Complete conceptual understanding of the $1/f$ noise

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Electron noise whose power spectral density $S(f)$ is inversely proportional to frequency f has been obtained by: 1) The reduction of the nonlinear Boltzmann equation with electron-electron $(e-e)$ interaction to a Fokker-Planck (FP) equation; 2) The steady state solution $f_0(v)$ of $e-e$ FP, which depends on the square of acceleration $\mathbf{a}$; 3) $f_0(v)$ becomes similar to the Fermi-Dirac distribution function if a^2 is caused by the zero-point field (ZPF) of QED. It is just because of $a^2_{\rm ZPF}$ that there is a small interval δv for the electron speed v where runaways occur; 4) In this δv range, the time-dependent Green solution of the $e-e$ FP decreases as $\tau^{-\varepsilon}$ with $\varepsilon < 0.01$. Then, $S(f) \propto 1/f^{1-\varepsilon}$ and also depends on the electron concentration, thus closely fitting the experimental data; 5) In a finite sample, fluctuactions are remembered because back diffusion is much more rapid than drift velocity.

In 1921 Johnson performed the first measurements of a steady-state electrical noise in order to verify the shot noise predicted by Schottky for triodes[1]. Johnson found the shot noise but together with another unexplicable noise he called "additional" or "flicker noise", afterwards, and so far, denoted as $1/f$ noise because its power spectral density $S(f)$ is inversely proportional to the frequency f. In the past 84 years more than thousand papers have been written on that subject but never a real explanation has been found regarding the physical origin of the $1/f$ noise for the conduction current (usually in semiconductors). We only mention the conduction current because the other cases of $1/f$noise (in earthquakes, in Nile floodings, in biophysics, etc.) present such behaviour only in a limited range of frequencies and have a lower cutoff[1]. After 1921 other electrical noises in steady-state conditions have been detected, the majority of them being theoretically predicted. In any case, all of them, but the $1/f$ noise, have received a clear and rapid explanation. On experimental bases[2], we may classify the electron noises in two categories: I) Noise due to the fluctuations of the number $\mathcal{N}$ of the charge carriers (usually electrons) contained between the two electrodes applied to the sample; II) Noise due to fluctuations of the mobility μ_m. With regards to the physical causes, there is a further subdivision. Point I) (fluctuations of $\mathcal{N}$) may be divided into: a) Shot noise, due to the random generation of electrons emitted from a hot cathode in a vacuum diode (or triode); b) Generation-recombination noise, due to random changes in the number of free electrons in the conduction band because of traps; c) Diffusion noise, due to the diffusive relaxation of $\mathcal{N}$ when $\mathcal{N} \neq \mathcal{N}_{\rm ions}$. Similarly, point II) (fluctuations of μ_m) may be divided into: a) Thermal noise, due to rapid changes of free carrier velocities (or momenta) when the carriers collide with the ion lattice; b) Convective noise due to changes of speed (or of energy); c) $1/f$ noise shown in the following, and in the companion paper[3], to be a convective noise in runaway conditions. The latter ones occur when the collision frequency $\nu(v)$ is proportional to v^{-n} with $n \geq 1$, at least in a small interval δv. In fact, when the speed increases because of an external electric field , $\nu(v)$ decreases in such a way that the electrons become almost collisionless, thus eliminating diffusion in the speed space.

All the physical noises are Gaussian and are therefore characterized by the two-times correlation function $C(\tau)$, defined as the ensemble average of the product of the same variable $v_x(t)$, having zero mean value $\langle v_x(t)\rangle = 0$, at two different times

$$C_{vx}(\tau) = \langle v_x(t)\, v_x(t+\tau)\rangle \ . \tag{1}$$

Being steady-state the stochastic process, C_{vx} does not depend on the chosen time t. If v_x, for example the x component of the carrier velocity $\mathbf{v}$, has a nonzero average, we must consider the variable $g_x(t) = v_x(t) - \langle v_x(t)\rangle$.

The Fourier transform of the correlation function is the power spectral density

$$\begin{aligned} S_{vx}(f) &= \int_{-\infty}^{\infty} d\tau\, C_{vx}(\tau)\, \exp(-i2\pi f\tau) \\ &= 2\int_0^{\infty} d\tau\, C_{vx}(\tau) \cos(2\pi f\tau) \end{aligned} \tag{2}$$

where the last step is called the Wiener-Khintchine relationship, and is a consequence of the time symmetry of $C_{vx}(\tau)$, i.e., $C_{vx}(\tau) = C_{vx}(-\tau)$. For all the noises but two of them, it is $C_{vx} \propto \exp(-\tau/\tau_0)$ so that

$$S_{vx}(f) \propto \left[1 + (2\pi f \tau_0)^2\right]^{-1} \ , \tag{3}$$

denoted as Lorentzian. The generation-recombination noise is given by a sum of Lorentzians, and the $1/f$ noise by a series of Lorentzians with a uniform distributions of τ_0 up to $\tau_0 \to \infty$, or at least $\tau_0 > 40$ days as experimentally verified. Apart from the difficulty of having traps with such long times, a noise due to traps is of the kind I), i.e., it would be due to fluctuations of the number $\mathcal{N}$ of carriers, while the $1/f$ noise, as shown by Hooge[2], is due to fluctuations of the mobility μ_m (second considered kind). In any case, what matters is the physical origin of the $1/f$ noise.

In the past, the only attempt to explain the $1/f$ noise as a mobility fluctuation was that of Handel[4–7], who suggested that its origin could be a quantum effect leading to an infrared catastrophe. Precisely, Handel supposed that the $1/f$ noise were due to low-frequency photons emission by part of electrons scattered by an impurity potential. The current modulation seemed to be a "beating" (oscillating self-interference) term in the electron density. However, Kiss and Heszler[8] have shown by rigorous quantum electrodynamics that the value of the beating term is zero at any given time. Consequently, Handel' hypothetical type of $1/f$ noise does not exist. The same conclusion applies to Ngai's theory[9] of $1/f$ noise and dielectric response which is an elaboration of Handel's theory. The second criticism is as radical as the one of Kiss and Heszler[8]: it is the "cage effect"[2,10], i.e., the screening that the considered sample, and the set-ups surrounding it, put to the soft photons at extremely small frequencies (up to 1/month) necessary to produce the scattering with the conduction electrons. These photons should have a wave length $\simeq 10^4$ times the distance between the Sun and the Earth and the boundary conditions eliminate them, as well known from the Casimir effect (for recent work see Ref. 11), the Van der Waals forces[12,13], and the microcavities which prevent atomic transitions with wave lengths larger than the cavity sizes[14]. Another important criticism is that electron scattering with the lattice and other electrons is completely neglected, thus implying zero resistance for any conductor. In fact, scattering prevents the long coherence time between electrons and soft photons[10]. For other mathematical and physical inconsistencies of Handel theory, see Ref. 10.

Other theories, published after Hooge's last review[2], give $1/f$ noise only in a limited f range. For example, Ref. 15 gives at maximum 5 decades instead of the experimental 10 decades if we take the upper frequency f_2 of the order of electron collision frequencies, i.e., $f_2 \simeq 10^{13}$ s^{-1}. But then the maximum frequency f_1, taking the classical relaxation of energy with a decay time constant 10^5 times the momentum relaxation turns out to be $f_1 \simeq 10^8$ s^{-1} instead of the minimum frequency $f_{\rm min} \simeq$1/month$\simeq 4 \times 10^{-7}$ s^{-1}. Moreover, there is no physical reason for the long time decay of the considered Brownian motion. Similarly, the "extreme value statistics"[16] is a purely mathematical model and the authors themselves recognized that "though we do not see a physical reason that necessitates this mathematical result, we speculate that it may be a key feature that underlies a unified treatment of systems displaying $1/f$ noise". But all the systems, with the exception of the current conduction, display $1/f$ noise in limited f ranges[1]. Moreover, those last attempts regards fluctuations of $\mathcal{N}$ and not of μ_m.

The mathematical models can be divided in the same two groups used for the physical causes: I) those implying $\Delta\mathcal{N}$ fluctuations, and II) mobility μ_m fluctuations. Among I), the most natural is McWhorter's model[2,17], where $1/f$ noise is obtained by a distribution of Lorentzians with weights inversely proportional to the relevant time constant τ_s of each sth-exponential decay. An almost equivalent way of stating McWhorter criterion is by fractals[18,19]. However, $\Delta\mathcal{N}$ fluctuations as an origin of $1/f$ noise have been disproved by experiments[2,20]. Only II) is viable and here the only way to obtain $1/f^{1-\varepsilon}$ with $\varepsilon < 0.01$ is to have a correlation $C_G(\tau)$ of $g(t) = G(t) - \langle G(t)\rangle$ (where $G(t)$ is the fluctuating conductance of the considered sample) that decays according to a power law

$$C_G(\tau) \propto \tau^{-\varepsilon} \tag{4}$$

instead of an exponential. The point is to show that the $\tau^{-\varepsilon}$ decay is just the time-dependent solution of the nonlinear Boltzmann equation. In any case , what matters is not the mathematics to obtain $1/f^{1-\varepsilon}$ dependence, which simply requires that Eq. (4) be satisfied. What matters is to find the numerical agreement with the experimental values and the correct dependence on some parameters for the power spectral density $S_G(f)$ of the conductance G because, as emphasized by Milotti[1] and by Hooge[2], only $C_G(\tau)$ have the same τ dependence as in Eq. (4), without any experimental cut-off at low frequency. Hooge's celebrated empirical formula, at absolute temperature $T = 0$, is

$$S_G(f)/G^2 = \alpha_{\rm latt}/(\mathcal{N} f) \ , \tag{5}$$

where $\mathcal{N}$ is the number of charge carriers and $\alpha_{\rm latt}$ a dimensionless parameter (due to scattering with the lattice) dependent on T. However, Eq. (5) does not fit well the experimental data. The reason is that $\alpha_{\rm latt}$ is not meaningful because the leading scattering is due to $e-e$ interactions; on the contrary, our α_ε also depends on the electron concentration N as[3]

$$\alpha_\varepsilon(T=0) = \frac{1.8 \times 10^{15}}{N + 10^{19}} \left(1 + \frac{N}{1.8 \times 10^{23}}\right) \ . \tag{6}$$

We found that Eq. (6) closely fits the experimental data, as shown in Fig. 1. In particular our values are close to Hooge's only for $N \simeq 10^{19}$ m^{-3} but they decrease for $N > 10^{19}$ m^{-3} up to an asymptotic value which is four order of magnitude lower than Hooge's. The lack of N dependence in Eq. (5) implies the great dispersion of the

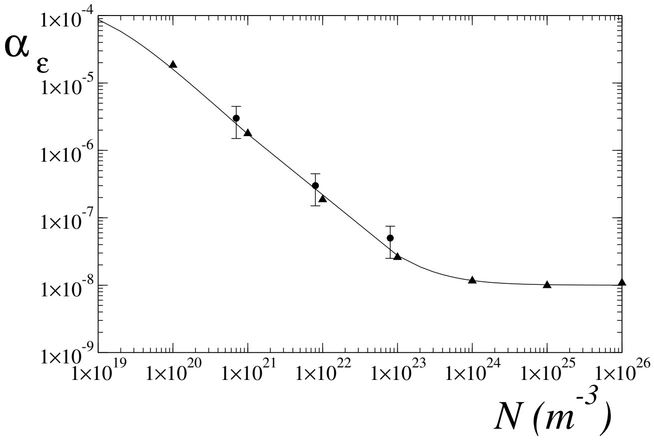

FIG. 1: **Plot of α_ε vs the concentration N.** The solid line interpolates the calculated values (triangles). The experimental data $\alpha_{\rm exp}$ are denoted by black circles with error bars.

experimental data Hooge found[2]. Moreover, we do not find exactly $S_G(f) \propto f^{-1}$, which presents an unphysical logarithmic divergence for the noise power $\int_0^\infty df\, S_G(f)$ at the limit $f \to 0$ (the divergence for $f \to \infty$ is eliminated by other scatterings). We find $S_G(f) \propto f^{-0.995}$ which is experimentally indistinguishable from f^{-1} but whose integral does not lead to a divergence for $f_{\rm min} \to 0$.

Let us now give the foundation and the main ideas of our theory, whose technical details are given in the companion paper[3]. We here develop the five points of the abstract.

1) In principle, any kind of electrical noise requires the solution of the nonlinear Boltzmann equation with $e-e$ interaction, or its equivalent Fokker-Planck equation ($e-e$ FP) . Fortunately, all the noises, but $1/f$, are sufficiently explained by the solutions of a Fokker-Planck (FP) equation with no $e-e$ interaction corresponding to the linear Boltzmann equation. The first achievement for the explanation of the $1/f$ noise was therefore the reduction of the nonlinear Boltzmann equation (with $e-e$ interaction) to an FP equation, obtained by one of us in a previous paper[21].

2) In a subsequent paper[22] we applied the $e-e$ FP equation (found in Ref. 21) to doped semiconductors, obtaining explicit expressions for the two collisions frequencies $\nu_1(v)$ and $\nu_2(v)$ appearing in the $e-e$ FP (while the usual FP contains a single $\nu(v)$). We have also found the solution of the $e-e$ FP in steady state conditions, as a function of $\nu_1(v)$, $\nu_2(v)$ and a^2, where $\mathbf{a} = e\mathbf{E}/m$ is the electron acceleration due to the electric field $\mathbf{E}$.

3) The present main idea is that

$$a^2 = a^2_{\rm D.C.} + a^2_{\rm ZPF} \; , \tag{7}$$

where $a_{\rm D.C.}$ is the acceleration due to the external D.C. electric field applied to the sample, and $a_{\rm ZPF}$ the acceleration due to ubiquitous zero-point field (ZPF) of quantum electrodynamics (QED) whose power spectral density is

$$S_{\rm ZPF} = 2hf^3/c^3 \; , \tag{8}$$

where h is the Planck constant, and c the speed of light.

It is the ZPF that explains in classical terms the already mentioned Casimir effect[11], the Van der Waals forces[12,13], the stability of the atoms[23], and the neutrino mass experiment performed at Troitsk[24]. Actually, an electron revolving around a proton in the fundamental state radiates a power given by Larmor formula. Assuming for simplicity a circular orbit with radius r and denoting e and v the electron charge and speed, respectively, it is

$$P_{\rm rad} = \frac{2}{3}\frac{e^2}{c^3}\left(\frac{v^2}{r}\right)^2 \; . \tag{9}$$

Yet an electron in its circular motion absorbs a power from the ZPF given by

$$P_{\rm abs} = n\frac{2}{3}\frac{e^2}{m}\pi^2 S_{\rm ZPF}(f) = \frac{8\pi^2}{3}\frac{e^2}{m}\frac{h}{c^3}\left(\frac{v}{2\pi r}\right)^3 \; , \tag{10}$$

where m denotes the electron mass, and $n = 2$ is the number of harmonic oscillators necessary to reproduce a circular motion. Equating Eq. (9) to Eq. (10) we obtain $mvr = h/(2\pi)$, which is the Bohr condition for hydrogen's fundamental state. The $S_{\rm ZPF}(f)$ given by Eq. (8) can also be derived in a classical way as due to the spin motion radiation in an expanding universe[25] so that h can be related to e, c, the average concentration $N_{e,u}$ of electrons in the universe, and the Hubble constant. With $S_{\rm ZPF}$ and spin, it is also possible to derive the Schroedinger equation in a classical way[26–30] (instead of postulating it in a quantum way). In our case it is $a^2_{\rm ZPF}$ that keeps the free electrons in motion even at absolute zero temperature $T = 0$, and with an average kinetic energy $\langle mv^2/2\rangle$ that, if equated to $3kT/2$, would give, for a conductor, $T = 10^4$ K. Actually, it is $\langle mv^2/2\rangle = \langle mv_0^2/2\rangle + 3kT/2$, where

$$U_F = \frac{m}{2}\langle v_0^2\rangle = \frac{\hbar^2}{2m\langle r^2\rangle} \tag{11}$$

is the average kinetic energy due to the ZFF at $T = 0$, which is also the Fermi energy. In the last step of Eq. (11), we have used Bohr condition [derived from Eq. (10)] applied to fluctuating quantities. Now $\langle r^2\rangle^{1/2}$, for two electrons in a cell, and taking into account the interactions with the electrons of the valence band (constrained around their nuclei), should be roughly

$$\langle r^2\rangle^{1/2} \simeq \frac{1}{4N^{1/3}} \; . \tag{12}$$

Substituting Eq. (12) into Eq. (11), we obtain

$$U_F = \frac{\hbar^2}{2m}(64N)^{2/3} \; , \tag{13}$$

which has the same dependence on $\hbar$, m, N as the exact U_F^{ex}, whose coefficient inside the round bracket is $6\pi^2$, close to our 64. With the N values reported in Fig. 1, it is $a^2_{ZPF} \gg a^2_{D.C.}$ so that a^2 given by Eq. (7) no longer depends on the external $E_{D.C.}$.

4) With $a^2 \simeq a^2_{ZPF}$ we have found that there is always a small interval δv for the electron speed v where $\nu_1 \propto \nu_2 \propto 1/v$, condition that is at the threshold of the runaways. Moreover in the effective δv interval our $e-e$ FP becomes similar to the usual FP equation (with no $e-e$ interaction), whose solution for the electron distribution function $f_0(v,\tau)$ has been found by Stenflo[31], and is of the kind $f_0 = K^2\tau^{-\varepsilon}$ with

$$\varepsilon = 3\left[1 - \nu_1 K^4(\nu_2 a^2)^{-1}\right] \ . \tag{14}$$

Notice that ε can be in general either positive or negative, either large or small, because it depends on ν_1, ν_2, K^2, and mainly on a^2. But, with $a^2 \simeq a^2_{ZPF}$, Eq. (14) gives $0.004 \le \varepsilon < 0.007$, whence, with the use of Eq. (2), $S_G(f) \propto 1/f^{0.995}$ which is experimentally indistinguishable from $1/f$.

5) The preceding four ideas and results are sufficient to explain in a satisfactory way the $1/f^{0.995}$ noise in an indefinite medium. The point is that the $\mathcal{N}$ electrons between the two electrodes (where a fluctuating voltage is measured) L apart from each other, take a time interval L/w (where w is the drift velocity, only due to $\mathbf{a}_{D.C.}$) to traverse L. How can the memory of a fluctuation can be preserved beyond L/w? This question clearly shows that a physical theory (purely mathematical equations are of no interest) must explain even this puzzle. Experimentally, the electron distribution function $f_0(v,\tau)$ (hence the correlation) must decay as $\tau^{-0.005}$ for more than one month, which is much larger than L/w. Moreover, there is no evidence of a tiny change in the decay after L/w. The solution is due to two results:

i) The electron-electron ($e-e$) scattering is dominant for the generation of $1/f^{1-\varepsilon}$ noise, as emphasized in Fig. 2 where $q(v_r) = 4\pi v_r^2 f_0(v_r)\nu_2(v_r)$ is reported, $v_r = v/\langle v^2\rangle^{1/2}$ being the reduced velocity. The lines interpolating the triangles and the black circles give the two products without and with the $e-e$ scattering, respectively. It appears that the contribution of $e-e$ scattering in the neighbourhoods of the δv range (useful for the production of $1/f^{1-\varepsilon}$ noise and centered at the reduced velocity $v_r = 2.07$) is almost three decades larger than all the other scattering processes.

ii) The $e-e$ scattering also preserves the memory of a fluctuaction much beyond the average transit-time L/w. In fact, free electrons are subjected to the drift velocity due to $\mathbf{a}_{D.C.}$, but also to diffusion due to $\mathbf{a}_{ZPF}$. Now, during the time of information transmission given by Eq. (21) of Ref. 3, the average displacement due to the drift velocity is

$$\delta x = w\tau_m = \mu_m E_{D.C.}\tau_m \ , \tag{15}$$

where the mobility $\mu_m = e\langle\mu(v)\rangle_v/m$ is expressed by Eqs. (2) and (3) of Ref. 3, and the values of $\langle\mu(v)\rangle_v$ for

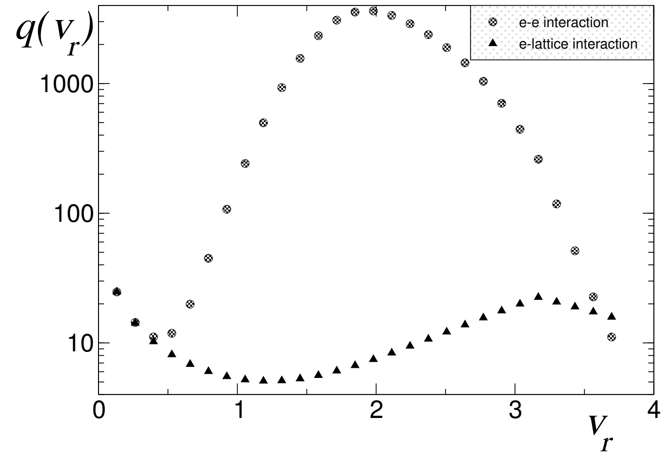

FIG. 2: **Plot of $q(v_r) = 4\pi v_r^2 f_0(v_r)\nu_2(v_r)$ vs the reduced velocity $v_r = v\langle v^2\rangle^{-1/2}$.** The circles represent the contribution due to $e-e$ interaction, and the triangles those due to $e-$lattice interaction.

the seven considered N, can be read in Table II of Ref. 3.

The most probable displacement in the same time interval, due to the longitudinal diffusion coefficient D_L for the electrons in the useful δv range, is[22,32]

$$\delta r = [4D_L(v_1)\tau_m]^{1/2} \ , \tag{16}$$

where v_1 is the smallest velocity of the useful δv range. Now, as shown by Parker and Lowke[32], when $\nu \propto v^{-1}$, i.e., at the threshold of runaways, D_L diverges. In fact, the expression of ν considered after Eq. (13), p.293 of Ref. 32, is $\nu = \nu_0(\epsilon/\epsilon_0)^{(l+1)/2} = \nu_0(v/v_0)^{l+1}$ (since $\epsilon = mv^2/2$) and they found

$$D_L/D_T = (l+3)/[2(l+2)] \ , \tag{17}$$

D_T being the transversal diffusion coefficient. We see clearly that Eq. (17) diverges for $l \to -2$, corresponding to $\nu \propto v^{-1}$. Parker and Lowke[32] obtained Eq. (17) by a semiquantitative model (their Sec. III), and their quantitative theory implies a still stronger divergence as can be seen comparing Eq. (17) with Table I of Ref. 32. Since D_L is defined as $\lim_{t\to\infty}\langle(x-x_0)^2/t\rangle$, the divergence means that $(x-x_0)^2 \propto t^2$, i.e., the diffusion becomes ballistic and $\delta r \simeq v_1\tau_m$. The ratio between δr and δx given by Eq. (16) becomes $\delta r/\delta x = v_1/(\mu_m E)$ which is very large. In our case the diffusion is practically due to the ZPF, which has random directions. For each ZPF wave train we have a different direction, so that we can no longer distinguish D_L from D_T, but speak of a single diffusion coefficient $D \simeq D_L/3$. In fact, $D_T \ll D_L$ (for each e.m. train), and the factor 1/3 comes from an average of a component on one axis of a three-dimensional diffusion along stochastic directions. Since $\delta r/\delta x = v_1/(3\mu_m E) \gg 1$, the transmission of information due to the electrons at the threshold of runaways is essentially caused by diffusion, the drift being negligible compared to diffusion. That is why the memory of a fluctuaction is preserved independently of the transit-time L/w. It was just the divergence of D_L when $\nu \propto v^{-1}$ that suggested to one of us (G. Cavalleri) the idea of the possible origin of $1/f$ noise when $\nu_2 \propto v^{-1}$, because of the connection between noise power spectral densities and generalized diffusion coefficients[33].

The $1/f$ noise is therefore valid for both an indefinite medium and a small sample.

Concluding, when a fluctuation produces a pimple in the distribution function of the electron speeds, the pimple tends to diffuse and drift in the speed space because of collisions. However, at the threshold of runaways there is a kind of counter-diffusion and counter-drift in the speed space, so that the pimple appears as almost crystallized, decaying as $\tau^{-0.005}$. Moreover, this result is independent of the transit time L/w of the electrons. What is more, our theoretical expression (6) fits the experimental data much better than Hooge's empirical formula, because we find a dependence on the electron concentration N besides the total number $\mathcal{N}$ of electrons in the considered sample.

The $1/f$ noise is therefore fully explained from both the experimental and theoretical points of view. The reason why this long standing problem challenged all the previous attempts is that it required a catena of successive achievements.

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