v^{2'}{}_j, v^{3'}{}_j\right)$ the relation

$$\frac{d}{dt'}\left(av'_l\right) = -k\sum_{j\neq l} M_j \frac{x_l - x_j}{\|x_l - x_j\|^3} \tag{3.3}$$

for all mass points l. The equations (3.3) imply for all objects in the solar system (sun, planets and spacecraft) by the use of (2.7)

$$\frac{d}{dt'}v'_l = -H_0 v'_l - k\sum_{j\neq l} M_j \frac{x_l - x_j}{\|x_l - x_j\|^3}. \tag{3.4}$$

Relation (3.4) yields that by virtue of the smallness of the Hubble constant the first expression of the right hand side is too small to can be realized by the observer. Hence, the observer cannot measure an anomalous acceleration of the spacecraft in the universe.

Let us now consider the energy of a photon emitted at time t_e from an atom in the universe moving away from the observer with velocity $v(t_e)$. Then, the formulae (2.12) and (2.13) are rewritten by the use of the time t' of the observer

$$\gamma = \left(1 - \left(\frac{v'(t'_e)}{c}\right)^2\right)^{-1/2} \tag{3.5a}$$

and

$$E(0, t_e) = \gamma^{-1}\left(1 + \frac{v'(t'_e)}{c}\right)^{-1} a(t_e)E_0 \tag{3.5b}$$

where t_e must be replaced by t'_e using (2.15). The corresponding frequencies are received from (3.5b) and the use of Planck´s law $E = h\nu$ where h is the Planck constant, i.e.

$$\nu(0, t_e) = \gamma^{-1}\left(1 + \frac{v'(t'_e)}{c}\right)^{-1} a(t_e)\nu_0 = \left(\frac{1 - v'(t'_e)/c}{1 + v'(t'_e)/c}\right)^{1/2} a(t_e)\nu_0 \approx \left(1 - \frac{v'(t'_e)}{c}\right)a(t_e)\nu_0. \tag{3.6}$$

Here, $\nu(0, t_e)$ is the arriving frequency measured by the observer and ν_0 is the frequency emitted at present time by the same atom at rest in the universe, i.e. the reference frequency. In the general case where the velocity vector and the line of sight enclose an angle ϑ the velocity $v'(t'_e)$ must be multiplied by $\cos\vartheta$. The total Doppler frequency shift in the universe is now given by the use of (3.6)

$$\frac{d}{dt'_e}\left(\nu_{obs}(t'_e) - \nu(0, t_e)\right) \approx \frac{d}{dt'_e}\left(\nu_{obs}(t'_e) - \left(1 - \frac{v'(t'_e)}{c}\right)\nu_0\right) + \frac{d}{dt'_e}\left(\left(1 - \frac{v'(t'_e)}{c}\right)\left(1 - a(t_e)\right)\nu_0\right) \tag{3.7}$$

where $\nu_{obs}(t'_e)$ is the observed received frequency. The first expression of (3.7) of the right hand side has been studied by several authors (see e.g. [1-3]) and the calculated frequency shift of this expression is

$$\frac{d}{dt'_e}\left(\nu_{obs}(t'_e) - \left(1 - \frac{v'(t'_e)}{c}\right)\nu_0\right) = \dot{\nu} \approx 6\cdot 10^{-9}\ Hz/s \tag{3.8}$$

(see [3]). Here, the application of the post-Newtonian approximation of a perfect fluid to several mass points is used neglecting the universe. The last expression of formula (3.7) is calculated by Taylor expansion and the use of (2.7)

$$\frac{d}{dt'_e}\left(\left(1-\frac{v'(t'_e)}{c}\right)(1-a(t_e))\nu_0\right) \approx \frac{d}{dt'_e}((1-a(t_e))\nu_0) \approx -H_0\nu_0. \qquad (3.9)$$

The used reference frequency ν_0 (see [1]) is

$$\nu_0 = 2.29 \cdot 10^9 \, Hz. \qquad (3.10)$$

The presently assumed Hubble constant

$$H_0 \approx 70 \frac{km}{\sec Mpc} \approx 2.3 \cdot 10^{-18} 1/\sec \qquad (3.11)$$

gives for the second expression of formula (3.7) by relation (3.9)

$$-H_0\nu_0 \approx -5.3 \cdot 10^{-9}. \qquad (3.12)$$

Hence, the total Doppler frequency shift (3.7) is by the use of (3.8), (3.9) and (3.12) about zero. Therefore, the calculated frequency shift (3.8) does not imply an anomalous acceleration but the total frequency shift is zero if the solar system and spacecraft are considered in the universe. The values of (3.8) and (3.12) give not exactly zero for the total Doppler frequency shift but the the value of (3.8) is either a little too large or the Hubble constant too small or both of them. The total Doppler frequency shift would be exactly zero if the value of (3.8) is accepted and the Hubble constant is given by

$$H_0 \approx 80 \frac{km}{\sec Mpc}. \qquad (3.13)$$

References

[1] J.D.Anderson, P.A.Laing, E.L.Lau, A.S.Liu, M.M.Nieto and S.G.Turyshev, Phys.Rev. D**65**, 082004 (2002).
[2] C.B.Markwardt, arXiv: gr-qc/0208046.
[3] S.G.Turyshev, M.M.Nieto and J.D.Anderson, arXiv: gr-qc/0503021.
[4] J.L.Rosales and J.L.Sanchez-Gomez, arXiv: gr-qc/9810085.
[5] J.L.Rosales, arXiv: gr-qc/0401014.
[6] A.F.Ranada and A.Tiemblo, arXiv: gr-qc/0602003.
[7] W.Petry, arXiv: physics/0509173.
[8] W.Petry, Gen.Rel.Grav. **13,** 865 (1981).
[9] W.Petry, in: Recent Advances in Relativity Theory, Vol.II (Eds.M.C.Duffy and M.Wegener), Hadronic Press, Palm Harbor FL 2002, p.196.
[10] W.Petry, Z.Naturforsch. **60a**, 255 (2005).
[11] W.Petry, Astrophys. and Space Sci. **254**, 305 (1997).
[12] W.Petry, in Proceedings: Physical Interpretations of Relativity Theory VI (Ed. M.C.Duffy), London, Sept.1998, University of Sunderland, p.252.
[13] W.Petry, in Proceedings: Physical Interpretations of Relativity Theory VIII (Ed. M.C.Duffy), London , Sept. 2002, University of Sunderland, p.386.
[14] W. Petry, Astrophys. and Space Sci. **272**, 353 (2000).