A NON-EINSTEINIAN DEVELOPMENT OF THE PCC RELATIVITY THEORY

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The paper is devoted to the memory of the great scientist and wonderful person, late Professor Dr

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Abstract.

The goal is to present a non - Einsteinian (a non - Lorentz - Einstein - Poincaré) development of the Partially Compatible but Consistent (PCC) relativity theory, which is based on the inherent characteristics of *time*, to show its consequences in the fundamentals of physics and influences on the further development of the relativity theories.

It was explained in the former papers that *time* is the unique physical variable, which agrees with Newton's explication of the absolute sense of *time*. Its value has been changing continuously monotonously increasingly equally and uniformly in all directions independently of anybody and anything. This should be reflected in the transformations of the temporal co-ordinates, which is a priori rejected in the Einsteinian relativity theory. The *time* scaling coefficients were a priori accepted equal in the Lorentz transformations, hence in the Einsteinian relativity theory, by restricting its validity only to such cases. In order to express the *time* independence, all the scaling coefficients will be permitted a priori different.

Another characteristics of Einstein's approach, which characterises the Einsteinian relativity theory, is the a priori acceptance of a light signal for the reference spatial point, hence of the light speed for the reference spatial speed, and the determination of the scaling coefficients exclusively for such a case. This implied partial pairwise compatibility of the transformations and represents another restriction on the validity of the Einsteinian relativity theory. In order to weaken this restriction, an arbitrary point with an arbitrary speed will be the spatial reference point and the spatial reference speed.

Einstein accepted the assumption on the invariance of the light speed relative to inertial frames (and tacitly relative to all *time* axes). He lifted it to the level of the uncontested postulate that has been the keystone of the Einsteinian relativity theory, which has also caused that the Einsteinian relativity theory is a singular case among relativity theories. The non-Einsteinian approach of this paper relaxes the theory of such a restrictive assumption. The values of all quantities will be consistently expressed with respect to the corresponding units.

The general case of the co-ordinate transformations is free of all constraints a priori accepted in the Einsteinian relativity theory. Consequently, new formulae are substantially different from those in the Einsteinian relativity theory. In the special and the singular case the formulae become of the form known in the Einsteinian relativity theory, but still more general. They reduce to the results of the Einsteinian relativity theory in the particular singular case.

A result of the paper is that for every speed, and not only for the light speed, we can find co-ordinate transformations such that the speed is invariant relative to such transformations. Such transformations for the light speed are the Lorentz transformations. This illustrates that the Lorentz-Einstein invariance of the light speed is not the property of light, hence not of its speed.

The paper opens a new avenue for research in physics, in the relativity theory, in mathematics and in the theory of dynamical systems with multiple *time* scales.

Keywords: Basic physics, co-ordinate transformations, dynamical systems Lorentz transformations mathematical physics, relativity theory, systems with multiple *time* scales, *time*.

1. INTRODUCTION

The basic assumption of Einstein's relativity theory, which Einstein lifted to the unquestionable postulate, is the invariance of the light speed. Another keystone of his theory is *time* dependence of the frames, hence of the space. He equalised *time* (the temporal variable) with its value. He did not distinguish between variable (*time*) and its value (*time* value). Consequently, he treated relativity of the numerical *time* value, of *time* unit and of *time* scale as the relativity of *time* itself. [4] – [6].

Lorentz, Einstein and Poincaré used the Voigt transformations in the form known as the Lorentz transformations in which the spatial transfer speed is invariant relative to all *time* axes (relative to zero instant, initial instant, *time* unit, and *time* scale). They a priori accepted also a light signal to be the reference temporal and spatial point, and its speed to be the reference and generic speed. [4] - [6], [29] – [34], [49].

The a priori accepted assumptions of Einstein's relativity theory constrain its validity so sharply that it represents a singular case, rather than to hold in general [17] - [25].

If we consider Einstein's explanation of *time* relativity only in the sense of the relativity of a zero *time* value, of an initial *time* value, of a *time* unit, and of a *time* scale, then it is in Newton’s sense (Scholium I on the page 8 in [48]). However, if we interpret Einstein's explanation of *time* relativity as the relativity of the variable - *time*, then it opposes crucially Newton's explication of *time* and of its relativity, which are essentially correct [13], [14], [17] – [25].

Time does not force any clock to function. *Time* does not depend on the clock speed. It is a kind of energy (exchange) that forces the clock to work. If the clock is movable, then it is a kind of energy, not *time*, which causes its movement. What clock hands show, it is not *time*, but it is a number associated with the *time* value relative to the chosen initial *time* value, *time* scale and *time* unit. A change of the speed of the clock hand, whatever is its cause, induces automatically the corresponding change of the *time* unit, but not any variation of *time* itself. Various speeds of various clock hands correspond to various *time* scales and *time* units. They do not, and cannot represent various *time* speeds. A change of the speed of a clock hand does not express a change of the *time* speed. Marmet presented excellent energy - matter explanations of these facts, [35], and discovered other serious drawbacks of the Einsteinian relativity theory (which is the Lorentz - Einstein - Poincaré theory of *time* relativity and its developments), [35] - [46]. Other physical explanations, mathematical proofs and more details are given in [25].

The recent papers [22] - [24] explained the importance of compatibility of the co-ordinate transformations. More details are in [25]. They present the definitions of partial and complete, either pairwise or entire, compatibility of the transformations. The Lorentz transformations are restrictively pairwise compatible rather than completely pairwise compatible, [22] - [25]. Besides, these references explain inconsistency of the use of the values of speeds in the Lorentz transformations and in, from them deduced, velocity and acceleration transformations.

This opened new problems in the relativity theory - the problem of *complete (pairwise, entire) compatibility of the transformations*, the problem of the consistent use of values of all variables in the transformations, and raised the following questions:

Q1 What are the necessary and sufficient conditions for the temporal co-ordinate transformations and for the spatial co-ordinate transformations to be (partially or completely, pairwise and/or entire) compatible?

Q2 What are the necessary and sufficient conditions for the velocity transformations to be (partially or completely) compatible?

Q3 What are the necessary and sufficient conditions for the acceleration transformations to be (partially or completely) compatible?

The new results obtained in [22] - [25] provoke also the following questions:

Q4 Does the constancy of the light speed in vacuum means also its invariance?

Q5 What is the justification for the invariance of the spatial transfer speed value in the Lorentz transformations?

Q7 What are consequences of the consistent application of the values of all variables relative to the frames used in the co-ordinate, velocity and acceleration transformations?

Q7 What are implications of different *time* scales on the co-ordinate, velocity and acceleration transformations, hence, on the relativity theory?

Q4 What are implications on dynamical systems with multiple *time* scales?

Q5 What are implications on speed, mass and energy?

The papers [22] - [24], contribute with full replies to these questions in the framework of the uniform transformations, and the book [25] replies to the questions regardless of non-uniformity or uniformity of the transformations. Uniformity of the transformations expresses their property that the same initial moment, the same *time* scale and *time* unit, i. e. the same *time* set and *time* axis, hold over the whole space. This is the temporal uniformity over the space. Another kind of the uniformity is the spatial uniformity. It is achieved in the Einsteinian relativity theory by accepting a priori a light signal for the spatial reference point.

The new co-ordinate, velocity and acceleration transformations in [22] - [25] represent either Newtonian generalisations of the Lorentz transformations or novel transformations essentially different from the Lorentz transformations and, from them deduced, velocity and acceleration transformations. They are based on the properties of *time*, which we discover in the physical reality.

The original form of the basic postulate of Einstein's relativity theory is Einstein's principle of the constancy of the light speed in vacuum relative to inertial frames. This is surprisingly interpreted and used after Einstein himself as the uncontested postulate on the invariance of the (numerical) value of the light speed with respect to a choice of an inertial spatial frame and of a *time* axis. Surprisingly, because the numerical value of the relative speed of anybody or of anything depends on both the speed of the inertial frame and on the units of both *time* and length. Although this holds also for the speed of light, it has not been a priori taken into account in the Einsteinian relativity theory [4] – [8], [29] – [34], [49]. Final results show that the value of the speed of light does not change when the Lorentz transformations are applied. This is the consequence of the equality of the *time* and space scaling coefficients in the Einsteinian relativity theory. Their equality ensures the changes of a *time* unit and of a length unit in the same ratio. Their changes do not influence the value of the quotient of the length passed by a light signal over *time*. In what follows it will be shown that this is not a property of light or of its speed, but that it is a property of the formula. For every constant speed there are co-ordinate transformations, and from them resulting velocity transformations, which ensure the speed invariance under such transformations. This will be presented in the sequel.

The Lorentz - Einstein relativity theory starts with the a priori demands for equality of *time* scaling coefficients and for equality of space scaling coefficients in the Lorentz transformations. This is another sever constraint on the validity of the Einsteinian relativity theory. Their values are determined exclusively for a priori accepted speed of an arbitrary point to be the light speed [4] – [8], [29] – [34], [49].

The aim of this paper is to present new co-ordinate and velocity transformations with the consistent use of the numerical values of all variables. They will be relaxed of the a priori accepted restrictions of the Einsteinian relativity theory. The reference spatial point will be freely chosen and fixed, rather than to be a light signal that is a priori accepted in the Einsteinian relativity theory. Consequently, the new transformations are partially compatible and consistent. They belong to the new mathematical relativity theory called *Partially Compatible but Consistent* (for short: *PCC*) *Relativity Theory* [25]. It overcomes the drawbacks of the Einsteinian relativity theory by new solutions to the posed problems.

2. NOTATION

We may accept any *time* scale for a *time* axis T. Different *time* scales can be associated with different *time* axes: "an original *time* scale" T that is not indexed, and "k"-*time* scale T_k. A *time* value (instant, moment) measured in T-scale and in T_k-scale is designated by t and t_k, respectively, $t \in T$ and $t_k \in T_k$, k = 1, 2, …, s. The indices i, j are arbitrary and fixed, $i, j \in \{1, 2, ..., s\}$. They may be equal, i = j, or different and then i < j. If they are equal, which is the trivial case, the equations become identities.

The Cartesian product set TxR^n is denoted by I, $I = T\text{x}R^n$. It is the *(1+n)-dimensional real integral vector space*, for short *the integral space*. A pair (t, x) is called *an event in I* [24] through [26]. It can occur exactly once due to the properties of *time*. The integral space corresponding to the *time* axis T_k and to the frame R_k^n is I_k: $I_k = T_k \text{ x } R_k^n$, k = 1, 2, …, s.

The vector $\mathbf{u}$ is an arbitrarily chosen and fixed constant unity vector $\|\mathbf{u}\| = 1$, where $\|.\|$ is the Euclidean (or any other scalar) norm. It is elementwise different from the zero vector.

Let origin O_k of the co-ordinate system R_k^n, hence R_k^n itself, move with a constant velocity $\mathbf{v}_{Ok}$ relative to the origin O of R^n and measured with respect to $t \in T$. The norm $\|.\|$ of the velocity $\mathbf{v}_{Ok} = v_{Ok}\mathbf{u}$ is denoted by v_{Ok}, $v_{Ok} = \|\mathbf{v}_{Ok}\|$ if measured relative to $t \in T$, k = 1, 2, …, s. It is the speed of the origin O_k relative to the origin O of R^n.

The value of the light speed in vacuum is denoted as usually by c [5: p. 15], [6: p. 26]. It is the light speed value with respect to a stationary frame in vacuum and measured relative to I, for short: *the light speed value*.

We can accept to consider a light signal and a translation of O_i together with R_i^n in a direction and sense of the unity vector **u**. The constant unity vector **u** is used to represent symbolically also the direction and the sense of a movement of the space R_j^n. Let sign(x) = $|x|^{-1}x$ for $x \neq 0$, and sign 0 = 0. Without losing in generality, [25], we represent a position of an arbitrary point P relative to the origins O and O_i of R^n and R_i^n at the same moment by vectors $\mathbf{r}_P \in R^n$ and $\mathbf{r}_P^i \in R_i^n$ as $\mathbf{r}_P = r_P\mathbf{u}$, and $\mathbf{r}_P^i = r_P^i\,\mathbf{u}$, i = 1, 2, …, *s*, respectively. Their lengths and senses are expressed by their norms $\rho_P = \|\mathbf{r}_P\|$ and $\rho_P^i = \|\mathbf{r}_P^i\|$, and by their algebraic values $r_P = \rho_P \mathrm{sign}(\mathbf{u}^T\mathbf{r}_P)$ and $r_P^i = \rho_P^i\,\mathrm{sign}(\mathbf{u}^T\mathbf{r}_P)$, respectively, which may vary in *time*:

$\mathbf{r}_P(t_{(.)};t_{(.)0}) = r_P(t_{(.)};t_{(.)0})\mathbf{u}$ is the position vector of the point P with respect to $O_{(.)}$ at $t_{(.)}$, (.) = i, j.

Their velocities depend on the reference integral space in general:

c_j^i is the scalar value of the light speed measured with respect to the origin O_j of R_j^n and relative to t_i (rather than relative to t_j if $T_i \neq T_j$),

c^{ij} denotes both c_i^i and c_j^j if and only if $c_j^j = c_i^i$: $c^{ij} = c^{ji} = c_i^i = c_j^j$, which is denoted simply by c in the Lorentz transformations (L1) through (L4) because it has been considered in the Einsteinian relativity theory as invariant with respect to a choice of the integral space,

$c_{(.)}^{(.)}$ is the light speed relative to the integral space $I_{(.)} = T_{(.)}\mathrm{x}R_{(.)}^n$, (.) = i, j; i, j ∈ {1, 2, … , *s*}, i ≤ j,

P an arbitrarily chosen point in the space R^n,

P_R a freely chosen and then fixed temporal reference point,

P_{SU} a freely chosen and then fixed spatial reference point in the space R^n,

$\mathbf{v}_P^{O_i}(t_i;t_{i0}) \equiv \mathbf{v}_P^i(t_i;t_{i0}) \equiv v_P^i(t_i;t_{i0})\mathbf{u}$ is the instantaneous velocity (the speed vector) of the point P with respect to O_i at t_i and measured relative to t_i provided the initial moment was t_{i0}; $\mathbf{v}_P^i(t_i;t_{i0}) \equiv \mathbf{v}_P^i \equiv v_P^i\mathbf{u}$ if and only if the velocity $\mathbf{v}_P^i(t_i;t_{i0})$ is constant vector, and

$\mathbf{v}_P^i(t_i;t_{i0}) \equiv \mathbf{v}_P^i(t_i) \equiv v_P^i(t_i)\mathbf{u}$ if and only if t_{i0} is known and fixed,

$\mathbf{v}_{O_k}^m \equiv v_{O_k}^m\mathbf{u}$ is the constant velocity of O_k relative to O measured in terms of t_m, k, m ∈{1, 2, …, *s*},

$v_{O_i}^r \leq v_{O_j}^r$ is accepted, r∈{*i*, *j*},

$\boldsymbol{v}_{ji}^k \equiv v_{ji}^k\,\mathbf{u} \equiv (v_{O_j}^k - v_{O_i}^k)\mathbf{u}$ is the constant relative velocity of O_j with respect to O_i measured all in terms of t_κ, k ∈ {i, j}; notice that $\boldsymbol{v}_{ji}^i \equiv -\boldsymbol{v}_{ij}^i$, that $\boldsymbol{v}_{ji}^i \neq \boldsymbol{v}_{ji}^j \equiv (v_{Oj}^j - v_{Oi}^j)\mathbf{u} \equiv -\boldsymbol{v}_{ij}^j$ is possible if $T_i \neq T_j$, that $\boldsymbol{v}_{ji}^{(.)} = v_{ji}^{(.)}\mathbf{u}$ is called *the spatial transfer velocity* and that $v_{ji}^{(.)}$ is called *the spatial transfer speed*,

$\boldsymbol{v}_{ji} \equiv v_{ji}\,\mathbf{u}$ denotes both $\boldsymbol{v}_{ji}^i$ and $\boldsymbol{v}_{ji}^j$ if and only if they are equal: $\boldsymbol{v}_{ji}^i = \boldsymbol{v}_{ji}^j = \boldsymbol{v}_{ji} = -\boldsymbol{v}_{ij}$. In the Lorentz transformations there is v_{ji} denoted by *v* because it is considered invariant. If, and only if *i* = *j* then $v_{ji}^i = -v_{ij}^i = v_{ji}^j = -v_{ij}^j = v_{ji} = -v_{ij} = v = 0$,

$v_{SU}^{(.)}$ is the known constant speed of the spatial reference point P_{SU}; $v_{SU}^{(.)} \in R^+$; $v_{SU}^{(.)} \in \{c_{(.)}^{(.)}, v_P^{(.)}\}$ is permitted but not required,

$q^{(.)}, \omega^{(.)}$ are the generic speeds, the values of which are measured relative to $T_{(.)}$, and which obey the following:

$q^{(.)}, \omega^{(.)} \in R^+$,

and we permit

$q^{(.)}\omega^{(.)} \in \left\{\left[c_{(.)}^{(.)}\right]^2, \left[v_P^{(.)}\right]^2, \left[v_{SU}^{(.)}\right]^2\right\}$,

(.) = i, j; i, j ∈ {1, 2, … , *s*}, i ≤ j,

If and only if the point P represents a light signal L, then:

$\mathbf{r}_P^O = \mathbf{r}_L = \mathbf{r} \in R^n$, $\mathbf{r}_L = r_L\mathbf{u}$, $\mathbf{r}_P^{O_{(.)}} = \mathbf{r}_L^{(.)} = \in R_{(.)}^n$, and

$\mathbf{v}_P^{O_{(.)}} = \mathbf{v}_L^{O_{(.)}} = \mathbf{v}_L^{(.)} = \mathbf{c}_{(.)}^{(.)} = c_{(.)}^{(.)}\mathbf{u}$,

(.) = *i*, *j*; *i*,*j*∈{1, 2,…, *s*}, *i* ≤ *j*.

3. *TIME* FEATURES

When we speak about relativity then we think of the relativity of *time*. What is *time*? Is it that what the clock hands show as Einstein claimed? What are its features? We cannot speak exactly about its relativity before we reply to these questions.

Our knowledge of, the common sense based on, and the experience with the physical reality led to the following axiomatic form of the characterisation of *time*, which expresses its crucial properties, agrees with Newton's explanation of *time* [48], and which has permitted new fundamental results in the relativity theory [13], [14], [17] – [25]:

Axiom 1.

a) ***Time***

Time* (*temporal variable*)** denoted by t (or by τ) *is* ***an independent physical variable. *Its value occupies (covers, encloses, impregnates, is over and in, and penetrates) equally, always and everywhere, beings, energy, matter, objects, and the space. Its value has been and will be smoothly and strictly monotonously continuously increasing always equally in all directions in space and independently of beings, energy, matter, objects and space, independently of all other (e.g. physical and mathematical) variables, independently of all movements, of all processes, of all events and of the space.*

b) **Physical dimensionality of *time***

Time is its own unique component. It is a ***basic physical variable****. Its physical dimensionality is a basic physical dimensionality denoted by "T", t[T], where "T" means "time".*

c) ***Time* value and its flow**

The ***value of time*** *is called* ***moment*** *or* ***instant.*** *It is the shortest possible duration that is the duration of a single time value. It can happen exactly once and then it is the same for all beings, energy, matter, for all objects, in the whole space, for all events, movements and processes. It is denoted by t and a subscript, e.g.* t_2*. An arbitrary instant will be denoted as time itself by t. The time value is determined accurately up to an unknown additive constant. A* ***sequence of time values*** *determines uniquely the order of events happening.*

A flow *(i. e.* ***an oriented variation****)* ***of time values*** *(for short:* ***a temporal flow****) from one time value to another one is the* ***duration between them*** *(for short:* ***duration****). Its orientation is the temporal orientation (the temporal sense, the temporal arrow), which is from a smaller (from a past) time value to a bigger (to a future) time value: dt > 0.*

There can be assigned exactly one real number to every moment, and vice versa. The set T of all moments is in one-to-one correspondence with the set R of all real numbers. An accepted rule of the correspondence determines ***a relative zero numerical value of time****,* ***a time scale*** *and* ***a time unit****, and vice versa.*

A total zero moment $t_{ZeroTotal}$ *has not existed and will not occur. Any instant may be accepted for* ***a relative zero time value*** t_{zero}.

d) ***Time interval***

The temporally ordered connected set of all instants between two different time values is a ***time interval****. The time interval* $[t_0, t_{trm}[$, $t_{trm} > t_0$*, reflects the duration from the initial instant* t_0 *to the terminal instant* t_{trm} *either of the existence or of the non-existence of the related being, or of the related form of energy, of the related kind of matter, of the related object, of the related event, of the related movement, of the related process or of the related rest.*

e) **Age**

Moment *(****instant****) reflects an instantaneous temporal situation of somebody or of something, which is called her/his or its* ***age****.* ***Time value difference*** $t - t_0$, $t > t_0$*, (or equivalently,* ***time interval*** $[t_0, t]$*) expresses, and it is used to measure* ***the duration*** *of the (non-) existence of a being, of a form of energy, of a kind of matter, of an object, of a movement, of a process, of a rest; it is used to measure duration of the (non-) existence of somebody or of something, relative to an accepted* ***initial moment*** t_0*. If* t_0 *is the instant of the beginning of the (non-)existence of a being, of a form of energy, of a kind of matter, of an object, of a movement, of a process, of a rest, of somebody or of something, then the time value difference* $t - t_0$, $t > t_0$*, (or equivalently, the time interval* $[t_0, t]$*) represents* ***the age*** *of the being, of the form of energy, of the kind of matter, of the object, of the movement, of the process, of the rest, of the somebody or of the something.*

A co-ordinate axis used for a *time* axis is in the sequel immovable relative to the environment. It is denoted by *T*. It is a geometrical representation of the ***time set*** denoted also by *T*, which is the temporally ordered set of all instants:

$T = \{t: t \in R,\ t \in C^{(1)}(R),\ dt > 0\}$.

Evidently, $T \neq R$ due to dt > 0.

Once a *time* axis has been accepted with a fixed *time* scale including a fixed *time* unit, then the (relative) zero moment $t_{zero} = 0$ should have been also accepted. Afterwards, we can select any instant $t_0 \in T$ for an initial instant. We accept that it has been chosen, known and fixed. It can be $t_0 = 0$, but need not.

The complete characterisation of *time* and its detailed explanation of its properties are presented in [25].

We should recognise the fact, which has been often ignored, that **Newton** himself [48: pp. 8 – 10] introduced and explained both the absolute and relative sense of *time*. Moreover, we should appreciate the truth that Newton's explanation of the relativity of *time* incorporates that of Einstein [4, p. 20], [6, pp. 26 – 27], [7, pp. 23 – 40], which was explained and proved in [13], [14], [17] – [25].

4. DYNAMICAL SYSTEMS WITH MULTIPLE *TIME* SCALES

Different processes can propagate with different speeds in different beings or systems, which can hold also in the same being or system at its different points or parts. Therefore, different *time* scales, different initial moments and/or different *time* units can be assigned to different material objects and/or to different parts of the same material object giving a relative meaning to *time* (in this sense that is Newtonian). This was recognised and used in the theory of dynamical systems with multiple *time* scales, which include singularly perturbed systems.

A large class of dynamical systems is adequately mathematically modelled by (S1) and (S2), [10] – [12], [15],

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}_1(t,\mathbf{x},\mathbf{y},M), \mathbf{x} \in R^p, \mathbf{y} \in R^s, \qquad \text{(S1)}$$
$$\mathbf{f}_1(.): R \times R^p \times R^s \times R^{s\times s} \to R^p, M \in R^{s\times s},$$

$$M\frac{d\mathbf{y}}{dt} = \mathbf{f}_2(t,\mathbf{x},\mathbf{y},M), \qquad \text{(S2)}$$
$$\mathbf{f}_2(.): R \times R^p \times R^s \times R^{s\times s} \to R^s.$$

The matrix M = diag$\{\mu_1 \ \mu_2 \ ... \ \mu_s\}$ contains different (possibly, but not necessarily, small) parameters μ_k that enable the introduction of s different *time* scales and/or units, $\mu_k \in R^+$, k = 1, 2, ... , s, $R^+ =]0,\infty[$:

$$t_k - t_{k0} = \mu_k(t - t_0), t_{k0} = \mu_k t_0, \mu_k \in R^+, \ k = 1,2,...,s. \qquad (1)$$

The link between the theory of dynamical systems with multiple *time* scales and the relativity theory is crucial to overcome the drawbacks and the constraints of the Einsteinian relativity theory. It enables the system approach to the relativity theory [13], [14], [17] - [25]. The *time* scaling coefficients μ_k establish the link.

5. COMPATIBILITY OF THE TRANSFORMATIONS

The *time* scaling coefficients α^i_j and α^j_i, and the space scaling coefficients λ^i_j and λ^j_i should be positive real valued. They intervene in the following generic co-ordinate transformations (2) through (5) :

$$t_i - t_{i0} = \alpha^i_j[(t_j - t_{j0}) + \frac{v^j_{ji}}{q^j\omega^j} r_j(t_j;t_{j0})],$$
$$i,j \in \{-, 1, 2, \ldots, s\}, i \le j, \qquad (2)$$

$$t_j - t_{j0} = \alpha^j_i[(t_i - t_{i0}) - \frac{v^i_{ji}}{q^i\omega^i} r_i(t_i;t_{i0})],$$
$$i,j \in \{-, 1, 2, \ldots, s\}, i \le j, \qquad (3)$$

$$\mathbf{r}_i(t_i;t_{i0}) = \lambda^i_j[\mathbf{r}_j(t_j;t_{j0}) + \boldsymbol{v}^j_{ji}(t_j - t_{j0})],$$
$$i,j \in \{-, 1, 2, \ldots, s\}, i \le j, \qquad (4)$$

$$\mathbf{r}_j(t_j;t_{j0}) = \lambda^j_i[\mathbf{r}_i(t_i;t_{i0}) - \boldsymbol{v}^i_{ji}(t_i - t_{i0})],$$
$$i,j \in \{-, 1, 2, \ldots, s\}, i \le j. \qquad (5)$$

The *time* scaling coefficients α^i_j and α^j_i, and the space scaling coefficients λ^i_j and λ^j_i are a priori mutually different, which expresses *time* independence of the space.

The co-ordinate transformations (2) through (4) do not contain the value of the speed of light. In a special case $q^{(.)}\omega^{(.)} = \left[c^{(.)}_{(.)}\right]^2$ that characterises the Lorentz transformations. Then the co-ordinate transformations contain the value of the speed of light.

The condition for the preservation of the generalised length in integral spaces will be used in its general form (6),

$$\begin{bmatrix} \mathbf{r}^{O_i}_P \\ (t_i - t_{i0})\mathbf{v}^{O_i}_P(t_i) \end{bmatrix}^T G \begin{bmatrix} \mathbf{r}^{O_i}_P \\ (t_i - t_{i0})\mathbf{v}^{O_i}_P(t_i) \end{bmatrix} \equiv$$
$$\begin{bmatrix} \mathbf{r}^{O_j}_P \\ (t_j - t_{j0})\mathbf{v}^{O_j}_P(t_j) \end{bmatrix}^T G \begin{bmatrix} \mathbf{r}^{O_j}_P \\ (t_j - t_{j0})\mathbf{v}^{O_j}_P(t_j) \end{bmatrix}. \qquad (6)$$

The matrix G = blockdiag{A -B} in (6), and the matrices A and B are positive definite matrices with A = B possible but not required, $A \text{ and } B \in R^{n\times n}$. The block diagonal form of the matrix G reflects the *time* independence of the space (Axiom 1).

The temporal co-ordinate equations (1) are inherent for multiple *time* scale dynamical systems [13], [14], [17], [20]. The condition (6) expresses the Gaussian transformation of the *time*-space length. It is a crucial condition of Einstein's general relativity theory for validity of the co-ordinate transformations defined by (2) through (5), [4], [5].

If, and only if the transformations (1) through (5) obey (6) then they form ***the Poincaré group***.

The generic co-ordinate transformations (2) - (5) can be set into the form of the homogeneous linear equations with variable gains [25] in general. The same holds for the Lorentz transformations.

Definition 1. [24], [25]

a) The temporal co-ordinate transformations (2) and (3) are ***compatible*** if, and only if they yield an identity as soon as one temporal co-ordinate and the corresponding (with the same subscript) spatial co-ordinate are eliminated from them [without using the spatial co-ordinate transformations (4) and (5)]. Otherwise, they are ***incompatible***.

Likewise, the spatial co-ordinate transformations (4) and (5) are ***compatible*** if, and only if they yield an identity as soon as one spatial co-ordinate and the corresponding (with the same subscript) temporal co-ordinate are eliminated from them [without using the temporal co-ordinate transformations (2) and (3)]. Otherwise, they are ***incompatible***.

b) The transformations (2) through (5) are ***pairwise compatible*** if, and only if both the temporal co-ordinate transformations (2) and (3) are compatible and the spatial co-ordinate transformations (4) and (5) are compatible. Otherwise, they are ***pairwise incompatible***.

c) The transformations (2) through (5) are ***entirely compatible*** if, and only if both 1) and 2) hold:
1) The temporal co-ordinate transformations (2) and (3) yield, by means of the spatial co-ordinate transformations (4) and (5), an identity as soon as temporal and spatial co-ordinates with the same subscripts are eliminated from them.
2) The spatial co-ordinate transformations (4) and (5) yield, by means of the temporal co-ordinate transformations (2) and or (3), an identity as soon as temporal and spatial co-ordinates with the same subscripts are eliminated from them.

Otherwise they are ***entirely incompatible***.

d) The transformations (2) through (5) are ***partially*** (i.e. ***restrictively***) [***pairwise***, ***entirely***] ***compatible*** if, and only if they are, respectively, [pairwise, entirely] compatible exclusively when the arbitrary point P moves with the speed restricted by certain conditions (which do not allow its arbitrary non-zero value), e. g. with the light speed, or when the generic speeds $q^{(.)}$ and $\omega^{(.)}$ should obey certain constraints.

e) The transformations (2) through (5) are ***completely*** (***pairwise***, ***entirely***) ***compatible*** if, and only if they are, respectively, (pairwise, entirely) compatible regardless of the speed of the arbitrary point P different from the light speed and if there is not any constraint on $q^{(.)}$ and $\omega^{(.)}$.

Definition 2. [24], [25]
The co-ordinate transformations (2) through (5) are *consistent* if, and only if the same *time* scale, the same *time* unit, the same length scale, and the same length unit are applied to the values of all the variables related to the same integral space.

6. CONSTRAINTS OF THE EINSTEINIAN RELATIVITY THEORY

The original Lorentz transformations were determined under the following a priori accepted conditions, which were also a priori adopted by Einstein, hence in the Einsteinian relativity theory [3] - [8], [29] - [34], [49]:

Constraint 1. All the *time* scaling coefficients α^{i}_{j} and α^{j}_{i} are equal: $\alpha^{i}_{j} \equiv \alpha^{j}_{i} \equiv \alpha$.

Constraint 2. All the space scaling coefficients λ^{i}_{j} and λ^{j}_{i} are equal: $\lambda^{i}_{j} \equiv \lambda^{j}_{i} \equiv \lambda$.

Constraint 3. In the proofs of the Lorentz transformations it is accepted that the arbitrary point P moves exclusively with the velocity of light: $\mathbf{v}^{(.)}_{P}(t_{(.)}) \equiv \mathbf{c}^{(.)}_{(.)} \equiv c^{(.)}_{(.)}\mathbf{u}$, (.) = i,j; i,j$\in${-,1,2,…, s},

Constraint 4. The numerical value of every speed, including the value $c^{(.)}_{(.)}$ of the light speed, is invariant relative to *time* axes: $v^{(.)}_{(..)}(t_{(.)}) \equiv v_{(..)}(t_{(.)})$, $c^{(.)}_{(.)} \equiv c_{(.)}$, (.) = i,j; i,j$\in${-,1,2,…, s}, i $\le$ j,
and the value of the light speed, hence its numerical value as well, is also invariant with respect to inertial spatial co-ordinate systems:
$c_{(.)} \equiv c$, (.) = i,j; i,j$\in${-,1,2,…, s}, i $\le$ j.

Constraint 5. In the proofs of the Lorentz transformations it is accepted that the position of the arbitrary point P is the position of a light signal: $\mathbf{r}_P(t_{(.)};t_{(.)0}) \equiv r_L(t_{(.)};t_{(.)0})\mathbf{u} \equiv c(t_{(.)} - t_{(.)0})\mathbf{u}$, $r_L(t_{(.)};t_{(.)0}) = \rho_L(t_{(.)};t_{(.)0}) \in R_+$, $\forall t_{(.)} \in T_{(.)}$, $(.) = i,j$; $i,j \in \{-,1,2,\ldots,s\}$, $i \leq j$.

Constraint 6. The matrices A and B in G, (6), are equal, A=B → G = blockdiag{A -A}.

In Einstein's special relativity they are the identity matrix I, A = B = I.

These constraints will be omitted a priori in the sequel.

7. CASE OF THE PARTIALLY COMPATIBLE BUT CONSISTENT (PCC) RELATIVITY THEORY

7.1. Problem statement

We will distinguish two essentially different cases with respect to the mutual relationship among the scaling coefficients, which are constant in either case:

General case: $\alpha_i^j \neq \alpha_j^i$ and/or $\lambda_i^j \neq \lambda_j^i$.

Special case: $\alpha_i^j = \alpha_j^i = \alpha_{ji} = \alpha_{ij}$ and

$\lambda_i^j = \lambda_j^i = \lambda_{ij} = \lambda_{ji}$.

A particular sub-case of the special case concerns the invariance of the light speed and of the transfer speed relative to a choice of a *time* axis. Besides, $\alpha_{ji} = \alpha_{ij} = \alpha$ and $\lambda_{ij} = \lambda_{ji} = \lambda$. It is the ***singular case***.

Problem statement. *Determine the values of the scaling coefficients* α_j^i, α_i^j, λ_j^i, λ_i^j, *and* μ_i, *(1) – (5), for the movement of the arbitrary point P with an arbitrary velocity, and for an arbitrarily chosen and then fixed spatial reference point* P_{SU} *moving with a known constant velocity* $\mathbf{v}_{SU}^{(.)} = v_{SU}^{(.)}\mathbf{u}$, *so that they are positive real numbers, that the equations (2) through (5) hold, and that they together with (1) imply the identity (6). Verify compatibility of the resulting co-ordinate transformations.*

7.2 Problem solutions for the general case

We will allow for the (numerical) value of any speed, hence of the light speed, to depend on an integral space with respect to which it is determined.

Theorem 1. Let the *time* scaling coefficients μ_i be defined by (1). Let the spatial reference point P_{SU} be arbitrarily chosen and then fixed moving with a known constant velocity $\mathbf{v}_{SU}^{(.)} = v_{SU}^{(.)}\mathbf{u}$, $v_{SU}^{(.)} \in R^+$. If the speed of the arbitrary point P is arbitrary, then, in order for the scaling coefficients $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j \neq \alpha_j^i$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j \neq \lambda_j^i$, determined for the speed $v_{SU}^{(.)}$ of P_{SU}, to be constant and to obey the equations (2) through (5), and for (1) through (5) to imply (6), it is necessary and sufficient that the following equations hold for any choice of the *time* scaling coefficients $\mu_i \in R^+$ and $\mu_j \in R^+$**:**

$$\alpha_j^i = \frac{\mu_i}{\mu_j}\frac{1}{1+\dfrac{v_{ji}^j v_{SU}^j}{q^j\omega^j}} = \frac{v_{SU}^j}{v_{SU}^i}\frac{1}{1+\dfrac{v_{ji}^j v_{SU}^j}{q^j\omega^j}},$$

$$i,j \in \{-, 1, 2, \ldots, s\},\ i \leq j, \tag{7}$$

$$\alpha_i^j = \frac{\mu_j}{\mu_i}\frac{1}{1-\dfrac{v_{ji}^i v_{SU}^i}{q^i\omega^i}} = \frac{v_{SU}^i}{v_{SU}^j}\frac{1}{1-\dfrac{v_{ji}^i v_{SU}^i}{q^i\omega^i}},$$

$$i,j \in \{-, 1, 2, \ldots, s\},\ i \leq j, \tag{8}$$

$$\lambda_j^i = \frac{1}{1+\dfrac{v_{ji}^j}{v_{SU}^j}},\ i,j \in \{-,1,2,\ldots,s\},\ i \leq j, \tag{9}$$

$$\lambda_i^j = \frac{1}{1-\dfrac{v_{ji}^i}{v_{SU}^i}},\ i,j \in \{-,1,2,\ldots,s\},\ i \leq j, \tag{10}$$

$$0 \leq v_{ji}^{(.)} < min\left\{v_{SU}^{(.)}, \frac{q^{(.)}\omega^{(.)}}{v_{SU}^{(.)}}\right\},\ (.) \in \{i,j\},$$

$$i,j \in \{-,1,2,\ldots,s\},\ i \leq j, \tag{11}$$

and

$$\frac{\mu_j}{\mu_i} = \frac{v_{SU}^i}{v_{SU}^j} = \frac{c_i^i}{c_j^j},\ v_{ji}^{(.)}, v_P^{(.)} \in R^+ \Rightarrow \frac{\mu_j}{\mu_i} = \frac{v_{SU}^i}{v_{SU}^j} = \frac{c_i^i}{c_j^j} =$$

$$= \frac{v_{ji}^i}{v_{ji}^j} = \frac{v_P^i}{v_P^j},\ i,j \in \{-,1,2,\ldots,s\},\ i \leq j. \tag{12}$$

The equations (7) – (10) give the next form to the equations (2) – (5):

$$t_i - t_{i0} = \frac{\mu_i}{\mu_j}\frac{(t_j - t_{j0}) + \dfrac{v_{ji}^j}{q^j\omega^j} r_P(t_j;t_{j0})}{1+\dfrac{v_{ji}^j v_{SU}^j}{q^j\omega^j}}, \tag{13}$$

$$t_j - t_{j0} = \frac{\mu_j}{\mu_i}\frac{(t_i - t_{i0}) - \dfrac{v_{ji}^i}{q^i\omega^i} r_P(t_i;t_{i0})}{1-\dfrac{v_{ji}^i v_{SU}^i}{q^i\omega^i}}, \tag{14}$$

$$\mathbf{r}_P(\mathrm{t_i};\mathrm{t_{i0}}) = \frac{\mathbf{r}_P(\mathrm{t_j};\mathrm{t_{j0}}) + \mathbf{v}_{ji}^{j}(\mathrm{t_j} - \mathrm{t_{j0}})}{1 + \frac{v_{ji}^{j}}{v_{SU}^{j}}}, \tag{15}$$

$$\mathbf{r}_P(\mathrm{t_j};\mathrm{t_{j0}}) = \frac{\mathbf{r}_P(\mathrm{t_i};\mathrm{t_{i0}}) - \mathbf{v}_{ji}^{i}(\mathrm{t_i} - \mathrm{t_{i0}})}{1 - \frac{v_{ji}^{i}}{v_{SU}^{i}}}, \tag{16}$$

The transformations (13) through (16) are partially both entirely and pairwise compatible.

Proof. *Necessity*. Let the spatial reference point P_{SU} be freely chosen and fixed. Let it move with a known constant speed $v_{SU}^{(.)} \in R^+$. Let the arbitrary point P be P_{SU}, $v_P^{(.)} \equiv v_{SU}^{(.)}$. Let the *time* scaling coefficients $\mu_i \in R^+$, $i = 1, 2, \ldots, s$, be defined by (1). Let the coefficients $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j \neq \alpha_j^i$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j \neq \lambda_j^i$, be constant and obey (2) through (5), and let (1) through (5) imply (6). Since $r_P(\mathrm{t_j};\mathrm{t_{j0}}) \equiv \equiv r_{SU}(\mathrm{t_j};\mathrm{t_{j0}})$ represents the position of the arbitrary point P = P_{SU} measured relative to the integral space $I_j = T_j \times R_j^n$, then $r_P(\mathrm{t_j};\mathrm{t_{j0}}) = v_{SU}^{O_j}(t_j - t_{j0})$, j= -, 1, 2, … , s. This, (1) and (2) yield:

$$\mathrm{t_i} - \mathrm{t_{i0}} = \mu_i(\mathrm{t} - \mathrm{t_0}) = \alpha_\mathrm{j}^\mathrm{i}\left(1 + \frac{v_{ji}^{j} v_{SU}^{O_j}}{q^j \omega^j}\right)(\mathrm{t_j} - \mathrm{t_{j0}}) =$$

$$= \alpha_\mathrm{j}^\mathrm{i}\left(1 + \frac{v_{ji}^{j} v_{SU}^{O_j}}{q^j \omega^j}\right)\mu_j(\mathrm{t} - \mathrm{t_0}),$$

so that:

$$\alpha_\mathrm{j}^\mathrm{i} = \frac{\mu_i}{\mu_j}\left[1 + \frac{v_{ji}^{j} v_{SU}^{O_j}}{q^j \omega^j}\right]^{-1}. \tag{17}$$

This implies the first equation in (7). The first equation in (8) is analogously proved by combining $r_P(\mathrm{t_i};\mathrm{t_{i0}}) = v_{SU}^{i}(\mathrm{t_i} - \mathrm{t_{i0}})$ with (1) and (3). From (1) and from (4) we deduce the following:

$$\mathbf{r}_P(\mathrm{t_i};\mathrm{t_{i0}}) = v_{\mathrm{SU}}^{\mathrm{O_i}}\mu_i(\mathrm{t} - \mathrm{t_0})\mathbf{u} = \lambda_\mathrm{j}^\mathrm{i}(v_{\mathrm{SU}}^{\mathrm{O_j}} + v_{ji}^{j})\mu_j(\mathrm{t} - \mathrm{t_0})\mathbf{u},$$

$$\lambda_\mathrm{j}^\mathrm{i} = \frac{\mu_i v_{\mathrm{SU}}^{\mathrm{O_i}}}{\mu_j v_{\mathrm{SU}}^{\mathrm{O_j}}}\left(1 + \frac{v_{ji}^{j}}{v_{\mathrm{SU}}^{\mathrm{O_j}}}\right)^{-1}. \tag{18}$$

We transform the right hand side of (6) as follows by using (1) and $r_P(\mathrm{t_k};\mathrm{t_{k0}}) = v_{SU}^{O_k}(t_k - t_{k0})$, k= 1, 2, …, s:

$$\begin{bmatrix} \mathbf{r}_P(\mathrm{t_i};\mathrm{t_{i0}}) \\ (\mathrm{t_i} - \mathrm{t_{i0}})\mathbf{v}_\mathrm{P}^{\mathrm{O_i}} \end{bmatrix}^\mathrm{T} G \begin{bmatrix} \mathbf{r}_P(\mathrm{t_i};\mathrm{t_{i0}}) \\ (\mathrm{t_i} - \mathrm{t_{i0}})\mathbf{v}_\mathrm{P}^{\mathrm{O_i}} \end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix} \mathbf{r}_{SU}(\mathrm{t_i};\mathrm{t_{i0}}) \\ (\mathrm{t_i} - \mathrm{t_{i0}})\mathbf{v}_\mathrm{SU}^{\mathrm{O_i}} \end{bmatrix}^\mathrm{T} G \begin{bmatrix} \mathbf{r}_{SU}(\mathrm{t_i};\mathrm{t_{i0}}) \\ (\mathrm{t_i} - \mathrm{t_{i0}})\mathbf{v}_\mathrm{SU}^{\mathrm{O_i}} \end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix} v_{SU}^{O_j}(v_{SU}^{O_j})^{-1}\mu_\mathrm{j}^{-1}\mu_\mathrm{j}\mu_\mathrm{i}(\mathrm{t} - \mathrm{t_0})v_\mathrm{SU}^{\mathrm{O_i}}\mathbf{u} \\ v_{SU}^{O_j}(v_{SU}^{O_j})^{-1}\mu_\mathrm{j}^{-1}\mu_\mathrm{j}\mu_\mathrm{i}(\mathrm{t} - \mathrm{t_0})v_\mathrm{SU}^{\mathrm{O_i}}\mathbf{u} \end{bmatrix}^\mathrm{T} G \bullet$$

$$\bullet \begin{bmatrix} v_{SU}^{O_j}(v_{SU}^{O_j})^{-1}\mu_\mathrm{j}^{-1}\mu_\mathrm{j}\mu_\mathrm{i}(\mathrm{t} - \mathrm{t_0})v_\mathrm{SU}^{\mathrm{O_i}}\mathbf{u} \\ v_{SU}^{O_j}(v_{SU}^{O_j})^{-1}\mu_\mathrm{j}^{-1}\mu_\mathrm{j}\mu_\mathrm{i}(\mathrm{t} - \mathrm{t_0})v_\mathrm{SU}^{\mathrm{O_i}}\mathbf{u} \end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix} v_{SU}^{O_i}(v_{SU}^{O_j})^{-1}\mu_\mathrm{i}\mu_\mathrm{j}^{-1}\mathbf{r}_P(\mathrm{t_j};\mathrm{t_{j0}}) \\ v_{SU}^{O_i}(v_{SU}^{O_j})^{-1}\mu_\mathrm{i}\mu_\mathrm{j}^{-1}(\mathrm{t_j} - \mathrm{t_{j0}})\mathbf{v}_{SU}^{O_j} \end{bmatrix}^\mathrm{T} G \bullet$$

$$\bullet \begin{bmatrix} v_{SU}^{O_i}(v_{SU}^{O_j})^{-1}\mu_\mathrm{i}\mu_\mathrm{j}^{-1}\mathbf{r}_P(\mathrm{t_j};\mathrm{t_{j0}}) \\ v_{SU}^{O_i}(v_{SU}^{O_j})^{-1}\mu_\mathrm{i}\mu_\mathrm{j}^{-1}(\mathrm{t_j} - \mathrm{t_{j0}})\mathbf{v}_{SU}^{O_j} \end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix} \mathbf{r}_{SU}(\mathrm{t_j};\mathrm{t_{j0}}) \\ (\mathrm{t_j} - \mathrm{t_{j0}})\mathbf{v}_{SU}^{O_j} \end{bmatrix}^\mathrm{T} G \begin{bmatrix} \mathbf{r}_{SU}(\mathrm{t_j};\mathrm{t_{j0}}) \\ (\mathrm{t_j} - \mathrm{t_{j0}})\mathbf{v}_{SU}^{O_j} \end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix} \mathbf{r}_P(\mathrm{t_j};\mathrm{t_{j0}}) \\ (\mathrm{t_j} - \mathrm{t_{j0}})\mathbf{v}_P^{O_j} \end{bmatrix}^\mathrm{T} G \begin{bmatrix} \mathbf{r}_P(\mathrm{t_j};\mathrm{t_{j0}}) \\ (\mathrm{t_j} - \mathrm{t_{j0}})\mathbf{v}_P^{O_j} \end{bmatrix}.$$

These identities imply:

$$\left(\mu_i v_{SU}^{O_i}\right)\left(\mu_j v_{SU}^{O_j}\right)^{-1} = 1. \tag{19}$$

This proves the first equality in (12). Since the point P_{SU} is arbitrary, then it can represent the light signal so that then $v_{SU}^{O_{(.)}} = c_{(.)}^{(.)}$, (.) = i, j, which implies the second equation in (12) in view of the preceding result. For the same reason, the third and the fourth equation in (12) hold as soon as $v_P^{O_{(.)}} \in R^+$ and $v_{ji}^{(.)} \in R^+$. The equations (18) and (19) yield:

$$\lambda_\mathrm{j}^\mathrm{i} = \left(1 + \frac{v_{ji}^{j}}{v_{\mathrm{SU}}^{\mathrm{O_j}}}\right)^{-1}.$$

This proves the equation (9). The equation (10) is proved in the same manner. The equations (7) through (10), real values of the scaling coefficients, and the definition of $v_{ji}^{(.)}$ imply the inequalities in (11). The first equations in (7) and (8) together with (12) result, respectively, in the second equations in (7) and (8). The equations (7) through (10) transform (2) through (5) into (13) through (16).

Sufficiency. Let the spatial reference point P_{SU} be freely chosen and fixed. Let it move with a known constant speed $v_{SU}^{(.)} \in R^+$. Let the arbitrary point P be P_{SU},

$v_P^{(.)} \equiv v_{SU}^{(.)}$. Let *time* scaling coefficients $\mu_k \in R^+$, k = 1, 2, ... , s, be defined by (1). Let (7) – (12) and all the conditions of the theorem statement hold. We start with (1), (7) and (12):

$$t_i - t_{i0} = \mu_i (t - t_0)\alpha_j^i (\alpha_j^i)^{-1} =$$

$$= \mu_i (t - t_0)\alpha_j^i \frac{\mu_j}{\mu_i}\left(1 + \frac{v_{ji}^j v_{SU}^j}{q^j \omega^j}\right),$$

or, by using (1) for k = j, and $r_P(t_j; t_{j0}) = = v_{SU}^j (t_j - t_{j0})$:

$$t_i - t_{i0} = \alpha_j^i \left(1 + \frac{v_{ji}^j v_{SU}^j}{q^j \omega^j}\right)(t_j - t_{j0}) =$$

$$= \alpha_j^i \left[(t_j - t_{j0}) + \frac{v_{ji}^j}{q^j \omega^j} r_P(t_j; t_{j0})\right].$$

This proves the validity of (2). The equation (3) is proved analogously by starting with $t_j - t_{j0}$. We transform $\mathbf{r}_P(t_i; t_{i0}) = v_P^{O_i}(t_i - t_{i0})\mathbf{u} = v_{SU}^{O_i}(t_i - t_{i0})\mathbf{u}$ by using $r_P(t_i; t_{i0}) = v_P^{O_i}(t_i - t_{i0}) = v_{SU}^{O_i}(t_i - t_{i0})$, (1), (9) and (12):

$$\mathbf{r}_P(t_i; t_{i0}) = v_P^{O_i}(t_i - t_{i0})\mathbf{u} = v_P^{O_i}\lambda_j^i (\lambda_j^i)^{-1}(t_i - t_{i0})\mathbf{u} =$$

$$= v_P^{O_i}\lambda_j^i \left(1 + \frac{v_{ji}^j}{v_{SU}^{O_j}}\right)(t_i - t_{i0})\mathbf{u} = v_P^{O_i}\lambda_j^i \left(1 + \frac{v_{ji}^j}{v_P^{O_j}}\right)(t_i - t_{i0})\mathbf{u} =$$

$$= v_P^{O_i}\lambda_j^i \left(1 + \frac{v_{ji}^j}{v_P^{O_j}}\right)\frac{\mu_i}{\mu_j}(t_j - t_{j0})\mathbf{u} =$$

$$= v_P^{O_j}\lambda_j^i \left((t_j - t_{j0}) + \frac{v_{ji}^j}{v_P^{O_j}}(t_j - t_{j0})\right)\mathbf{u} =$$

$$= \lambda_j^i \left(r_P(t_j; t_{j0}) + v_{ji}^j (t_j - t_{j0})\right)\mathbf{u} =$$

$$= \lambda_j^i \left(\mathbf{r}_P(t_j; t_{j0}) + v_{ji}^j (t_j - t_{j0})\mathbf{u}\right).$$

This proves (4). The equation (5) is analogously proved by starting with

$$\mathbf{r}_P(t_j; t_{j0}) = v_P^{O_j}(t_j - t_{j0})\mathbf{u} = = v_{SU}^{O_j}(t_j - t_{j0})\mathbf{u}.$$

Let us now transform the left-hand side of (6) as follows by using

$$\mathbf{r}_P(t_i; t_{i0}) = v_P^{O_i}(t_i - t_{i0})\mathbf{u} = v_{SU}^{O_i}(t_i - t_{i0})\mathbf{u}.,$$

$$\mathbf{v}_P^{O_i} = v_P^{O_i}\mathbf{u} = \mathbf{v}_{SU}^{O_i} = v_{SU}^{O_i}\mathbf{u},$$

in view of the constancy of the velocity $\mathbf{v}_{SU}^{O_i}$, and (1):

$$\begin{bmatrix} \mathbf{r}_P(t_i; t_{i0}) \\ (t_i - t_{i0})\mathbf{v}_P^{O_i} \end{bmatrix}^T G \begin{bmatrix} \mathbf{r}_P(t_i; t_{i0}) \\ (t_i - t_{i0})\mathbf{v}_P^{O_i} \end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix} \left[\mathbf{r}_P(t_j; t_{j0}) + \mathbf{v}_{ji}^j (t_j - t_{j0})\right]\left(1 + \frac{v_{ji}^j}{v_{SU}^j}\right)^{-1} \\ \frac{\mu_i}{\mu_j} \frac{(t_j - t_{j0}) + \frac{v_{ji}^j}{q^j \omega^j} r_P(t_j; t_{j0})}{1 + \frac{v_{ji}^j v_{SU}^j}{q^j \omega^j}} v_P^{O_i}\mathbf{u} \end{bmatrix}^T G \bullet$$

$$\bullet \begin{bmatrix} \left[\mathbf{r}_P(t_j; t_{j0}) + \mathbf{v}_{ji}^j (t_j - t_{j0})\right]\left(1 + \frac{v_{ji}^j}{v_{SU}^j}\right)^{-1} \\ \frac{\mu_i}{\mu_j} \frac{(t_j - t_{j0}) + \frac{v_{ji}^j}{q^j \omega^j} r_P(t_j; t_{j0})}{1 + \frac{v_{ji}^j v_{SU}^j}{q^j \omega^j}} v_P^{O_i}\mathbf{u} \end{bmatrix} \equiv$$

This, $\mathbf{r}_P(t_j; t_{j0}) = v_P^{O_j}(t_j - t_{j0})\mathbf{u}$, $\mathbf{v}_P^{O_j} = v_P^{O_j}\mathbf{u} = = \mathbf{v}_{SU}^{O_i} = v_{SU}^{O_i}\mathbf{u}$, (1) and (12) imply:

$$\begin{bmatrix} \mathbf{r}_P(t_i; t_{i0}) \\ (t_i - t_{i0})\mathbf{v}_P^{O_i} \end{bmatrix}^T G \begin{bmatrix} \mathbf{r}_P(t_i; t_{i0}) \\ (t_i - t_{i0})\mathbf{v}_P^{O_i} \end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix} \left(1 + \frac{v_{ji}^j}{v_{SU}^j}\right)\left(1 + \frac{v_{ji}^j}{v_{SU}^j}\right)^{-1} \mathbf{r}_P(t_j; t_{j0}) \\ \frac{1 + \frac{v_{ji}^j}{q^j \omega^j} v_{SU}^j}{1 + \frac{v_{ji}^j v_{SU}^j}{q^j \omega^j}}(t_j - t_{j0}) v_P^{Oj}\mathbf{u} \end{bmatrix}^T G \bullet$$

$$\bullet \begin{bmatrix} \left(1 + \frac{v_{ji}^j}{v_{SU}^j}\right)\left(1 + \frac{v_{ji}^j}{v_{SU}^j}\right)^{-1} \mathbf{r}_P(t_j; t_{j0}) \\ \frac{1 + \frac{v_{ji}^j}{q^j \omega^j} v_{SU}^j}{1 + \frac{v_{ji}^j v_{SU}^j}{q^j \omega^j}}(t_j - t_{j0}) v_P^{Oj}\mathbf{u} \end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix} \mathbf{r}_P(t_j; t_{j0}) \\ (t_j - t_{j0})\mathbf{v}_P^{O_j} \end{bmatrix}^T G \begin{bmatrix} \mathbf{r}_P(t_j; t_{j0})\mathbf{r}_0 \\ (t_j - t_{j0})\mathbf{v}_P^{O_j} \end{bmatrix}.$$

This proves (6) and completes the proof of sufficiency.

Compatibility. We verify compatibility of the transformations (13) through (16) as follows. We allow arbitrary speed $v_P^{(.)} \in R^+$ of P. We replace $(t_j - t_{j0})$ from (14) into (13):

$$t_i - t_{i0} = \frac{\mu_i}{\mu_j}\left[\frac{\mu_j}{\mu_i}\frac{(t_i - t_{i0}) - \dfrac{v_{ji}^i}{q^i\omega^i} r_P(t_i;t_{i0})}{1-\dfrac{v_{ji}^i v_{SU}^i}{q^i\omega^i}} + \frac{v_{ji}^j}{q^j\omega^j} r_P(t_j;t_{j0})\right]\bullet$$

$$\bullet\left(1+\frac{v_{ji}^j v_{SU}^j}{q^j\omega^j}\right)^{-1},$$

and we use $r_P(t_{(.)};t_{(.)0}) = v_P^{(.)}(t_{(.)} - t_{(.)0})$ for (.) = i, j, and (1),

$$t_i - t_{i0} \equiv \frac{\mu_i}{\mu_j}\left[\begin{array}{l}\dfrac{\mu_j}{\mu_i}\left(1-\dfrac{v_{ji}^i v_P^i}{q^i\omega^i}\right)(t_i - t_{i0})\left(1-\dfrac{v_{ji}^i v_{SU}^i}{q^i\omega^i}\right)^{-1} + \\ + \dfrac{v_{ji}^j v_P^j}{q^j\omega^j}(t_j - t_{j0})\end{array}\right]\bullet$$

$$\bullet\left(1+\frac{v_{ji}^j v_{SU}^j}{q^j\omega^j}\right)^{-1} \equiv$$

$$\equiv\left[\left(1-\frac{v_{ji}^i v_P^i}{q^i\omega^i}\right)\left(1-\frac{v_{ji}^i v_{SU}^i}{q^i\omega^i}\right)^{-1} + \frac{v_{ji}^j v_P^j}{q^j\omega^j}\right]\left(1+\frac{v_{ji}^j v_{SU}^j}{q^j\omega^j}\right)^{-1}\bullet$$

$$\bullet(t_i - t_{i0}).$$

For these identities to hold it is necessary and sufficient that $v_P^{(.)} \equiv v_{SU}^{(.)}$. Hence, (13) and (14) are only partially compatible. In order to verify compatibility of the spatial co-ordinate transformations we replace at first $\mathbf{r}_P(t_j;t_{j0})$ by the right-hand side of (16) into (15), we use $\mathbf{r}_P(t_i;t_{i0}) = \mathbf{v}_P^{O_i}(t_i - t_{i0})$ and afterwards we apply (1) and (12):

$$\mathbf{r}_P(t_i;t_{i0}) \equiv \frac{\dfrac{\mathbf{r}_P(t_i;t_{i0}) - v_{ji}^i(t_i - t_{i0})\mathbf{u}}{1-\dfrac{v_{ji}^i}{v_{SU}^{O_i}}} + v_{ji}^j(t_j - t_{j0})\mathbf{u}}{1+\dfrac{v_{ji}^j}{v_{SU}^{O_j}}} \equiv$$

$$\equiv \frac{\dfrac{\left(1-\dfrac{v_{ji}^i}{v_P^{O_i}}\right)\mathbf{r}_P(t_i;t_{i0})}{1-\dfrac{v_{ji}^i}{v_{SU}^{O_i}}} + v_{ji}^j\dfrac{\mu_j}{\mu_i}\mu_i(t - t_0)\mathbf{u}}{1+\dfrac{v_{ji}^j}{v_{SU}^{O_j}}} \equiv$$

$$\equiv \frac{\dfrac{1-\dfrac{v_{ji}^i}{v_P^{O_i}}}{1-\dfrac{v_{ji}^i}{v_{SU}^{O_i}}} + \dfrac{v_{ji}^i}{v_P^{O_i}}}{1+\dfrac{v_{ji}^j}{v_{SU}^{O_j}}}\mathbf{r}_P(t_i;t_{i0}).$$

For these identities to hold it is necessary and sufficient that $v_P^{(.)} \equiv v_{SU}^{(.)}$. Hence, (15) and (16) are only partially compatible. Altogether, the transformations (13) through (16) are partially pairwise compatible. We will test their complete entire compatibility as follows. We replace, respectively, $(t_j - t_{j0})$ and $\mathbf{r}_P(t_j;t_{j0})$ from (13) and (16) into (15):

$$\mathbf{r}_P(t_i;t_{i0}) \equiv \left(1+\frac{v_{ji}^j}{v_{SU}^{O_j}}\right)^{-1}\bullet$$

$$\bullet\left[\mathbf{r}_P(t_j;t_{j0}) + v_{ji}^j\frac{\mu_j}{\mu_i}\frac{(t_i - t_{i0}) - \dfrac{v_{ji}^i}{q^i\omega^i}r_P(t_i;t_{i0})}{1-\dfrac{v_{ji}^i v_{SU}^i}{q^i\omega^i}}\mathbf{u}\right] \equiv$$

$$\equiv\left(1+\frac{v_{ji}^j}{v_{SU}^{O_j}}\right)^{-1}\bullet$$

$$\bullet\left[\mathbf{r}_P(t_j;t_{j0}) + v_{ji}^j\frac{\mu_j}{\mu_i}\frac{1-\dfrac{v_{ji}^i v_P^i}{q^i\omega^i}}{1-\dfrac{v_{ji}^i v_{SU}^i}{q^i\omega^i}}(t_i - t_{i0})\mathbf{u}\right] \equiv$$

$$\equiv \left(1+\frac{v_{ji}^{j}}{v_{\mathrm{SU}}^{\mathrm{O_j}}}\right)^{-1} \bullet$$

$$\bullet \left[\frac{\mathbf{r}_P(\mathrm{t_i};\mathrm{t_{i0}}) - v_{ji}^{i}(\mathrm{t_i} - \mathrm{t_{i0}})\mathbf{u}}{1-\frac{v_{ji}^{i}}{v_{\mathrm{SU}}^{\mathrm{O_i}}}} + v_{ji}^{j}\frac{\mu_j}{\mu_\mathrm{i}}(\mathrm{t_i} - \mathrm{t_{i0}})\mathbf{u}\right] \equiv$$

$$\equiv \left(1+\frac{v_{ji}^{j}}{v_{\mathrm{SU}}^{\mathrm{O_j}}}\right)^{-1}\left(\frac{1-\frac{v_{ji}^{i}}{\mathrm{v_P^{O_i}}}}{1-\frac{v_{ji}^{i}}{\mathrm{v_{SU}^{O_i}}}} + \frac{\mu_j}{\mu_i}\frac{v_{ji}^{j}}{v_\mathrm{P}^{\mathrm{O_i}}}\right)\mathbf{r}_i(\mathrm{t_i};\mathrm{t_{i0}}) \equiv$$

$$\equiv \left(1+\frac{v_{ji}^{j}}{v_{\mathrm{SU}}^{\mathrm{O_j}}}\right)^{-1}\left(\frac{1-\frac{v_{ji}^{i}}{\mathrm{v_P^{O_i}}}}{1-\frac{v_{ji}^{i}}{\mathrm{v_{SU}^{O_i}}}} + \frac{v_{ji}^{j}}{v_\mathrm{P}^{\mathrm{O_j}}}\right)\mathbf{r}_i(\mathrm{t_i};\mathrm{t_{i0}}).$$

For these identities to be valid it is necessary and sufficient that $v_P^{(.)} \equiv v_{SU}^{(.)}$. Hence, (13), (15) and (16) are only partially entirely compatible. We prove in the same way partial entire compatibility among (14), through (16). Altogether, the transformations (13) through (16) are partially entirely compatible. Q. E. D.

The equations (13) through (16) are beyond the Lorentz - Einstein relativity theory. The former are rational functions of $v_{ji}^{i}/q^i\omega^i$, $v_{ji}^{j}/q^j\omega^j$, $v_\mathrm{ji}^\mathrm{i}/\mathrm{v_P^{O_i}}$ or $v_\mathrm{ji}^\mathrm{j}/\mathrm{v_P^{O_j}}$, while the latter are not. The former do not contain square roots or squared speeds, while the latter do both. The former contain the relative values of all speeds, but not the latter. The former are consistent, but the latter are inconsistent. The former are partially pairwise and entirely compatible, while the latter are completely entirely compatible. The former do not contain the light speed in general, while the latter do necessarily. Therefore, the former show that the light speed is not a necessary generic speed, while the latter demand that exclusively.

The preceding theorem results from the characterisation of *time* based on the physical reality and our experience (Axiom 1). The *time* independence is expressed by different scaling factors in equations (2) through (5), and by A ≠ B in G, (6). The obtained scaling coefficients, (7) through (10), differ essentially from those by Lorentz. The above result is general.

The special case will be examined in the sequel.

The preceding theorem takes the following form in the case the reference frames R_i^n and R_j^n move in parallel with the same velocity: $\mathbf{v}_\mathrm{ji}^\mathrm{i} = \mathbf{v}_\mathrm{ji}^\mathrm{j} = \mathbf{0}$. This is important for dynamical systems (S1), (S2).

Corollary 1. If the reference frames R_i^n and R_j^n move with the same velocity in the same direction and sense, and if the speed of the arbitrary point P is arbitrary, then, in order for the scaling coefficients α_i^j, α_j^i, $\alpha_i^j \neq \alpha_j^i$, λ_i^j and λ_j^i, to be constant and to obey the equations (2) through (5), and for (1) through (5) to imply (6), it is necessary and sufficient that the following equations hold for any choice of $\mu_\mathrm{i} \in R^+$ and $\mu_j \in R^+$**:**

$$\alpha_j^i = \frac{\mu_i}{\mu_j} = \frac{v_{SU}^j}{v_{SU}^i} = \frac{c_j^j}{c_i^i}, \qquad \alpha_i^j = \frac{\mu_j}{\mu_i} = \frac{v_{SU}^i}{v_{SU}^j} = \frac{c_i^i}{c_j^j},$$

$$\lambda_j^i = \lambda_i^j = 1,\ i, j = 1,2,..., \mathrm{n}.$$

The coordinate transformations (2) through (5) reduce to

$$t_i - t_{i0} = \frac{\mu_i}{\mu_j}\left(t_j - t_{j0}\right), \mathbf{r}_\mathrm{P}\left(t_i;t_{i0}\right) = \mathbf{r}_\mathrm{P}\left(t_j;t_{j0}\right).$$

The (numerical) value of the light speed need not be the same with respect to $I_i = T_i \times R_i^n$ and $I_\mathrm{j} = T_\mathrm{j} \times R_\mathrm{j}^n$ as soon as the *time* scales of T_i and T_j are different: $\mu_i \neq \mu_j$.

The space scaling factors $\lambda_i^j = \lambda_j^i = 1$ because R_i^n and R_j^n move with the same speed.

The preceding corollary is significant for dynamical systems (S1), (S2) with multiple *time* scales. It links the relativity theory with the theory of these systems. It establishes the conditions under which the dynamical system (S1), (S2) can have different *time* scales.

7.3 *Problem solution for the special case*

In the case the relative values c_i^i and c_j^j of the light speed are mutually equal, $c_i^i = c_j^j$, then they are denoted by c^{ij} or by $c^{ji}, c_i^i = c_j^j = c^{ij} = c^{ji}$. This designates the same relative value of the light speed with respect to I_i and I_j. Then, v_ji denotes that v_ji^i and v_ji^j are mutually equal:

$$v_{ji}^i = v_{ji}^j = v_{ji} = -v_{ij} = -v_{ij}^i = -v_{ij}^j.$$

Analogously,

$$v_{SU}^{i} = v_{SU}^{j} \Rightarrow v_{SU}^{i} = v_{SU}^{j} = v_{SU}^{ij} = v_{SU}^{ji},$$
$$q^{i}\omega^{i} = q^{j}\omega^{j} \Rightarrow q^{i}\omega^{i} = q^{j}\omega^{j} = (q\omega)^{ij} = (q\omega)^{ji}.$$

Theorem 2. Let the *time* scaling coefficients $\mu_i \in R^+$, i = 1, 2, ... , s, be defined by (1). Let the spatial reference point P_{SU} be arbitrarily chosen and then fixed moving with a known constant velocity $\mathbf{v}_{SU}^{(.)} = v_{SU}^{(.)}\mathbf{u}$, $v_{SU}^{(.)} \in R^+$. If the speed of the arbitrary point P is arbitrary, then, in order for the scaling coefficients $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j = \alpha_j^i = \alpha_{ij} = \alpha_{ji}$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j = \lambda_j^i = \lambda_{ij} = \lambda_{ji}$, determined for the speed $v_{SU}^{(.)}$ of P_{SU}, to be positive real numbers and to obey (2) through (5), and for (1) through (5) to imply (6) it is necessary (but not sufficient) that the following equations hold for any choice of the *time* scaling coefficient $\mu_i \in R^+$:

$$v_{SU}^{O_{(.)}} = \sqrt{q^{(.)}\omega^{(.)}},\; i,j \in \{-, 1, 2, \ldots, s\},\; i \le j, \tag{20}$$

$$\frac{v_{ji}^{i}}{v_{ji}^{j}} \equiv \frac{v_P^{i}}{v_P^{j}} \equiv \frac{c_i^{i}}{c_j^{j}},\quad i,j \in \{-, 1, 2, \ldots, s\},\; i \le j, \tag{21}$$

$$\alpha_{ij} = \frac{1}{\sqrt{1-\left(\frac{v_{ji}^{i}}{\sqrt{q^{i}\omega^{i}}}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{v_{ji}^{j}}{\sqrt{q^{j}\omega^{j}}}\right)^2}},$$
$$i,j \in \{-, 1, 2, \ldots, s\},\; i \le j, \tag{22}$$

$$\lambda_{ij} = \frac{1}{\sqrt{1-\left(\frac{v_{ji}^{i}}{v_{SU}^{i}}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{v_{ji}^{j}}{v_{SU}^{j}}\right)^2}} = \alpha_{ij},$$
$$i,j \in \{-, 1, 2, \ldots, s\},\; i \le j, \tag{23}$$

$$\frac{\mathbf{u}^T A\mathbf{u}}{\mathbf{u}^T B\mathbf{u}} = \left(\frac{v_{ji}^{i}}{v_{ji}^{j}}\right)^2 = \left(\frac{v_P^{i}}{v_P^{j}}\right)^2 = \left(\frac{c_i^{i}}{c_j^{j}}\right)^2,$$
$$i,j \in \{-, 1, 2, \ldots, s\},\; i \le j, \tag{24}$$

$$v_{ji}^{(.)} < v_{SU}^{(.)},\; (.) \in \{i,j\},\; i,j \in \{-, 1, 2, \ldots, s\},\; i \le j, \tag{25}$$

$$\mu_j = \mu_i\sqrt{\frac{1-\frac{v_{ji}^{j}}{v_{SU}^{j}}}{1+\frac{v_{ji}^{j}}{v_{SU}^{j}}}} = \mu_i\sqrt{\frac{1-\frac{v_{ji}^{i}}{v_{SU}^{i}}}{1+\frac{v_{ji}^{i}}{v_{SU}^{i}}}},$$
$$i,j \in \{-, 1, 2, \ldots, s\},\; i \le j. \tag{26}$$

The equations (2) through (5) become the equations (27) through (30),

$$t_i - t_{i0} = \frac{t_j + \frac{v_{ji}^{j}}{q^{j}\omega^{j}} r_P(t_j; t_{j0})}{\sqrt{1-\left(\frac{v_{ji}^{j}}{\sqrt{q^{j}\omega^{j}}}\right)^2}},$$
$$i,j \in \{-, 1, 2, \ldots, s\},\; i \le j, \tag{27}$$

$$t_j - t_{j0} = \frac{t_i - t_{i0} - \frac{v_{ji}^{i}}{q^{i}\omega^{i}} r_P(t_i; t_{i0})}{\sqrt{1-\left(\frac{v_{ji}^{i}}{\sqrt{q^{i}\omega^{i}}}\right)^2}},$$
$$i,j \in \{-, 1, 2, \ldots, s\},\; i \le j, \tag{28}$$

$$\mathbf{r}_P(t_i; t_{i0}) = \frac{\mathbf{r}_P(t_j; t_{j0}) + v_{ji}^{j}(t_j - t_{j0})\mathbf{u}}{\sqrt{1-\left(\frac{v_{ji}^{j}}{v_{SU}^{j}}\right)^2}},$$
$$i,j \in \{-, 1, 2, \ldots, s\},\; i \le j, \tag{29}$$

$$\mathbf{r}_P(t_j; t_{j0}) = \frac{\mathbf{r}_P(t_i; t_{i0}) - v_{ji}^{i}(t_i - t_{i0})\mathbf{u}}{\sqrt{1-\left(\frac{v_{ji}^{i}}{v_{SU}^{i}}\right)^2}},$$
$$i,j \in \{-, 1, 2, \ldots, s\},\; i \le j. \tag{30}$$

If additionally A = B in G, (6), then for the coefficients $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j = \alpha_j^i = \alpha_{ij} = \alpha_{ji}$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j = \lambda_j^i = \lambda_{ij} = \lambda_{ji}$, determined for the speed $v_{SU}^{(.)}$ of P_{SU}, to be positive real numbers and to obey (2) through (5), and for (1) through (5) to imply (6) it is necessary and sufficient that the equations (20) – (26) and the following equations hold for any choice of the *time* scaling coefficient $\mu_i \in R^+$:

$$v_{ji}^{i} = v_{ji}^{j} = v_{ji},$$
$$v_{SU}^{i} = v_{SU}^{j} = v_{SU}^{ij} = v_{SU}^{ji} =$$
$$= \sqrt{q^{i}\omega^{i}} = \sqrt{q^{j}\omega^{j}} = \sqrt{(q\omega)^{ij}} = \sqrt{(q\omega)^{ji}}$$
$$i,j \in \{-, 1, 2, \ldots, s\},\; i \le j, \tag{31}$$

$$c_i^{i} = c_j^{j} = c^{ij} = c^{ji},\; i,j \in \{-, 1, 2, \ldots, s\},\; i \le j, \tag{32}$$

The transformations (27) through (30) are partially both pairwise and entirely compatible.

Proof. The proof of this theorem is very long. It is therefore omitted due to the space limitation. Its methodology is the same as that of the proof of Theorem 2 in [21]. It is worked out in [25].

The equations (22), (23) for the scaling factors contain consistently the relative values of all the speeds, hence of the light speed. They generalise the Lorentz formulas for α and λ, which have been the fundamental of the Einsteinian relativity theory.

Notice that for A ≠ B in G, (6), the above theorem provides only the necessary, but not the sufficient, conditions.

Note 1. If A = B in G, (6), then the equations (27) and (28) can be set into the following forms:

$$t_i - t_{i0} = \frac{t_j + \dfrac{v_{ji}}{\left(v_{SU}^{ji}\right)^2} r_P(t_j; t_{j0})}{\sqrt{1-\left(\dfrac{v_{ji}}{v_{SU}^{ji}}\right)^2}},$$

$i,j \in \{-, 1, 2, \ldots, s\}$, $i \leq j$, (27b)

$$t_j - t_{j0} = \frac{t_i - t_{i0} + \dfrac{v_{ji}}{\left(v_{SU}^{ij}\right)^2} r_P(t_i; t_{i0})}{\sqrt{1-\left(\dfrac{v_{ji}}{v_{SU}^{ij}}\right)^2}},$$

$i,j \in \{-, 1, 2, \ldots, s\}$, $i \leq j$. (28b)

7.4 Problem solutions for the singular cases

In the singular cases the Constraint 1 through the Constraint 6 are valid. They give the following form to the Theorem 2:

Corollary 2. Let the Constraint 1 through the Constraint 3, and the Constraint 6 be valid. Let the *time* scaling coefficients $\mu_i \in R^+$, i = 1, 2, ... , s, be defined by (1). Let the spatial reference point P_{SU} be arbitrarily chosen and then fixed moving with a known constant velocity $\mathbf{v}_{SU}^{(.)} = v_{SU}^{(.)}\mathbf{u}$, $v_{SU}^{(.)} \in R^+$. In order for the coefficients $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j = \alpha_j^i = \alpha_{ij} = \alpha_{ji} = \alpha$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j = \lambda_j^i = \lambda_{ij} = \lambda_{ji} = \lambda$, determined for the speed $v_{SU}^{(.)}$ of P_{SU}, to be positive real numbers and to obey (2) through (5), and for (1) through (5) to imply (6) it is both necessary and sufficient that the equations (20) through (30) hold for any choice of the *time* scaling coefficient $\mu_i \in R^+$, for $v_{SU}^{ij} = v_{SU}^{ji} = v_{SU}$ and for $v_{ji} = -v_{ij} = v$. The equations (2) through (5) become the Non - Lorentz transformations (NL1) through (NL4):

$$\mathrm{t_i} - \mathrm{t_{i0}} = \frac{(\mathrm{t_j} - \mathrm{t_{j0}}) + \dfrac{v}{v_{SU}^2} r_P(\mathrm{t_j}; \mathrm{t_{j0}})}{\sqrt{1-\left(\dfrac{v}{v_{SU}}\right)^2}}, \quad \text{(NL1)}$$

$$\mathrm{t_j} - \mathrm{t_{j0}} = \frac{(\mathrm{t_i} - \mathrm{t_{i0}}) - \dfrac{v}{v_{SU}^2} r_P(\mathrm{t_i}; \mathrm{t_{i0}})}{\sqrt{1-\left(\dfrac{v}{v_{SU}}\right)^2}}, \quad \text{(NL2)}$$

$$\mathbf{r}_P(\mathrm{t_i}; \mathrm{t_{i0}}) = \frac{\mathbf{r}_P(\mathrm{t_j}; \mathrm{t_{j0}}) + \boldsymbol{v}(\mathrm{t_j} - \mathrm{t_{j0}})}{\sqrt{1-\left(\dfrac{v}{v_{SU}}\right)^2}}, \quad \text{(NL3)}$$

$$\mathbf{r}_P(\mathrm{t_j}; \mathrm{t_{j0}}) = \frac{\mathbf{r}_P(\mathrm{t_i}; \mathrm{t_{i0}}) - \boldsymbol{v}(\mathrm{t_i} - \mathrm{t_{i0}})}{\sqrt{1-\left(\dfrac{v}{v_{SU}}\right)^2}}. \quad \text{(NL4)}$$

The transformations (NL1) through (NL4) are partially both pairwise and entirely compatible.

The Non-Lorentz transformations (NL1) through (NL4) incorporate the Lorentz transformations as a singular case. The former reduce to the latter when the spatial reference point moves with the light speed *c*, i.e. for $v_{SU} = c$. The following Corollary shows this explicitly:

Corollary 3. Let Constraint 1 through Constraint 6 be valid. Let the *time* scaling coefficients $\mu_i \in R^+$, i = 1, 2, ... , s, be defined by (1). In order for the coefficients $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j = \alpha_j^i = \alpha_{ij} = \alpha_{ji}$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j = \lambda_j^i = \lambda_{ij} = \lambda_{ji}$, determined for the light speed $c_{(.)}^{(.)}$ of P_{SU}, to be positive real numbers and to obey (2) through (5), and for (1) through (5) to imply (6) it is both necessary and sufficient that the equations (20) through (30) hold for any choice of the *time* scaling coefficient $\mu_i \in R^+$, for $v_{SU}^{ij} = v_{SU}^{ji} = v_{SU} = c$ and for $v_{ji} = -v_{ij} = v$. The equations (2) through (5) become the Lorentz transformations (L1) through (L4):

$$\mathrm{t_i - t_{i0}} = \frac{(\mathrm{t_j - t_{j0}}) + \dfrac{v}{c^2} r_P(\mathrm{t_j;t_{j0}})}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}}, \tag{L1}$$

$$\mathrm{t_j - t_{j0}} = \frac{(\mathrm{t_i - t_{i0}}) - \dfrac{v}{c^2} r_P(\mathrm{t_i;t_{i0}})}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}}, \tag{L2}$$

$$\mathbf{r}_P(\mathrm{t_i;t_{i0}}) = \frac{\mathbf{r}_P(\mathrm{t_j;t_{j0}}) + \boldsymbol{v}(\mathrm{t_j - t_{j0}})}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}}, \tag{L3}$$

$$\mathbf{r}_P(\mathrm{t_j;t_{j0}}) = \frac{\mathbf{r}_P(\mathrm{t_i;t_{i0}}) - \boldsymbol{v}(\mathrm{t_i - t_{i0}})}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}}. \tag{L4}$$

Corollary 3, hence the fact that the Lorentz transformations (L1) through (L4) are obtained by starting with Axiom 1 on *time*, show that there is not any collision among them. The Lorentz transformations (L1) through (L4) may not be used, and cannot be used, to state any of the following essentially and fundamentally wrong claims, [25]:

Wrong claim 1. *Time* is a dependent variable.

Wrong claim 2. *Time* depends on spatial frames, hence on the space.

Wrong claim 3. There are many different *times*.

Wrong claim 4. There are many different speeds of the *time* value evolution (i.e. *time* speeds).

Wrong claim 5. The *time* speed changes due to a change of the clock speed.

Comment 1. It is not *time* that forces a clock to operate. Any variation of the speed of a clock hand corresponds only to an adequate variation of the *time* unit, but not to any variation of the speed of the *time* value evolution. It is energy that forces the clock to operate. Marmet [35] - [40] explained why a variation of the speed of a clock itself causes a variation of the speeds of the clock hands.

Let us repeat once more, a change of the speed of the clock hand means nothing else than the corresponding change of the *time* unit. Once we determine the scaling coefficients among the *time* scales corresponding to different speeds of the clock hands, then we can use the equation (1) to verify that all the clocks show always the same moment, provided they are identical, operate identically and perfectly, accurately and precisely, in the same conditions, their hands were set at the same initial position and they started to operate at the same instant. The book [25] exposes more details and proofs in this regard.

Comment 2 . It is not *time* that causes a variation of the kinetic energy of a moving body in general, i.e. of the mass of the moving body in the Lorentzian inertial frames, when the body speed varies. It is a variation of the energy exchange between the body and its environment, which causes a variation of the kinetic energy of a moving body in general, hence, which causes the variation of the mass of the body in the Lorentzian inertial frames. For the energy - mass explanation see the papers by Marmet [35] - [40]. The book [25] presents the general kinematic results in this connection.

8. VELOCITY TRANSFORMATIONS

8.1. General case

Theorem 3. Let the *time* scaling coefficients $\mu_i \in R^+$, i = 1, 2, ... , s, be defined by (1). Let the scaling coefficients μ_i, $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j \neq \alpha_j^i$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j \neq \lambda_j^i$, be positive real numbers and obey (2) through (5), and let (1) through (5) imply (6). Then, a constant non-zero velocity $\mathbf{v}_\mathrm{P}^{\mathrm{O_i}}$ of the arbitrary point P with respect to the origin $\mathrm{O_i}$ of R_i^n and relative to $\mathrm{t_i}$ and the velocity $\mathbf{v}_\mathrm{P}^{\mathrm{O_j}}$ of the same point P with respect to the origin $\mathrm{O_j}$ of R_j^n and relative to $\mathrm{t_j}$ are interrelated as follows:

$$\mathbf{v}_P^{O_i} = \frac{\mu_j}{\mu_i} \frac{1+\dfrac{v_{ji}^j v_{SU}^j}{q^j \omega^j}}{1+\dfrac{v_{ji}^j}{v_{SU}^j}} \frac{\mathbf{v}_P^{O_j} + \mathbf{v}_{ji}^j}{1+\dfrac{v_{ji}^j v_\mathrm{P}^{\mathrm{O_j}}}{q^j \omega^j}}, \tag{31}$$

$$\mathbf{v}_P^{O_j} = \frac{\mu_i}{\mu_j} \frac{1-\dfrac{v_{ji}^i v_{SU}^i}{q^j \omega^j}}{1-\dfrac{v_{ji}^i}{v_{SU}^i}} \frac{\mathbf{v}_P^{O_i} - \mathbf{v}_{ji}^i}{1-\dfrac{v_{ji}^i v_\mathrm{P}^{\mathrm{O_i}}}{q^j \omega^j}}. \tag{32}$$

In order for them to be compatible it is necessary and sufficient that

$v_\mathrm{P}^{\mathrm{O_i}} \equiv v_\mathrm{SU}^\mathrm{i}, v_\mathrm{P}^{\mathrm{O_j}} \equiv v_\mathrm{SU}^\mathrm{j}$.

The transformations are partially compatible.

Proof. Let all the conditions be valid. Hence, the Theorem 1 holds. The velocity is defined as:

$$\mathbf{v}_{\mathrm{P}}^{\mathrm{O}_{(.)}}(\mathrm{t}_{(.)}) = \frac{d\mathbf{r}_{(.)}(\mathrm{t}_{(.)};\mathrm{t}_{(.)0})}{d\mathrm{t}_{(.)}}, (.) = \mathrm{i, j}. \tag{33}$$

By applying (13) and (15) to the right hand side of the preceding equation and by using (1), $dr_P(\mathrm{t}_j;\mathrm{t}_{j0})/d\mathrm{t}_j = v_P^{O_j}$, $v_{ji}^j\mathbf{u} = \boldsymbol{v}_{ji}^j$ and (12) we find:

$$\mathbf{v}_P^{O_i} = \frac{d\left[\dfrac{\mathbf{r}_P(\mathrm{t}_j;\mathrm{t}_{j0}) + v_{ji}^j(\mathrm{t}_j - \mathrm{t}_{j0})\mathbf{u}}{1+\dfrac{v_{ji}^j}{v_{SU}^{O_j}}}\right]:dt_j}{d\left[\dfrac{\mu_i}{\mu_j}\dfrac{(\mathrm{t}_j - \mathrm{t}_{j0}) + \dfrac{v_{ji}^j}{q^j\omega^j}r_P(\mathrm{t}_j;\mathrm{t}_{j0})}{1+\dfrac{v_{ji}^j v_{SU}^j}{q^j\omega^j}}\right]:dt_j} =$$

$$= \frac{\dfrac{\mathbf{v}_P^{O_j} + v_{ji}^j\mathbf{u}}{1+\dfrac{v_{ji}^j}{v_{SU}^j}}}{\dfrac{\mu_i}{\mu_j}\dfrac{1+\dfrac{v_{ji}^j v_P^{O_j}}{q^j\omega^j}}{1+\dfrac{v_{ji}^j v_{SU}^j}{q^j\omega^j}}} = \frac{\mu_j}{\mu_i}\frac{1+\dfrac{v_{ji}^j v_{SU}^j}{q^j\omega^j}}{1+\dfrac{v_{ji}^j v_P^{O_j}}{q^j\omega^j}}\frac{\mathbf{v}_P^{O_j} + v_{ji}^j\mathbf{u}}{1+\dfrac{v_{ji}^j}{v_{SU}^j}}.$$

This proves (31). The equation (32) is analogously proved by using (1), (12), (14), (16), (33), $dr_P(\mathrm{t}_i;\mathrm{t}_{i0})/dt_i = v_P^i$ and $v_{ji}^i\mathbf{u} = \boldsymbol{v}_{ji}^i$. It is easy to check that for (31) and (32) to be compatible it is necessary and sufficient that $v_{\mathrm{P}}^{\mathrm{O_i}} \equiv v_{\mathrm{SU}}^{\mathrm{i}}, v_{\mathrm{P}}^{\mathrm{O_j}} \equiv v_{\mathrm{SU}}^{\mathrm{j}}$. Q. E. D.

The equations (31) and (32) confirm the equations (12).

The velocity transformation equations (31) and (32) are beyond the Lorentz – Poincaré - Einstein relativity theory. The speed value ratios in the denominators of the right hand side quotients in (31) and (32) do not contain the light speed value, which appears in the ratio $"vv_{\mathrm{P}}^{\mathrm{O}_{(.)}}/c"$ in the velocity transformations resulting from the Lorentz transformations, [30] - [32]. If we accept $\mathbf{v}_{\mathrm{SU}}^{(.)} \equiv \mathbf{c}_{(.)}^{(.)}$, then the formulas (31) and (32) contain the light speed values $\mathrm{c}_{\mathrm{i}}^{\mathrm{i}}$ and $\mathrm{c}_{\mathrm{j}}^{\mathrm{j}}$ relative to I_i and I_j, respectively. However, the light speed value is independent of *time* scale and of *time* unit in the Lorentz – Poincaré - Einstein relativity theory [3] – [7], [30] - [33], [50].

Note 2. The equations (31) and (32) show that
- the light speed is not invariant in general,
- the light speed value does not restrict the values of other speeds,
- the light speed obeys the general rules,
- the light speed is not an exceptional speed from the kinematic point of view.

8.2. Special case

Theorem 4. Let the *time* scaling coefficients $\mu_i \in R^+$, i = 1, 2, ... , s, be defined by (1). Let A = B in G. Let the scaling coefficients μ_i, $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j = \alpha_j^i = \alpha_{ij} = \alpha_{ji}$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j = \lambda_j^i = \lambda_{ij} = \lambda_{ji}$, be positive real numbers and obey (2) through (5), and let (1) through (5) imply (6). Then, a constant non-zero velocity $\mathbf{v}_{\mathrm{P}}^{\mathrm{O_i}}$ of the arbitrary point P with respect to the origin $\mathrm{O_i}$ of R_i^n and relative to $\mathrm{t_i}$ and the velocity $\mathbf{v}_{\mathrm{P}}^{\mathrm{O_j}}$ of the same point P with respect to the origin $\mathrm{O_j}$ of R_{j}^n and relative to $\mathrm{t_j}$ are interrelated as follows:

$$\mathbf{v}_P^{O_i} = \frac{\mathbf{v}_P^{O_j} + \mathbf{v}_{ji}^j}{1+\dfrac{v_{ji}^j \mathrm{v}_P^{O_j}}{\left(v_{SU}^j\right)^2}}, \quad \mathbf{v}_P^{O_j} = \frac{\mathbf{v}_P^{O_i} - \mathbf{v}_{ji}^i}{1-\dfrac{v_{ji}^i \mathrm{v}_P^{O_i}}{\left(v_{SU}^i\right)^2}}. \tag{34}$$

Proof. Let all the conditions hold. Hence, the Theorem 2 and Note 1 are valid. The equations (27b), (28b), (29), (30) and (33) furnish (34). Q. E. D.

Note 3. When we accept $\mathbf{v}_{\mathrm{P}}^{(.)} \equiv \mathbf{c}_{(.)}^{(.)}$ in (34) then it results that the light speed is not invariant, but it obeys the general rule as all other speeds.

Theorem 5. Let the *time* scaling coefficients $\mu_i \in R^+$, i = 1, 2, ... , s, be defined by (1). Let A = B in G. Let the scaling coefficients μ_i, $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j = \alpha_j^i = \alpha_{ij} = \alpha_{ji}$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j = \lambda_j^i = \lambda_{ij} = \lambda_{ji}$, be positive real numbers and obey (7) through (10), and let (1) through (5) imply (6). Then,

a) The spatial reference speed $v_{\text{SU}}^{(.)}$ is invariant relative to the integral spaces I_i and I_j, $v_{\text{SU}}^{(.)} \equiv v_{\text{SU}}^{\text{ij}} \equiv v_{\text{SU}}^{\text{ji}}$.

b) For the light speed $c_{(.)}^{(.)}$ to be invariant relative to the integral spaces I_i and I_j, $c_{(.)}^{(.)} \equiv c^{ij} \equiv c^{ji}$, it is necessary and sufficient to be the spatial reference speed $v_{\text{SU}}^{(.)}$, $c_{(.)}^{(.)} \equiv v_{\text{SU}}^{(.)} \equiv v_{\text{SU}}^{\text{ij}}$.

Proof. Let all the conditions be satisfied.

a) Let the arbitrary point P move with the spatial reference velocity, $v_{\text{P}}^{(.)} \equiv v_{\text{SU}}^{(.)}$. Then the equations (34) become the following:

$$\mathbf{v}_P^{O_i} = \mathbf{v}_{SU}^i = \frac{\mathbf{v}_{SU}^j + \mathbf{v}_{ji}^j}{1 + \frac{v_{ji}^j \mathrm{v}_{SU}^j}{\left(v_{SU}^j\right)^2}} = \mathbf{v}_{SU}^j,$$

$$\mathbf{v}_P^{O_j} = \mathbf{v}_{SU}^j = \frac{\mathbf{v}_{SU}^i - \mathbf{v}_{ji}^i}{1 - \frac{v_{ji}^i \mathrm{v}_{SU}^i}{\left(v_{SU}^i\right)^2}} = \mathbf{v}_{SU}^i.$$

Hence, $v_{\text{SU}}^{(.)} \equiv v_{\text{SU}}^{\text{ij}} \equiv v_{\text{SU}}^{\text{ji}}$.

b) For the light speed to be invariant in view of (34) it is necessary and sufficient that

$$\mathbf{v}_P^{O_i} = \mathbf{c}_i^i = \frac{\mathbf{c}_j^j + \mathbf{v}_{ji}^j}{1 + \frac{v_{ji}^j \mathrm{c}_j^j}{\left(v_{SU}^j\right)^2}} = \mathbf{c}_j^j, \text{ and}$$

$$\mathbf{v}_P^{O_j} = \mathbf{c}_j^j = \frac{\mathbf{c}_i^i - \mathbf{v}_{ji}^i}{1 - \frac{v_{ji}^i \mathrm{c}_i^i}{\left(v_{SU}^i\right)^2}} = \mathbf{c}_i^i.$$

For these equations to hold it is necessary and sufficient that $\mathbf{c}_{(.)}^{(.)} \equiv \mathbf{v}_{SU}^{(.)} \equiv v_{\text{SU}}^{\text{ij}} \equiv v_{\text{SU}}^{\text{ji}} \equiv c^{ij} \equiv c^{ji}$.

Q. E. D.

8.3. Singular case

Theorem 6. Let the *time* scaling coefficients $\mu_\text{i} \in R^+$, $i = 1, 2, \ldots, s$, be defined by (1). Let A = B in G. Let $\mathbf{v}_{\text{ji}}^{(.)} \equiv \mathbf{v}$ and $\mathbf{v}_{\text{SU}}^{(.)} \equiv \mathbf{v}_{\text{SU}}$. Let the scaling coefficients μ_i, $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j \equiv \alpha_j^i \equiv \alpha$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j \equiv \lambda_j^i \equiv \lambda$, be positive real numbers and obey (2) through (5), and let (1) through (5) imply (6). Then, a constant non-zero speed $\mathbf{v}_{\text{P}}^{\text{O}_\text{i}}$ of the arbitrary point P with respect to the origin O_i of R_i^n and relative to t_i and the speed $\mathbf{v}_{\text{P}}^{\text{O}_\text{j}}$ of the same point P with respect to the origin O_j of R_j^n and relative to t_j are interrelated as follows:

$$\mathbf{v}_P^{O_i} = \frac{\mathbf{v}_P^{O_j} + \mathbf{v}}{1 + \frac{v\mathrm{v}_P^{O_j}}{v_{SU}^2}}, \quad \mathbf{v}_P^{O_j} = \frac{\mathbf{v}_P^{O_i} - \mathbf{v}}{1 - \frac{v\mathrm{v}_P^{O_i}}{v_{SU}^2}}. \tag{35}$$

Proof. Let all the conditions hold. Then the Corolary 2 holds. We replace the equations (27b), (28b), (29), (30) by the equations (NL1) through (NL4) in the proof of the Theorem 5, which then becomes the proof of the Theorem 6. Q. E. D.

Theorem 7. Let the *time* scaling coefficients $\mu_\text{i} \in R^+$, $i = 1, 2, \ldots, s$, be defined by (1). Let A = B in G. Let the scaling coefficients μ_i, $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j = \alpha_j^i = \alpha_{ij} = \alpha_{ji} = \alpha$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j = \lambda_j^i = \lambda_{ij} = \lambda_{ji} = \lambda$, be positive real numbers and obey (2) through (5), and let (1) through (5) imply (6). Let $\mathbf{v}_{\text{ji}}^{(.)} \equiv \mathbf{v}_{ji}$. Then,

c) The spatial reference speed $v_{\text{SU}}^{(.)}$ is invariant relative to all integral spaces, $v_{\text{SU}}^{(.)} \equiv v_{SU}$.

d) For the light speed $c_{(.)}^{(.)}$ to be invariant relative to all integral spaces, $c_{(.)}^{(.)} \equiv c$, it is necessary and sufficient to be the spatial reference speed $v_{\text{SU}}^{(.)}$, $c_{(.)}^{(.)} \equiv v_{\text{SU}}^{(.)} \equiv v_{SU}$.

Proof. Under the theorem statement, the Corollary 2 is valid. When we set $\mathbf{v}_{\text{ji}}^{(.)} \equiv \mathbf{v}_{ji}$ in the proof of the Theorem 5 then it becomes the proof of the Theorem 7 since the scaling coefficients and are the same for all integral spaces mutually related by the transformations (27) trough (30), which reduce in this case to (NL1) through (NL4), and for the light speed of the reference point they reduce to (L1) through (L4). Q. E. D.

Corollary 4. a) For every speed there are co-ordinate and velocity transformations under which the velocity is invariant.
b) For the light speed such transformations are the Lorentz transformations (L1) through (L4), and from them resulting Einstein's law of the composition of velocities.

Note 4. The light speed is not an exceptional speed. It obeys the general rules.

9. CONCLUSION

Our experience with, our understanding of, and our knowledge about the physical reality led to the axiomatic characterisation of the properties of *time* (Axiom 1). It is in Newton's sense. It expresses that *time* is a unique and a basic physical variable. There are not two or more different *times*. There are many different *time* scales and *time* units. Newton himself explained this in [48]. The numerical value of the speed of *time* value evolution (of the *time* speed) equals one (with respect to all *time* axes). It is invariant relative to a choice of a *time* axis [25]. It is independent of everybody and everything, hence, of the space. *Time* is the unique physical variable with such property of its speed [25].

The numerical value of the speed of light is not invariant with respect to a choice of a *time* axis.

The features of *time* enabled us to avoid all the constraints a priori accepted in the Einsteinian relativity theory after Lorentz and Einstein, and to start establishing fundamentals of a new relativity theory.

The co-ordinate transformations are *compatible* if, and only if the application of the inverse transformation to the transformation itself results in an identity. If, and only if this holds for both pairs, the pair of the temporal co-ordinate transformations and the pair of the spatial co-ordinate transformations, then the transformations are *pairwise compatible*. If, and only if this is true for the temporal and spatial transformations altogether, then they are *entirely compatible*. If, and only if this compatibility of the transformations is valid only for the light speed of an arbitrary point, or under restriction on the generic speeds, then the compatibility of the transformations is *partial (restrictive) compatibility*. Their compatibility is *complete* if, and only if it holds for arbitrary value of the speed of an arbitrary point, and. if there is not any restriction on the generic speeds.

Uniformity of the transformations means that the temporal co-ordinate transformations are independent of a choice of an arbitrary point (of its position and of its speed), i. e. that they hold uniformly over the space. Uniform co-ordinate transformations were established in [20] through [23], [25]. Otherwise, the transformations are *non-uniform*, which holds for the transformations (13) through (16) established herein.

The *time* scaling factors defined by the equation (1), which are used in the theory of dynamical systems with multiple *time* scales, are introduced in the relativity theory in the Theorem 1 in this framework.

The new mathematical relativity theory is partially compatible, but consistent. It is called *Partially Compatible but Consistent* (for short: *PCC*) *Relativity Theory*. The *time* scaling coefficients and the space scaling coefficients are a priori all permitted to be mutually different. This led to their new forms and to the new forms of the corresponding co-ordinate transformations. They are crucially different from the Lorentz transformations. They show that the relative light speed value should be used in computations, i. e. that the value of the light speed depends on the integral space relative to which it is measured. They have implied new formulas for speed. They are proved for the general case and their partial entire and pairwise compatibility is also verified. They are consistent.

The new scaling factors determined by (7) through (10), or by (22), (23), and the related new co-ordinate transformations (13) through (16), respectively, (27 through (30), do not contain the speed of light in general.

It is shown that starting with the Axiom 1 and by accepting Constraint 1 through Constraint 6, we get the Lorentz co-ordinate transformations. This clarifies that they may not, and cannot be used to claim either *time* dependence on the space or a variation of the speed of *time* value evolution. Such claims are wrong. *Time* does not forces any clock to work. A variation of the speed of clock hands means only the corresponding variation of the *time* units, and vice versa. It does not mean either a variation of the speed of *time* value evolution or the existence of different *time*s for clocks moving with different speeds. Energy, not *time*, forces the clock to operate.

The velocity transformations (31), (32) in the general case, (34) in the special case, (35) in the singular case, are essentially different from Einstein's velocity transformations known as Einstein's law of the velocity compositions.

The Theorems 5 and 7 discover that the light speed is not an exceptional speed. It is not invariant and it does not restrict other speeds, in general. The Theorem 7 shows also that for every speed we can construct co-ordinate transformations, and from them resulting velocity transformations, so that the given speed is invariant. For the light speed such transformations are the Lorentz transformations and Einstein's law of the velocity compositions.

However, the light speed is not invariant, and its value does not restrict the values of other speeds, in the Galilean - Newtonian spaces. This confirms the results of Goy [9, p. 98], Keswany [27, pp. 135, 138], Martin [47, pp. 53, 54], and Wesley [52, p. 261]. This does not contradict the famous experimental results of Fizeau, Michelson and Morley, because they do not prove the light speed invariance, which is shown in the book [25]. In this connection see also the works of Ceapa [1], [2], Kracklauer [28], Marmet [44], [46], Martin [47], and Wesley [52]. They all verify the obvious fact that the relative light speeds are different with respect to a railway station, and with respect to a fast moving train, with respect to a flying rocket, and with respect to another light signal, all moving in parallel and in the same sense in the Galilean - Newtonian space.

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