Multivalued logic and Quantum Mechanics

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Abstract

Probability theory has been widely used in quantum mechanics but today there are alternatives to probability theory: “fuzzy logic” which describes individuals, whereas probability theory describes populations. Nevertheless, probabilities are included in fuzzy logic and can help to model quantum uncertainty in a more precise way. This new approach would permit, for instance, a more consistent interpretation of Young's experiment and wave-particle duality. The purpose of this article is to attempt a connection between quantum mechanics and fuzzy logic, in order to have a new mathematical model for the spatio-temporal extension of particles, seen as "micro-objects" in the sense of A. Granik and H.J. Caulfield [1].

Keywords: quantum mechanics, multivalued logic, young's experiment, particle time extension, particle space extension.

Introduction

Quantum theory (QT), upon which our description of matter is built, uses Probability Theory. In fact, when QT was developed, uncertainty could be explained only with probabilities. The use of probabilities does not rely on what Nature is. Probabilities were the best mathematical tool to describe Uncertainty when Quantum mechanics has been created but today, there are alternatives.

Uncertainty could be modelled in a better way with Fuzzy logic. Fuzzy logic describes individual phenomena whereas probability theory describes populations. The fuzzy principle states that everything is a matter of degree [2]. In a fuzzy universe a particle's position would be known at all times, except that such a position would be ambiguous (a particle would be simultaneously "here" to some degree and "there" to some other degree) [3]

In this paper we are trying to study some aspects of Quantum mechanics using Fuzzy logic.

1. Description of some Quantum Mechanics paradoxes

1.1. Young's experiment

This experiment can be described as follows. A monochromatic light signal is emitted from a source S and passes through two slits g_1 and g_2. Then it lights a photographic screen P (cf Figure 1).

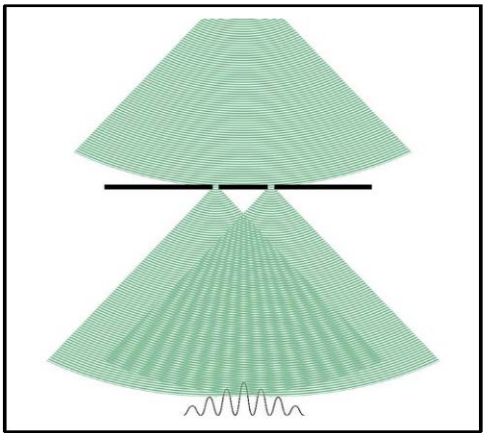

Fig.1: Young's experiment. ***Wikipedia.com***

-If we consider only one slit there is no interference. Light behaves as corpuscles. If we block one slit after the other then we obtain for each slit a distribution of lightning intensity $I_1(x)$ and $I_2(x)$, the diffraction patterns of g_1 and g_2.

When both slits are opened:

- At low source's intensity and at the beginning a few light flashes of electrons appeared.

It has been first shown by the Bologna group [4] who has photographed the number of a sensitive TV camera.

- At high source's intensity we have a system of interference fringes, but the intensity of which is not the sum of the intensities produced by g_1 and g_2 separately [5, 6]. Light behave as waves. The fringes were able to increase the storage timed up to values of minutes.

- At low source's intensity and after a certain time, Tonomura *and Al*. [7] demonstrate that a single electron build up on an interference pattern. If the number of electrons arrives practically one by one, we should not see any interference, but if the time increases there are fringes of interference [8]. The experiment was carried out from the beginning to the end with constant and extremely low electron intensities (< 1000 electrons/sec). This removed any probability that the fringe might be due to "classical" interactions between electrons.

The formation of fringes could be observed as a time series, the electrons were accumulated over time to gradually form an interference pattern on the monitor, the electrons arrived at random positions on the detector only one by one and it took more than 20 minutes.

If we reduce the source intensity until the electrons arrive practically one by one, we should not see any interference. In fact, when the electrons arrive slowly, if the time of the experiment is short, then the spots have a located impact, but if the time increases, there are fringes of interference [8].

The classical interpretation of this phenomenon does not allow us to choose between corpuscle or wave for an electron.

1.2. Measure's theory

Classical and quantum mechanics make predictions based on repetitive measurements which imply a certain spread of results.

If we consider a concept of a "thing in itself" and assume that it has a "fuzzy" and deterministic character, then in a series of experiments, designed to elicit its properties to the outside observers, it appears as a random set thus disguising its deterministic nature. The classical and the quantum mechanics can be viewed as different realization of a "fuzzy thing in itself".

Our basic assumption is that reality is fuzzy and non local not only in space but in time. Idealized points as in classical mechanics correspond to the ultimate sharpness of the fuzziness density and emerge in a process of interaction between different parts of wholeness. This process transforms a fuzzy reality into a crisp one.

According to the Copenhagen interpretation, the act of measurement is disturbing a quantum object and values extracts are called « observables ». The measurement consists of decomposing the wave packet, which is known as the collapse of ψ function. This interpretation can be described with fuzzy automata. The presentation will be given in the next chapter.

In Quantum mechanics [8] the problem of the measurement is fundamental and looms large on almost any study. The measurement theory, which is the result of studies from the Copenhagen school, is represented by a group of rules, which can be summed up as follows:

1-The result of the measurement of an « observable » “A” of random nature.

2-The value “a” that the result can take, belongs to the “A” spectrum. *(all values tacken by the observable A)*

3-In the simple case where “a” is a proper discrete and non degenerate value of “A”, its probability is given by $<a|\ \Psi_i >|^2$, Ψ_i is the wave function of the measured system at the beginning and |a> the proper function or vector associated to “a”.

The reducing rule of the wave function permits to define it after the measurement. It is easy to enounce the theory in the case of an ideal measurement where the measuring device can give the same result “a”, if the first measure is repeated immediately after. This means that the wave function Ψ_f after the first measurement is necessarily the proper function of “A” associated with “a”.

Some physicist, such as Einstein or Schrödinger, felt uncomfortable with the coherence of the theory. They wondered why among all the possibilities, a particular result that leads to drastic consequences, manifest itself rather than another.

Other people wondered if the wave function would be the expression of the information an observer could have. Then it was difficult to link this wave function, with dynamical reality. The main problem was to find why a measurement appears after measuring rather than the others which were still possible before measuring. The last word has not been said.

2. Fuzzy set logic

2.1. Nomenclature

Fuzzy set theory was created in the middle of the 60th century by the professor Lotfy Zadeh [9, 10]. This theory insists on the uncertainty which appears in phenomena and actual situations.

It is known that the world is perceived through human senses, but these perceptions are not precise. In addition, each person perceives something a little different from a similar fact or situation. In fact, precision is difficult to reach because things do not remain the same constantly [11]. Fact and individuals are not isolated and interact in imprecise ways.

Fuzzy logic is an approximate reasoning method. It is the application of a human mode of perception and deduction rather than the application of an electronic binary mode. This theory is confronted with three challenges:

- Real situations are difficult to define

-To describe a system we need more data than a human can identify, analyze and understand at the same time.

- Faculty of making precise meaningful conclusions decreases when systems are more complex [12].

With this theory, computer analysis can be done using rules coming from functions in a formulated expert knowledge.

Membership functions: The basic idea of fuzzy sets theory is the concept of « membership function ». It's a kind of extension of the concept of « characteristic function » in the classical sets theory. Let A be a non fuzzy set and μA its characteristic function μ_A is defined by:

$\mu A(x) = 1$ if $x \in A$

$\mu A(x) = 0$ if $x \notin A$

Let Ă be the fuzzy set on A and $\mu_{Ă}$ it's « membership function »
$\mu_{Ă}(x) = i$ with $0 \leq i \leq 1$ means that x belongs to A with a degree: i. [12].

According to certain authors, the balance principle of the membership function can be extended to other logic elements like the existential quantificator $\exists$ (x).

Let a set $X = \{x_1, x_2, ...x_i, ..., x_n\}$

$\exists_\alpha (X_1)$. such that $\alpha = 1$ means it exists a x_i
such that $\alpha = 0$ means it doesn't exist a x_i
such that $0 < \alpha < 1$ means that xi has a fuzzy existence

*2.2- F*uzzy *temporal logic*

Temporal logic formula (Khaled Ben Lamine)[13] has been used successfully for specifying properties for the purpose of concurrent systems.

The fuzzy temporal formulas are constructed from enumerable collection of propositions deriving from the Boolean connectives (OR, AND,…) and the temporal connectives:

O next
□ Always
◊ Eventually
U until

Truth of a proposition is a real value in the interval [0, 1]; For instance, the truth value of the proposition: ***ObservableMicroobject*** will be a real number in the interval [0, 1] reflecting our incapacity to draw clear boundaries between the true value and the falseness of a proposition. This allows us to include fuzzy statements such that;

Slightlyobservable or Completelyobservable

Partiallyexisting or Completelyexisting

If now, we consider for instance an event in the temporal domain T and the existential quantifier.

Let a set $T = \{\dots t_{-2}, t_{-1}, t_0, t_1, t_2, ... t_i, ..., t_n\}$

such that $\alpha = 1$ means that the actualised event is in the present

$$\exists_\alpha A(t_0)$$

such that $\alpha = 0$ means that the event is not actualised in the present
such that $0 < \alpha < 1$ means that the event is potential in the present

$$\exists_\alpha A(t_0 + k)$$

such that $0 < \alpha < 1$ means that the event is potential in the future α

$$\exists_\alpha A(t_0 - k)$$

such that $0 < \alpha \leq 1$ means that the event is in the past, potential or actualised

α = density of the presence, different of the presence probability.

Same as before, the time density will be distinguished from the probability. In this theory, we attribute “a fuzzy existence” to the potential events which have not the weight of the present. It is also another way of quantifying the time.

In fuzzy set theory we have different operators like in classical set theory (inclusion, union, intersection...). However, fuzzy set theory is one extension of « Boolean algebra » because of the membership aspects.

2.3-Probability and fuzzy set theory

Fuzzy set theory was sometimes assimilated with probability theory. But it is false. In fact some aspects are different. The sum of probabilities p(u) of elements u from U equals 1 but the sum of membership degree $\mu_A(u)$ of u from A (included in referential U) does not equal 1 and is superior to 1.

3-Space extension and time extension of particle

By linking the Quantum Mechanics with fuzzy logic we make the hypothesis that particle's position could be represented with fuzzy logic.

In crisp logic the position of a particle is a mapping from N^3 to {0,1}. The mapping confers the truth value 0 or 1 on any statement of the position of the particle, so each statement of position is certainly true or certainly false.

If $X = \{x_1, x_2, x_3, ..., x_n\}$ is a domain where the particle is located , we can write the position of the particle Po_x as:

$$Po_x \in x_1 \vee Po_x \in x_2 \vee Po_x \in x_3 \vee ..... \vee Po_x \in x_n$$

But in this representation Po_x is located somewhere in the interval.

In many valued logic, other trues values are possible, in fuzzy logic the position of a particle is a mapping from a subset of R3 to the real interval [0, 1] and expresses the level of certainty of each statement of position. In applying the fuzzy set theory we can write:

$$Po_x \underset{\alpha}{\in} x_1 \wedge Po_x \underset{\beta}{\in} x_2 \wedge Po_x \underset{\gamma}{\in} x_3 \wedge .... \wedge Po_x \underset{\omega_i}{\in} a_n$$

Where α, β, γ, $\ldots\omega_i$ represent degrees of membership.

The particle is everywhere in the spatial domain with a certain « density of presence » but not with a probability of presence. This means also that the particle is extended in space with different degrees. We can develop similar explanation for the time extension.
Quantum logic takes into consideration the empirical principle that measurement gives imprecise information about objective reality.

4-Interpretation with fuzzy logic

4.1 Young's experiment

We explained this experiment with the fuzzy logic in the following way. Particle could have both time and space extension in a certain equilibrium state.(Figure 2). The young's experiment shows both aspects. If the source sends a high flow of electrons, we observe interferences immediately. This can be explained by space extension of particles which lead to interferences.

In fact with a high flow; the particle's position couldn't be reduced to a corpuscle leading to interferences. The space extension predominates over time extension which is hidden.

But when the source sends a low flow of electrons,the interferences aren't visible immediately. The space expansion does not exist any more. However, we can see the time expansion (the interference appears progressively). In other terms the future of a particle could interact with the past of another particle. A particle leaves a track which can interact with another particle. With this suggestion we replace the duality wave - corpuscle by a duality time extension or space extension.

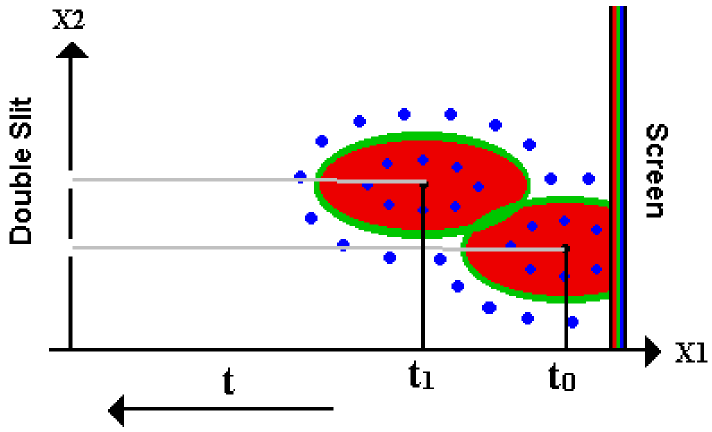

Fig. 2: Temporal extension and interaction between two micro objects.

4.2. Measure's theory and fuzzy automata

The emerging crisp reality (final step of measurement) carries less information that the underlying fuzzy reality : there is irreversible loss of information usually called collapse of the wave function, but from the point of view of Granik and Caufield it is not as much a collapse as a realisation of one of many possibilities existing within a fuzzy reality.
Any measurement rearranges the fuzzy reality leading to different detection outcomes according to the changing fuzziness.
If we assume the fuzzy nature of things then the apparent statistical character of physical phenomena would follow not from intrinsic randomness but from their fuzzy-deterministic nature.
Before one measurement all the solutions of the Schrödinger’s equations are possible. A measurement could be imagined as stopping a system that fluctuates on a definite domain. Fuzzy automata could model this idea [14, 15].

A fuzzy automaton is defined as a set of elements which are the states that represent the configuration of the system. For example, in a fuzzy world a particle located in a delimited space is everywhere but with a density of presence. Numerous possibilities of location exist which correspond to different configurations or different states of an automaton.

Concept of automata

The behaviour of an automaton is to alternatively wait in a state and perform a transition or event. We may think of the state as being information representing the "knowledge" of the automaton when in that state and the event as modifying that information. A state space will be a state ordered by information content while an event space will be a set of events ordered by time.

In the state and event space potential element have a fuzzy existence independantly of the probability.

Concept of fuzzy automata

A fuzzy automaton is a sextuple (Q, q_0, I, O, V, d, f) where :

Q = set of states (belong to states space)
q_0 = initial state
I = set of inputs symbols or input alphabet
V = valuation set [0.1]
d = transition function: d: Q x I x Q $\longrightarrow$ V
f = final state determination function f:: Q $\longrightarrow$ V

The states can represent the configuration of the system. For example, in a fuzzy world a particle located in a delimited space is everywhere in this delimited space but with a density of presence (valuation). Numerous possibilities of location exist which correspond to different configurations or different states of an automaton.

There is an initial state which corresponds to the first configuration of the automaton. Moreover, there are inputs which allow the system to change from a state to another. We can have one or several inputs.

For example, the initial state which represents the position of a particle which belongs together in 4 delimited regions (x_1, x_2, x_3, x_4) and is defined as follows:

	x_1	x_2	x_3	x_4
q_0	0.3	1	0.4	0

where the valuations represent membership degree of the particle to the regions. These numbers represent the spatial extension of the particle.

It means that the particle is a little in x_1 and in x_3, entire lies in x_2 and not in x_4.

We deal also with a concept of composite state.

We have one input represented as a vector. At each state we apply the input and the system changes. The automata act as follows:

Initial state q_0 new state:

$$[x_1 \quad x_2 \quad x_3 \quad x_4] \cdot \begin{bmatrix} \mathbf{0} & \mathbf{0.2} & \mathbf{0.8} & \mathbf{0} \\ \mathbf{0.8} & \mathbf{0.9} & \mathbf{0} & \mathbf{0.4} \\ \mathbf{0.3} & \mathbf{0.5} & \mathbf{0.5} & \mathbf{1} \\ \mathbf{1} & \mathbf{0.2} & \mathbf{0.3} & \mathbf{0} \end{bmatrix} = [x_1' \quad x_2' \quad x_3' \quad x_4']$$

Where the matrix represents the transition states of the automata and not a probability matrix.

x_1’ = max (min (0, x1), min (0.8, x2), min (0.3, x3), min (1, x4)) = 0.8
x_2’ = max (min (0.2, x1), min (0.9, x2), min (0.5, x3), min (0.2, x4)) = 0.9
x_3’ = max (min (0.8, x1), min (0, x2), min (0.5, x3), min (0.3, x4)) = 0.4
x_4’ = max (min (0, x1), min (0.4, x2), min (1, x3), min (0, x4)) =0.4

At the end of the measurement the particle is principally in the state x'_2 , a little less in x'_1 and more less in x'_3 and x'_4.
If the level **l** of observability wil be l ≥ 0,9 only the state x’2 will be observable.
The matrix which describes the state transition after a particular input is **not a probability matrix.** The sum of line or column is not necessary equal to 1.

5-Discussion

The fuzzy set theory permits to have a new definition of the interpretation of wave particles duality and in general of Quantum mechanics and relativity by the uses of the particle extension in space and in time.

It permits new perspectives to find an explanation of Young's experiment and of the measure's theory. Particle interactions create a sort of particular spatio-temporal extensions (deformations) which do not follow traditional representation.

Moreover, the application of fuzzy logic could lead to specify the notion of hidden variables which are the time and space extensions. We give thus the same interpretation as the one enounced by Einstein, De Broglie and Bohm.

Another way of viewing Young's experiments would be to use Mezey's formulation which explains that any piece of a molecule determines the properties of the whole [16].Therefore, when we throw a high flow of particles through two gaps, each flow of particles could have the same properties as the entire one.

In fact, this formulation could be the result of the matter's extension described below.

In conclusion, we can also link fuzzy set theory and the corresponding tools of artificial intelligence (developed in the "Laboratoire de Bioinformatique" of Tours) as knowledge engineering systems [17, 18, 19]. These applications are composed of three main components: a base of facts, a base of rules and an interference engine.

The Expert System could be among researchers a collective memory and "intelligence".

General conclusion

In this paper, we assume a fuzzy nature of micro-objects in quantum mechanics. This approach leads to six main conclusions regarding classical Quantum Mechanic issues:

Micro-object: we will not speak about particle or wave.

In view of this fuzzy interpretation of Quantum mechanics, it does not seem strange that a particle Micro – Object sometimes can exhibit wave's properties.

Degree of spatial membership of particle:

To adapt the concept of fuzziness location of a particle, we introduce the notion of particle membership in a spatial domain (2d or 3d), but also in a temporal domain (4d).

If μ = membership density of a particle, the degree of membership of the particle to an elementary domain Δ V is : $\mu \Delta$ V.

Wave function:

A wave function then is treated as a function describing a deterministic entity having a fuzzy character. A consequence of such an interpretation is that the complementarity's principle and wave particle duality can be abandoned in favour of a fuzzy deterministic Micro-object.

The double slit experiment:

The double slit experiment can be interpreted now as a Micro-Object's interference with itself (in space and time), because it has a simultaneous membership in all parts of space including elementary volumes containing both slit. (Figure 3). Since the total membership of a Micro-Object in a given finite volume, is fixed, any change of its membership in one of the slits affects the membership everywhere leading to the interference effect.

Measurement:
A measurement is defined as a continuous process of defuzzification whose final stage detection, is inevitably accompanied by a dramatic loss of information through the emergence of locality. As some authors say, fuzziness density varies from diffuse to sharp in Quantum mechanics and any measurement rearranges the fuzzy reality leading to different detections.

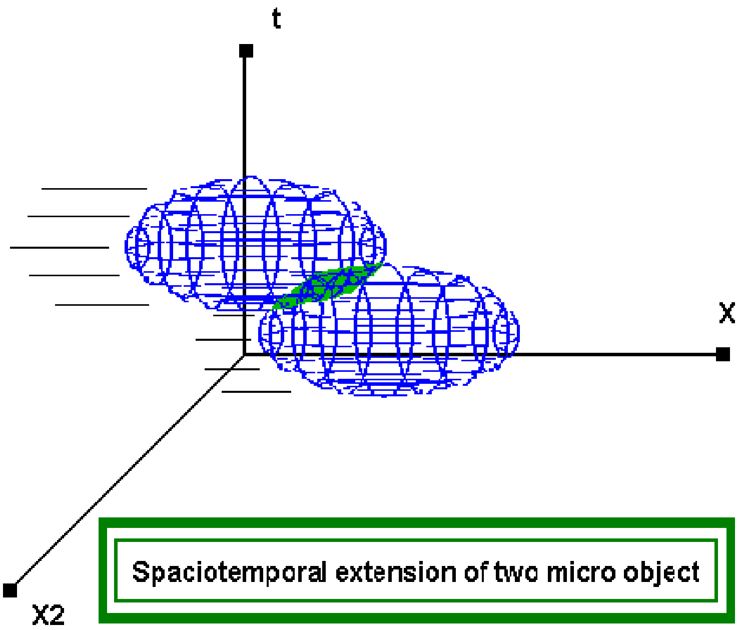

Fig. 3 : Spatio-temporal extension of two micro-object.

Reality:
Reality is fuzzy and non local not only in space but also in time, which means that any Micro-Object has membership in space and in both the future and the past (time extension). In this sense one might define the present as a time average over the membership density.

• Acknowledgements

We wish to thank Mr Joseph LEVY, Matthieu GOYAT, Antoine SAUTEL and Antoine Mathias for their suggestions and the CIO Bank for their support.

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Appendix 1- Fuzzy numbers

Imprecise statements about quantities can be regarded as fuzzy subsets of the real line whose elements are real numbers. From a set of integers we considered the fuzzy subset « position » which represents the location of a particle (figure 4).

Position: integers $\longrightarrow$ [0,1]

Such that the membership function $f_{position}(x)$ is a monotonic function of x , with $f_{position}(x) = 0$ when the particle isn't present here an $f_{position}(x) = 1$ when the particle is entirely localized at this point. There are also all the intermediate values.

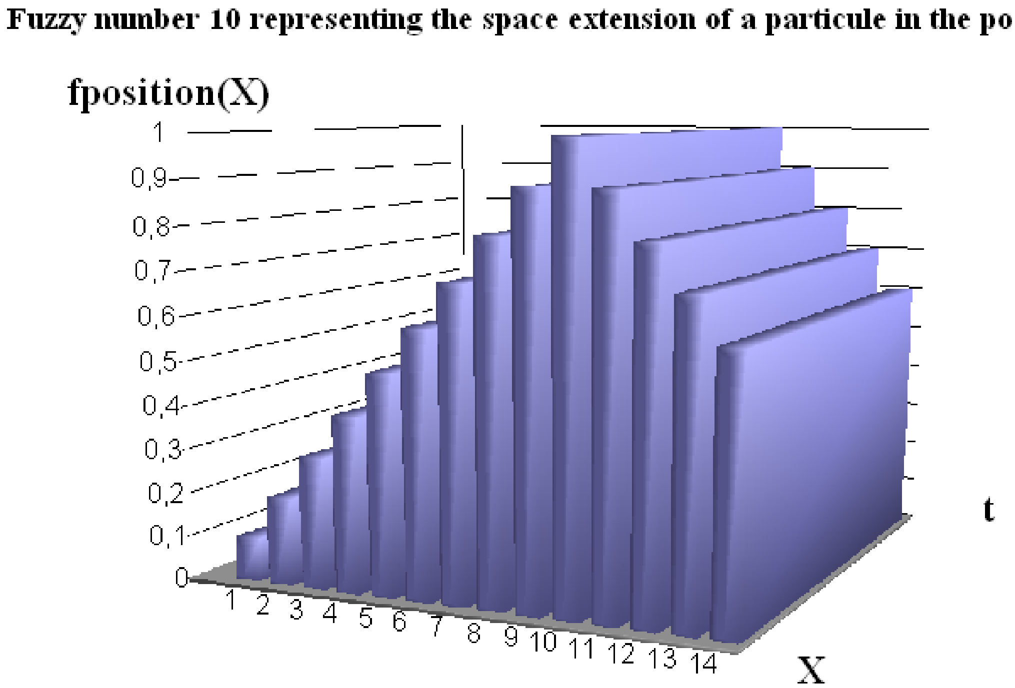

Fig 4 : Representation of fposition (x) invariante during the time t

The proposition: **"particle A is in a position x" = TRUE becomes**:

"**Particle A is in a position x" = $f_{position}$ (x)**

Quantum mechanics works with amplitudes which could be interpreted as complex-valued truth values.

The fuzzification of the proposition:

" **IF A is a x at time t AND has velocity v THEN A is at x+vdt at time t+dt"** would be:

$$d/dt\,(f_{position}(x,t)) = d/dx\,(f_{position}(x,t))$$

to get the Schrödinger equation , we would need in addition:

$$v = i\,d/dx\,(f_{position}(x,t))$$

The propositions developed supra constitute rules of the expert system.

The Schrödinger equation is somehow contained in the fuzzification of position.

Appendix 2 - knowledge bases system

In order to link physical experiments, classic models of Quantum Mechanics, new logical approaches, knowledge bases systems could be an interesting method.

We can also link fuzzy set theory and the corresponding tools of artificial intelligence (developed in the "Laboratoire de Bioinformatique") as knowledge engineering systems [13, 14, 15]. These applications are composed of three main components: a base of facts, a base of rules and an interference engine.

The base of facts can be defined as the collected data known by the expert system. **The base of rules** contains the physicist's expert know how expressed in the form of production rules. Expert system rules correspond approximately to the different elements of the expert's reasoning.

IF <condition> ≤

THEREFORE <conclusion>

In which <condition> is a fact or a gathering of premise facts of the rule and <conclusion> is a conclusion fact of the rule.

The function of the **inference engine** is to simulate the expert's reasoning by using data and rules existing in the base of knowledge of the system. Therefore, the inference engine determines if a rule must or must not be activated.

This system will permit to generate a set of inferences rules which can describe certain paradoxes and contradictions of Quantum Mechanics and give some solutions of these paradoxes.

An example of rules could be:

IF gap1 **AND** gap2 **AND** Flow of photons < 2/s **AND** Time x≤

THEREFORE Localized impact

IF gap1 **AND** gap2 **AND** Flow of photons < 2/s **AND** Time > x

THEREFORE Interferences

IF Diameter gap1 <= wavelength of light **AND** Diameter gap2 <= wavelength of light **THEREFORE** Diffraction

The Expert System could be among researchers a collective memory and "intelligence".