On the structure of space-time and matter as obtained from the Planck scale by period doubling in three and four dimensions

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One of the most interesting questions in modern physics is the possible relation of the Planck scale to our perceived world. The Planck energy ($10^{22}$ MeV) is extremely large as compared to the rest energies of the elementary particles and the Planck length ($10^{-35}$ m) is too short to be directly connected to any real world distances. Why the Planck scale is interesting is that it is absolute, as it is determined by the natural constants $h$, $c$, $G$ and $\varepsilon_0$. Another reason is that the Planck scale may represent the ultimate “graininess”, i.e. the basic structure, of the space-time and matter.

It is well known that nonlinear systems show universal behavior in the form of period doubling, which is the same as frequency and energy halving. If period doubling is applied to the Planck energy $E_0=h/c_0$, an absolute and unadjustable set of sublevels is borne. Spatial period doubling will correspondingly yield a set of increasing lengths.

The rest energy of the electron-positron pair is given directly by a Planck energy sublevel, whereas the nucleon rest energies originate from a sum energy of two adjacent sublevels.

It is also shown that the value of the elementary electric charge squared, which is proportional to energy, results from the Planck charge squared by the same period doubling process.

It is further shown that the planets in the Solar system occupy orbits, the radii of which can be calculated from the Planck length by spatial period doubling. A spectrum of velocities can be calculated from the speed of light by the same process. These velocities fit the consequent orbital velocities of the planets and the quantized redshifts of galaxies, if redshift is interpreted as velocity.

A hypothesis is made that the invariant properties and structures of matter are related to periodic structures obtained by a period doubling process in three and four dimensional nonlinear systems.

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1 INTRODUCTION

The Planck scale is a natural starting point for studies of space-time structures, since the Planck length and time are determined by natural constants. The difficulty is in that the Planck energy (10^{22} \text{ MeV}) is far too large as compared to e.g. the electron (0.511 \text{ MeV}) or proton (938 \text{ MeV}) to indicate any direct and simple connection between these energies. It has also been difficult to relate the Planck charge and the elementary charge, although their magnitudes are not so different.

Observations may point to some kind of a large-scale quantization. W. G. Tifft first pointed out that the redshifts of nearby galaxies are quantized \[1\]. There is also the Bode-Titius rule \[r_n=0.4+0.3\times2^n\], which may point to some underlying order in our Solar system.

M. Feigenbaum in his article \[2\] states that “various Hamiltonian systems ... such as the Solar system – can come under this discipline ... as an external parameter (temperature, for example) is varied” ... “... That is, the period has doubled to \(2^t\).”

The process of period doubling means exact frequency and energy halving according to the Planck relation \[E = hf = \frac{h}{\text{period}}\]. The fundamental frequency or energy of the system may be arbitrarily high, but the period doubling process will generate a series of subfrequencies and subenergies finally reaching energy levels encountered in our material world. Lengths or distances, which fit our perceived world, result from spatial period doubling.

Another important property of a non-linear system is the generation of sum and difference frequencies. This means that single subenergies mix and form new energy levels.

The universe is also dynamic and for a holistic picture we shall need both a model for the invariant properties of matter and the dynamics of the universe. T. Suntola \[3\] has recently developed a four-dimensional model of the dynamical universe, where the fourth dimension has a spatial nature and the rest energy of mass (\(E=mc^2\)) is due to the expansion of the universe in the fourth dimension at the speed of light. In this model our 3-dimensional universe is the surface of a 4-sphere. A fourth dimension will be needed for the electric charge in the model represented in this article.

The occurrence of a possible period doubling process in nature was studied by A. Lehto \[4,5\], who found out that several energies, distances and velocities fitted rather nicely into a doubling scheme in three dimensions. He interpreted the findings to show that time has a multidimensional periodic nature related to the stationary properties of matter.

It is the purpose of this article to suggest that invariant properties and structures of matter may result from a period doubling process in three and four dimensions, when applied to the Planck scale.

We shall first develop the mathematical model and then compare the model with the elementary particle properties, Solar system, redshifts of galaxies, cosmic background temperature, cosmic radiation and some other topics.

2 MODEL

The fundamental quantities are the Planck mass \(m_o\) and charge \(q_o\), from which the Planck length, period (and frequency), mass-energy, electrostatic Coulomb-energy and elementary electric charge can be derived.

The number of the Planck energy sublevels and their combinations is huge. Because there are only a few stable particles, we conclude that not all of the sublevels are occupied. Therefore selection rules for stable occupation must exist. By occupation we mean attaching electric
charge(s) to a sublevel in which process an energetic charged object is borne. The object is electrically neutral, if the charges cancel. We shall start the development of the model by defining the Planck scale needed.

2.1 The Planck scale units

The necessary Planck scale units are defined as:

\[ l_o = \sqrt{\frac{hG}{c^3}} \] (1)

\[ t_o = \frac{l_o}{c} \] (2)

\[ E_o = \frac{hc}{l_o} \] (3)

\[ q_o^2 = 4\pi\varepsilon_o hc \] (4)

\[ i_o = \frac{e}{t_o} \] (5)

\[ E_o^e = \frac{1}{4\pi\varepsilon_o} \frac{e^2}{l_o} \] (6)

Table I shows the values of the Planck scale units defined by equations (1) to (6) needed in this study.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Symbol</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>(l_o)</td>
<td>4.0513 \times 10^{-35} m</td>
<td></td>
</tr>
<tr>
<td>Time (period)</td>
<td>(t_o)</td>
<td>1.3513 \times 10^{-43} s</td>
<td></td>
</tr>
<tr>
<td>Single level energy</td>
<td>(E_{os})</td>
<td>3.0603 \times 10^{22} MeV</td>
<td></td>
</tr>
<tr>
<td>Double level energy</td>
<td>(E_{od})</td>
<td>6.9161 \times 10^{22} MeV</td>
<td>(E_{oe} + ) next level</td>
</tr>
<tr>
<td>Electrostatic energy</td>
<td>(E^E_o)</td>
<td>3.5543 \times 10^{19} MeV</td>
<td></td>
</tr>
<tr>
<td>Electric charge</td>
<td>(q_o)</td>
<td>4.7012 \times 10^{-18} As</td>
<td></td>
</tr>
<tr>
<td>Electric current</td>
<td>(i_o)</td>
<td>1.1856 \times 10^{24} A</td>
<td>(e/t_o)</td>
</tr>
<tr>
<td>Magnetic moment</td>
<td>(\mu_{oe})</td>
<td>1.5485 \times 10^{-46} Am²</td>
<td>for electron</td>
</tr>
<tr>
<td>Magnetic moment</td>
<td>(\mu_{op})</td>
<td>1.9104 \times 10^{-46} Am²</td>
<td>for proton, see text</td>
</tr>
<tr>
<td>Temperature</td>
<td>(T_o)</td>
<td>3.55 \times 10^{22} K</td>
<td></td>
</tr>
</tbody>
</table>

Equations (1) and (2) define the Planck scale for structures in the periodic space-time. Equation (3) is the Planck relation \(E=hf\), where \(f=c/l_o\) and \(l_o\) is interpreted as wavelength (=corresponding to one period). Equation (4) defines the Planck charge squared, because this is proportional to energy. The unit current, defined by equation (5), is not strictly a Planck scale current, since the elementary charge is used instead of the Planck charge.
Part of the rest energy of charged particles is contained in their electric fields and the electrostatic (Coulomb) energy for the elementary charge can be calculated from equation (6). The unit value is shown in Table I, although it is not strictly a Planck scale unit either.

The gravitational constant $G$ is the main source of inaccuracy in the Planck scale units. The relative standard uncertainty of $G$ is 150 ppm and hence that of the Planck energy, length, magnetic moment and temperature 75 ppm. The values of the natural constants have been adopted from the NIST table of natural constants [6].

### 2.2 Basic formulae

If $x_m$ and $x_n$ belong to a doubling sequence, then their ratio $R$ is of the form $R = x_m/x_n = 2^{z+n}$. It was shown in [4] that an observer perceives the geometric mean of the periods and therefore the perceived number of doublings is either of the form $n = N/3$, or $n = N/4$, where $N$, the total number of doublings, is an integer. The process of period doubling in three and four dimensions offers very simple formulae for the calculation of the perceived values of the physical properties of the objects corresponding to their periodic structure:

Energy level (3-d)  
$$E_n = 2^{-N/3} \cdot E_o = 2^{-n} \cdot E_o$$  
(7)

Energy level (4-d)  
$$E_n = 2^{-N/4} \cdot E_o = 2^{-n} \cdot E_o$$  
(8)

Charge squared (4-d)  
$$q^2_n = 2^{-N/4} \cdot q_o^2 = 2^{-n} \cdot q_o^2$$  
(9)

Magnetic moment (3-d)  
$$\mu_n = 2^{N/3} \cdot \mu_o = 2^n \cdot \mu_o$$  
(10)

Length (3-d)  
$$l_n = 2^{N/3} \cdot l_o = 2^n \cdot l_o$$  
(11)

Period (3-d)  
$$t_n = 2^{N/3} \cdot t_o = 2^n \cdot t_o$$  
(12)

Temperature (3-d)  
$$T_n = 2^{-N/3} \cdot T_o = 2^{-n} \cdot T_o$$  
(13)

Velocity (3-d)  
$$v_n = 2^{-N/3} \cdot c = 2^{-n} \cdot c$$  
(14)

Equation (14) can be derived as follows: $v_{ij} = l_i/t_j = 2^i l_i/2^j t_o = 2^{i-j} l_o/t_o = 2^{i-j} c = 2^n c$, where $c$ is the speed of light and $l_o/t_o = c$ according to equation (2).

### 2.3 Notation

In a three dimensional orthogonal coordinate system volume is $V_N = 2^i l_i \cdot 2^j l_j \cdot 2^k l_k = 2^{i+j+k} l_o^3 = 2^N V_o$, where $V_N$ is the volume of the object, $i$, $j$, $k$ numbers of doubling of the edge lengths and $V_o$, the initial or unit volume (Planck scale cube).

The total number of doublings $N = i+j+k$ will be denoted by $(i, j, k)$, which also refers to the structure of the object. The corresponding notation in 4-d is $(i, j, k, l)$. The perceived numbers of doublings are $n = (i+j+k)/3$ and $n = (i+j+k+l)/4$ for 3-d and 4-d correspondingly.

The geometric shape of the object or structure is normally parallelepiped but the shape is cubical, if $i$, $j$, $k$ or $i$, $j$, $k$, $l$ are all equal. The charge squared $(q_n^2)$ with $n = 9.75$ is called the elementary electric charge (squared) and denoted by $e^2$.

Magnetic moment will be denoted by $\mu$ and defined classically as current times loop area, or $\mu = iA$.  

The rest energy of particles may be determined by a single Planck energy sublevel. These particles will be called single-level particles. Particles, whose rest energy is determined by two sublevels, will be called double-level particles and so forth.

2.4 Separation of adjacent levels
In the three dimensional system the adjacent values $X_i=2^iX_0$ and $X_j=2^jX_0$ ($j=i+0.333$) are separated by a factor of $2^{0.333}$, which means that $(X_j-X_i)/X_i=260000$ ppm and $(X_i-X_j)/X_i=210000$ ppm. The separation is correspondingly $190000$ ppm and $160000$ ppm in four dimensions. $X$ is any quantity in equations (7)-(14). The difference between the calculated and experimental values (in chapter 3) may be compared with these level separations.

2.5 Density of states
The perceived density of the Planck energy sublevels or states, $D(E)=\Delta n/\Delta E_n$, can be calculated from equations (7) and (8) by solving for $n$. The (absolute) density of states is

$$D(E) = \frac{i}{E_n \ln 2} = \frac{i}{E_0 \ln 2} 2^n$$

where $i$ is 3 or 4 depending on the number of dimensions. Equation (15) shows that the density of states grows exponentially with $n$. The perceived Planck scale 3-d density of states is $D(E)=1.4 \times 10^{-28}$ (1/eV).

2.6 Superstability
The result of a period doubling process is a quantized system with exact values. Considering transitions it is customary to talk about initial and final states. From the operational viewpoint transition to a new state results from an operation on the initial state. The $1/x$ shape of both the Coulomb and gravitational potentials leads to $x^3=rt^2$ dependence of $x$ on $t$ (Kepler’s law). Let us now define volume $V$ as $V=x^3$ and consider the driving force or an operator $V$ acting on the period $t$. According to $V=t^2$ the operation is squaring the period. Let us further assume that there is a shortest period $t_0$, which doubles (as space expands) according to $t/t_0=2^i$, where $i$ is an integer.

The first operation of $V$ on $t/t_0=2^i$ yields $V_0(t/t_0)=2^{2i}$ (where o means operation), the second operation is $V_0[V_0(t/t_0)]=2^{2i+2i}=2^{4i}$, the third $V_0[V_0[V_0(t/t_0)]]=(2^{4i})^2=2^{8i}$, the fourth $V_0[V_0[V_0[V_0(t/t_0)]]]=(2^{8i})^2=2^{16i}$ and so forth. The resulting periods can be represented as

$$t_i = 2^{2i} t_0$$

where $i$ is an integer. This is the result of functional iteration leading to superstables, as shown by Feigenbaum in [2]. The total number of doublings is of the form $N=2^i$. The physical interpretation is that the final state of the previous doubling is the initial state for the next. We will later show in section 3 that the stable structures found in Nature obey equation (16).

2.7 Particle production
Electron-positron pair production from a 1.022 MeV gamma quantum is perhaps the best known example of materialization of energy. In this article it is assumed that this type of a pair conversion process is generally valid for the Planck energy levels obtained by period doubling. The particle production principle is illustrated in figure 1. A single sublevel splits into a pair of energy levels, which become particles when charges are attached. A double level first splits into a pair of half energy, which materialize into four particles.
2.8 Subcharges and magnetic moments

2.8.1 Subcharges

Because energy is proportional to charge squared, we shall define the subcharges using the Planck charge squared. Repeating equation (9) we obtain for the perceived subcharges:

\[ q_n^2 = 2^{-n} q_o^2 \quad (17) \]

It will be shown in chapter 3 that one of the superstable subcharges can be identified with the elementary electric charge.

2.8.2 Magnetic moment

Let us now define the magnetic moments \( \mu \) as classical current loops in the Planck scale. By definition magnetic moment equals current times the loop area. The loop current is obtained by dividing the elementary charge \( e \) by the period of orbital rotation. Two different loops, shown in figure 2, are defined: a) the orbital type denoted by \( \mu_{oe} \) with the Planck length \( l_o \) as the circumference, and b) the radial type denoted by \( \mu_{op} \) with half Planck length \( l_o/2 \) as the diameter of the loop (as shown by the dotted line in figure 2 b)).

The definition of the orbital type unit magnetic moment \( \mu_{oe} \) is:

\[ \mu_{oe} = \frac{e}{t_o} \pi \left( \frac{l_o}{2\pi} \right)^2 = \frac{e c^2}{4 \pi} \cdot t_o \tag{18} \]

The numeric value of \( \mu_{oe} \) is \( 1.5485 \times 10^{-46} \) (Am\(^2\)). The unit radial type magnetic moment \( \mu_{op} \) is correspondingly:

\[ \mu_{op} = \frac{e}{2 t_o} \pi \left( \frac{l_o}{4} \right)^2 = \pi e c^2 \cdot \frac{t_o}{32} \tag{19} \]

The numeric value of \( \mu_{op} \) is \( 1.9104 \times 10^{-46} \) (Am\(^2\)). The factor 2 in the denominator of equation (19) follows from the assumption that the speed of light cannot be exceeded. If the radius of the loop is \( l_o/4 \), then the circumference is \( 2\pi(l_o/4) \). Because \( l_o/t_o = c \), then \( \pi(l_o/2)/(\pi t_o/2) = c \), which means that the corresponding period in the loop must be \( (\pi/2) t_o \) or longer. The nearest period longer to \( (\pi/2) t_o = 1.57t_o \) allowed by doubling is 2, so the shortest period in this case is \( 2t_o \). Numerical calculations for stable elementary particles are shown in chapter 3.
3 EXPERIMENTAL

3.1 The elementary electric charge

The Planck charge $q_{o} = 4.701 \times 10^{-18}$ (As) is surprisingly close to the elementary electric charge $e = 1.602 \times 10^{-19}$ (As) differing only by a factor of about 29. Obviously equality would be the simplest case, but it seems that subcharges have been borne in process of time, as doubling (i.e. halving) process has continued. We shall now show that a particular subcharge is the elementary electric charge.

The perceived number of doublings for the elementary charge squared is:

$$n = \log\left(\frac{e^2}{q_{o}^2}\right) / \log(2) = -9.7499 = -\frac{39}{4}$$

which means $N = 39$ doublings in four dimensions. According to equation (20) a perceived charge $e^2$ is created:

$$e^2 = g \cdot q_{o}^2$$

where $g$ is $2^{-39/4} = 2^{-(1+2+4+32)/4}$, which is a superstable (1, 2, 4, 32) structure.

The electric force constant is called the fine structure constant alpha and defined as

$$\alpha = \frac{e^2}{2 \epsilon_{o} \hbar c}$$

By dividing both sides by $2\pi$, one obtains $\alpha/2\pi = e^2 / 4 \pi \epsilon_{o} \hbar c$ or

$$\frac{\alpha}{2\pi} = \frac{e^2}{q_{o}^2}$$

which is the ratio of the elementary charge and Planck charge squared. We further obtain

$$\alpha = 2\pi \frac{e^2}{q_{o}^2}$$

Since $e^2/q_{o}^2 = 2^{-39/4}$, the inverse of the fine structure constant $\alpha$ obtains a value of $\alpha^{-1} = 2^{39/4}/2\pi = 137.045$. The difference to the recommended value is 65 ppm.

From Eq. (21) the elementary charge obtains a value of

$$e = 1.60213 \times 10^{-19}$$

which differs by 30 ppm from the recommended value.

The process of volume doubling may also produce other values for the force constants. Some of these are shown in reference [5].
3.2 Rest energy and magnetic moment

3.2.1 Electron-positron pair
The superstable \( N=224 \) or (32, 64, 128) sublevel 1.021 MeV energy is almost exactly the same as the rest energy of an electron-positron pair. We may therefore assume that under proper conditions this sublevel may materialize into an electron-positron pair, when split into two equal parts and occupied by elementary electric charges. The total energy of the object is the sublevel energy added by the corresponding electric field Coulomb energy.

Because \( (32, 64, 128) \) sublevel is superstable, we must assume that the electron-positron pair forms a joint energy object, i.e. they remain connected.

Equations (10), (18) and (19) show that magnetic moment is directly proportional to period. Energy \( (E=\frac{\hbar}{t}) \), in turn, is inversely proportional to period. The split energies and magnetic moments of the \( N=224 \) sublevel then correspond to \( N=227 \) period doublings and the perceived number of doublings is \( n=75.67 \) for both the half-energies and the magnetic moments.

Tables II and III show the numeric values corresponding to the splitting of the \( N=224 \) or (32, 64, 128) sublevel into a pair of particles.

The model for an electron-positron pair is simply a Planck energy sublevel occupied by elementary electric charges. The differences between the calculated and experimental values may be compared to the separation of the 3-d sublevels in 2.4.

### TABLE II. Sublevel \( N=224 \) energies (MeV)

<table>
<thead>
<tr>
<th>Electron</th>
<th>Energy</th>
<th>( n )</th>
<th>( N )</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sublevel</td>
<td>1.0206</td>
<td>74.67</td>
<td>224</td>
<td>(32, 64, 128)</td>
</tr>
<tr>
<td>Coulomb</td>
<td>0.0012</td>
<td>74.67</td>
<td>224</td>
<td>(32, 64, 128)</td>
</tr>
<tr>
<td>sum</td>
<td>1.0218</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Half</td>
<td>0.5109</td>
<td>75.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Half</td>
<td>0.5109</td>
<td>75.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td>0.5110</td>
<td>75.67</td>
<td>electron</td>
<td></td>
</tr>
<tr>
<td>Difference</td>
<td>190 ppm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE III. Splitting of \( N=224 \) magnetic moment (Am\(^2\))

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>Perceived</th>
<th>( n )</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated</td>
<td>4.643E-24</td>
<td>74.67</td>
<td>(32, 64, 128)</td>
</tr>
<tr>
<td>Split</td>
<td>9.286E-24</td>
<td>75.67</td>
<td></td>
</tr>
<tr>
<td>Split</td>
<td>9.286E-24</td>
<td>75.67</td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td>9.285E-24</td>
<td>75.67</td>
<td>electron</td>
</tr>
<tr>
<td>Difference</td>
<td>-160 ppm</td>
<td></td>
<td>calc vs. measured</td>
</tr>
</tbody>
</table>

3.2.2 Electron magnetic moment anomaly
The magnetic moment \( \mu \) of a particle is traditionally considered as resulting from the charge \( e \), mass \( m \) and spin \( S (= \frac{\hbar}{2 \pi}) \) of the particle:

\[
\mu = g_s \cdot \frac{e}{2m} \cdot S
\] (25)

\( g_s \) is the gyromagnetic ratio of the particle.
where $eS/2m$ is called magneton (either Bohr or nuclear depending on the value of $m$). The dimensionless number $g_s$ is gyromagnetic ratio characteristic to the internal structure of the particle. The experimental values of $g_s$ for the electron, proton and neutron are $-1.001$, $+2.793$ and $-1.913$ respectively. The experimental value of the electron magnetic moment is a little larger than the Bohr magneton $\mu_B = eS/2m_e$. Magnetic moment anomaly is defined as $a_e = |\mu_e|/\mu_B - 1$, where $|\mu_e|$ is the measured electron magnetic moment. The anomaly can be measured very accurately and its current accepted value is $a_e = 0.00116$.

In this model the magnetic moment anomaly is obtained by replacing the Bohr magneton by the magnetic moment value given in table III for the electron. One obtains

$$a_e = \frac{|\mu_e|}{\mu_{\text{model}}} - 1 = -0.00016$$

which is negative and much smaller than the classical anomaly. The value of $a_e$ differs by 160 ppm from unity. This discrepancy may partly be due to the inaccuracy of $G$ in the Planck mass and hence the model magnetic moment. In principle the doubling process is exact.

### 3.2.3 Nucleons

The leptons and baryons differ from one another in that the leptons are considered as structureless particles, whereas the baryons have some kind of an internal structure. This difference is also reflected in the way their rest energies and magnetic moments are borne in the period doubling scheme.

The electron-positron pair is borne by dividing a single 1.021 MeV Planck sublevel (32, 64, 128) into two parts with half energy each. The magnetic moment of the electron is twice the magnetic moment of the (32, 64, 128) level. The (64, 64, 64) double sublevel with $2^{64}E_{od} = 3749.2$ MeV energy contains two levels and the splitting yields correspondingly a pair of equal energies, which split into four levels (figure 1.) and four magnetic moments, each four times the (64, 64, 64) magnetic moment. One fourth of the (64, 64, 64) sublevel energy is the same as (66, 66, 66) sublevel energy. This is shown in table IV.

<table>
<thead>
<tr>
<th>$n$</th>
<th>Proton</th>
<th>Neutron</th>
</tr>
</thead>
<tbody>
<tr>
<td>66.00</td>
<td>937.31</td>
<td>937.31</td>
</tr>
<tr>
<td>74.33</td>
<td>1.29</td>
<td></td>
</tr>
<tr>
<td>74.66</td>
<td>1.021</td>
<td></td>
</tr>
<tr>
<td>Sum</td>
<td>938.33</td>
<td>939.62</td>
</tr>
<tr>
<td>Difference</td>
<td>62 ppm</td>
<td>55 ppm</td>
</tr>
</tbody>
</table>

The experimental rest energies of the proton and neutron are 938.27 MeV and 939.57 MeV respectively. Table IV shows that 937.31 MeV energy is a little short of the rest energies of the nucleons. If we attach a superstable (32, 64, 128) sublevel structure and energy to the 937.31 MeV level, a very close agreement between the calculated and measured proton rest energies is obtained, as shown by the small ppm-difference.

The measured difference between the rest energies of the neutron and the proton is 1.28 MeV. Table IV shows that this energy corresponds to $n=74.33$ or $N=223$ single sublevel energy, adjacent to the superstable $N=224$ sublevel. In this model the elementary electric charges are introduced in the 937.31 MeV structure by adding an electron-positron pair.

The superstable $N=192$ or (64, 64, 64) radial type magnetic moment is $3.5240 \times 10^{-27}$ Am$^2$. If this value is multiplied by four (corresponding to (66, 66, 66) structure) the magnetic moment of a proton is obtained, as shown in table V.
According to measurements neutron’s charge is divided into a positively charged inner layer and a negatively charged outer layer. There are thus two current loops of opposite magnetic moments. The negative loop has larger magnetic moment than the positive one and the total (negative) magnetic moment is the sum of the moments of the two loops.

Simplest case is now assumed: The larger loop is identical to the proton’s loop (save the sign of the charge) and a concentric smaller positive loop $\mu_x$ is added, as shown in figure 3.

The measured magnetic moment of the neutron is minus $9.6624 \times 10^{-27}$ (Am$^2$). The unknown $\mu_x$ can now be calculated from $\mu_x = \mu_p + \mu_n$. It is found that

$$\mu_x = 2^{64.33} \cdot \mu_{op}$$

or

$$\mu_x = 2^{-1.67} \cdot \mu_p$$

which shows that the magnitude of $\mu_x$ results from equation (10), too. Neutron’s magnetic moment is:

$$\mu_n = (2^{64.33} - 2^{66.00}) \cdot \mu_{op}$$

and no traditional gyromagnetic ratio is needed. Table V shows the calculated magnetic moments of the nucleons.

<table>
<thead>
<tr>
<th>$n$</th>
<th>Nucleon</th>
<th>$\mu$ (Am$^2$)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>66.00</td>
<td>Proton</td>
<td>1.40959E-26</td>
<td>Positive loop</td>
</tr>
<tr>
<td>64.33</td>
<td>Neutron</td>
<td>4.43994E-27</td>
<td>Positive loop</td>
</tr>
<tr>
<td>66.00</td>
<td>$-1.40959E-26$</td>
<td>-9.6560E-27</td>
<td>Negative loop</td>
</tr>
</tbody>
</table>

The experimental magnetic moment of the proton differs from the calculated one by 720 ppm and that of the neutron by 660 ppm.

A sublevel is just an energetic object without any electrical charges. In the principle it can be neutral or occupied by one charge or two (opposite) charges. In this model the neutron is neutral, because it hosts two elementary charges forming the magnetic moment current loops but there is only one positive charge within the proton.

As hydrogen is the most abundant substance in the universe, it is plausible to think that the negative charge associated with the proton’s positive elementary charge remains attached to the proton, although rather loosely. The ground state orbital magnetic moment of hydrogen is the same as electron’s (intrinsic) magnetic moment. This means that the Bohr orbit magnetic moment has the same periodic structure as the electron’s magnetic moment.

The special feature with the proton is that the perceived number of doublings, i.e. $n=66.00$, is obtained from both a 3-d and a 4-d object ((66, 66, 66) and (66, 66, 66, 66)).

### 3.3 The Solar system

Equations (11) and (14) yield the model lengths and velocities. Figure 4 shows the distances and orbital velocities of the planets in $(r, v)$-space. The asteroids ($N=42$ in equation (14))
occupy the gap between Mars and Jupiter. The observed velocities are consequent values of $N$ from $N=38$ (Mercury) to $N=48$ (Pluto) in equation (14) with one exception. There is an unoccupied orbit with $N=44$ between Jupiter and Saturn, as if a planet were missing there, too.

The distances from the Sun have been calculated from equation (11). The elementary interpretation of the quantization of velocity and distance is that the periodic structures forming massive bodies synchronize.

FIG. 4. Planets in $(r,v)$ space. The asteroids occupy space around $N=42$ orbit and $N=44$ is empty. Note that $N=42$ and $N=44$ are on both sides of Jupiter ($N=43$). The $N$-values indicated belong to $v$ in Eq. (13).

3.4 Quantized galaxy redshifts

According to the accepted cosmological model space is expanding. This view is based on the observed redshifts of the galaxies, which is the larger the dimmer (i.e. farther away) the galaxy is. This observation led to the idea that redshift is due to the Doppler shift, meaning that all galaxies recess from us the faster the farther away they are.

Redshift is composed of three components, one of which is genuine Doppler shift, the others gravitational redshift and the third so called cosmological redshift due to the expansion of “space itself”.

W.G. Tifft of the University of Arizona has measured redshifts for over twenty-five years using both optical and radio spectra (at 21 cm wavelength). Careful measurements with large radio telescopes at 21 cm wavelength have given results far exceeding the accuracy of optical measurements. Tifft has found out with high S/N ratio that the redshifts are not only quantized but also variable [7].

Quantization has been verified by Napier and Guthrie [8] in 1996 and Napier [9] in 2003. These observations indicate that the redshift is not due to motion alone.

The most prominent redshift period corresponds to 73 km/s and its half. Practically all Tifft’s redshift periods follow Eq. (14) and its modification (Lehto-Tifft rule), where cube root is replaced by ninth root (allowing transitions between redshift states). According to the
cosmological principle all observers anywhere in the universe should observe the same redshift periods. The origin of the quantized redshift may not necessarily be velocity at all but an expression of the energetic state of the galaxy. The doubling process is inevitably function of time, if the volume of the universe is expanding. This property of the doubling process may explain the variable redshift periods observed by Tifft.

### 3.5 21 cm wavelength

According to current understanding a neutral hydrogen atom can experience an electron transition relative to the proton spin. This magnetic transition is observed in many astronomical sources and occurs at 21 cm wavelength. With $N=336$, or superstable $(16, 64, 256)$, equation (11) yields $\lambda=2^{12.006} \times 21.04$ cm. The earth laboratory wavelength is 21.12 cm, which corresponds to 1420.4058 MHz frequency.

### 3.6 Cosmic background temperature

The 3K cosmic background radiation corresponds to a black body, whose temperature is 2.73 K. Equation (13) yields $T=2.76$ K with $N=320$, or superstable $(64, 128, 128)$. The next number of volume doublings towards colder temperature is $N=321$, which corresponds to 2.189 K. It is interesting note that this temperature is very close to the $\lambda$-point 2.186 K of liquid helium $^4$He. This is the temperature, where helium becomes super fluid and its specific heat rises abruptly. This may be an accidental coincidence, but if it is not, then there might be a physical explanation. Since superfluidity means frictionlessness (or losslessness), then even a weak coupling to oscillating driving force at resonant frequency (i.e. Planck sublevel) creates large amplitudes.

### 3.7 Cosmic ray spectrum

The most energetic particles are found in the cosmic rays producing the muon showers. The energy spectrum of incident particles is shown schematically in figure 5 as function of energy per nucleus. The vertical axis is particle flux in an arbitrary logarithmic scale. The puzzling features in the curve are the “knee” at 4.5 PeV and the “ankle” at around 6 EeV, as pointed out by R. Ehrlich [10].

![FIG. 5. The knee and ankle in the cosmic ray spectrum.](image)
Equation (7) yields $E = 4.4$ PeV with $N = 128$, which is superstable ($32, 32, 64$). For $N = 96$ or superstable ($32, 32, 32$), we obtain $7.1$ EeV. These energies coincide with the knee and ankle energies, as shown in figure 5. The observed kinetic energies correspond to transitions from $N=125$ to $N=128$ sublevel and from $N=93$ to $N=96$ sublevels. It may be predicted that there ought to be another “knee” at $N=160$ (superstable ($32, 64, 64$)) corresponding to $2.7 \times 10^3$ GeV energy.

A review article of the present models of the knee and ankle is presented in [11]. None of the models is based on period doubling.

4 SUPERSTABILITY

The decimal part of the perceived number of doublings tells the number of dimensions of the structure or object, because $n=N/3$ points to three dimensions and $n=N/4$ to four dimensions. This information makes it possible to break $N$ down into components in 3- and 4-dimensions. The objects in Table VI are related to structures showing very high stability over time. The superstability condition $N_i=2^i$ is found with all these objects.

<table>
<thead>
<tr>
<th>TABLE VI. Possible breakdown of $N$ into components $N_i$. Superstability.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
</tr>
<tr>
<td>---------------------------------</td>
</tr>
<tr>
<td>Electron-positron pair rest energy</td>
</tr>
<tr>
<td>(p-e) magnetic moment</td>
</tr>
<tr>
<td>Elementary charge squared</td>
</tr>
<tr>
<td>Nucleon double level energy (3947 MeV)</td>
</tr>
<tr>
<td>Nucleon double level magnetic moment</td>
</tr>
<tr>
<td>21 cm spin-flip wavelength</td>
</tr>
<tr>
<td>3 K CBR</td>
</tr>
<tr>
<td>&quot;knee&quot;</td>
</tr>
<tr>
<td>&quot;ankle&quot;</td>
</tr>
</tbody>
</table>

Notation “(p-e) magnetic moment” means the sum of the absolute values of the magnetic moments of a positron and an electron. The role of the double levels is explained in chapter 3.2.3.

The number of dimensions of the elementary electric charge squared is an interesting exception. This may mean that electric interactions take place also via the fourth dimension.

5 FORCE RATIOS

The volume doubling process seems to be applicable to both spatial and temporal structures of matter. If we assume that the expansion of the universe is connected to the process of volume doubling then this process should be going on at the present time, too. As no smaller electric charge is known to exist than the elementary electric charge, we must conclude that the volume doubling process is at halt after 39 volume doublings in four dimensions. This means that an extremely stable configuration has been reached.

As comes to the Planck mass the doubling process seems to have halted at the electron-positron pair after $N=224$ volume doublings, because no lighter elementary particles than the electron and positron are known to exist.
The Planck mass and charge represent the same energy, because

\[ Gm_o^2/l_o = q_o^2/4 \pi \varepsilon_o l_o. \]

This means that in the beginning the energy gradients, or the gravitational force and the electrostatic (Coulomb) force were equal.

Because the Planck charge has experienced much fewer volume doublings than the Planck mass, its strength is consequently much larger. Figure 6 shows how the Planck mass and charge squared have developed in process of time as the volume doubling has proceeded.

![Graph showing the relative dilution of the Planck mass and charge squared as function of the perceived number of volume doublings in process of time. Initial value is taken as 1. Note that 2^74.67 = 149.33 (n of e-p pair mass squared).](image)

Figure 6 shows that the volume doubling process has proceeded much farther with mass than with the electric charge. The difference between the perceived numbers of doublings is 149.33 - 9.75 = 139.58. This means that the perceived ratio of the forces (which are proportional to mass and charge squared) is \(2^{(149.33 - 9.75)} = 10^{42}\). This ratio may be interpreted as the perceived order of magnitude ratio of the present strengths of the electric and gravitational interactions.

The strength of the so-called weak interaction is on the order of \(10^{-13}\). This is close to \(2^{-128/3} = 1.4 \times 10^{-13}\), which would relate this interaction to the superstable (32, 32, 64) sublevel.

6 DISCUSSION

The number today of known elementary particles is huge. In this article only the properties of the basic ones (i.e. electron, positron and the nucleons) have been investigated. A systematic analysis will be needed for further studies, since there are plenty of possibilities to add and subtract energies. The known conservation laws, like conservation of leptons and others, may help in finding the right schemes and ways to combine energy sublevels.
The period doubling process seems to be applicable to both mass and charge. Table VI shows that the difference between mass (energy) and charge is in the number of dimensions, as the electric charge squared is four dimensional.

The rest energies of the electron and the positron are found in the direct sequence of energy sublevels obtained by the doubling process from the Planck energy. The nucleons seem to be composite particles, and their rest energy is obtained from a double Planck energy sublevel. The magnetic moment structures are also different. Electron’s magnetic moment is of the Bohr type, whereas nucleon’s magnetic moment is radial.

The Bohr magneton seems to be applicable to single-level particles, because the number of doublings (or halvings) is the same for the (orbital type) magnetic moment and the corresponding Planck energy sublevel. The nuclear magneton is not applicable to the nucleons, because the nucleon rest energy is determined by the sum of two Planck energy sublevels and the magnetic moments by one or two radial type magnetic moments.

Spin was invented to explain the magnetic moments of particles and is traditionally considered as a fundamental property of particles. In this model it is the magnetic moment that is fundamental and spin has no special role and is not needed to explain the basic particle properties dealt with.

The analysis of the electron magnetic moment anomaly in chapter 3.2.2 showed that the anomaly is near zero according to this model. Because the doubling process is exact, there should be no anomalies.

The superstability condition applies to the electron-positron pair, not the electron or positron alone. This is interesting in the sense that it is the electron-positron pair that is stable. This implies that the electron and positron always co-exist forming a joint energy-object and being somehow connected. The pair can be spatially separated in the 3-dimensional world without altering their invariant properties. Freedom in the 3-dimensional space means that the connection is via the fourth dimension.

The non-removable connectedness of the electron-positron pair may be related to the EPR paradox, too.

W.G. Tifft has shown that the redshift period of galaxies may change from one period to the next (redder, i.e. lower energy) period in a few years. As the thicknesses and diameters of galaxies are typically 10,000-100,000 light years, no signal traveling at the speed of light would reach the whole galaxy within years. This implies that the galaxy is some kind of a superstructure and the change is due to the doubling process concerning the whole structure simultaneously. The Sun is also a superstructure in a smaller scale, as shown by the quantization of the Solar system. According to this model the change takes place in the fourth dimension.

If the hydrogen atom is considered as an “elementary particle” based on its structure (as explained earlier), then there is no need for speculations about the existence of antimatter worlds. The neutron hosts both an electron and a positron and the hydrogen atom does the same. Of course, the positions of the electron and positron can be artificially exchanged thus creating antimatter.

There is room in this model for the neutrinos. They may be related to the unoccupied sublevels and/or the fourth dimension.

Actually, this model does not distinguish between small or large, elementary or sophisticated, as all structures are obtained using the principle of multidimensional period doubling. In some sense the objects obtained are fractal, i.e. their structures have similar geometric properties despite of the number of doublings or their “size”.
Acknowledgments

The author would like to thank Dr. T. Suntola and Dr. H. Sipilä for many enlightening and fruitful discussions.

References

7. W.G. Tifft, Evidence for quantized and variable redshifts in the CBR rest frame, Astrophysics and Space Science, 244, 29 (1996)