SOLUTIONS FOR QCD, ASTROPHYSICS AND COSMOLOGY

S.B.M. Bell†

Interdisciplinary Quantum Group, Department of Computer Science

The University of Liverpool, Chadwick Building, Peach Street

Liverpool, L69 7ZF, United Kingdom

ABSTRACT

Scaling combinatorial hierarchies in which each level describes a circular motion have been found by us to be an intrinsic part of quantum electrodynamics and general relativity. From them we derive the combinatorial hierarchy that is the raison d'être of the Alternative Natural Philosophy Association (ANPA) [Kilmister 2003a]. This is used to derive the following (1) the electromagnetic theory, responsible for binding atoms together, from the gravitational, including the derivation of an approximate value for the fine structure constant, then (2) the weak interaction, responsible for radioactivity, including an approximate value for the Weinberg angle and finally (3) the strong interaction, responsible for binding the nuclei of atoms together, including an approximate value for the quark-gluon-quark coupling constant.

In the first of a trilogy of papers, we considered electromagnetism. We derived the masses of the *Z*-and *W*-bosons without introducing the Higgs

† Mail@sarahbell.org.uk

boson, the masses of the μ meson and the τ meson relative to the mass of the electron and a parameter associated with the Cabibbo-Kobayashi-Maskawa matrix in the second part of the trilogy. In the third part we derived some of the parameters associated with the strong interaction, the masses of the π meson and the proton relative to that of the electron, further parameters associated with the Cabibbo-Kobayashi-Maskawa matrix and discussed our description of the broken weak interaction. Here we show that quantum chromodynamics (QCD) is based on the same scaling hierarchy of spins as our quantum version of General Relativity, quantum electrodynamics and the Electro-weak interaction.

We also derive the mass of the proton from the fourth member of the ANPA combinatorial hierarchy alone, and thus the mass of the electron itself. This last entails some consideration of the structure of the solar system, where we derive the Titus-Bode empirical law for the orbital radius of the planets, the Milky Way Galaxy, where we derive the phenomena of Modified Newtonian Gravity (MOND), and the cosmos, where we derive the Red Shift, all from the quantum version of General Relativity. These successes provide evidence of the correctness of this particular theory of quantum gravity, which may not be unique.

At this point every parameter of the Standard Model derived by us, both here and previously, is given as an expression using only the coupling constants and the dimension bearing constants, *c*, the speed of light, the usual value of *h*, Planck's constant, and *G*, the gravitational constant. The coupling constants themselves appear as the four members of the ANPA combinatorial hierarchy, an idea which is itself a model of man's psychology, and, finally, it is this on which the structure rests.

1. PRELIMINARIES

1.1 General Introduction

This paper follows on from a trilogy of papers describing the ANPA combinatorial hierarchy. The ANPA combinatorial hierarchy is fascinating because it is a most unusual hierarchy derived from very simple criteria [Manthey 1993], [Kilmister 2003a&b], [Bastin and Kilmister 1995], [Parker-Rhodes 1981]. The hierarchy in its simplest form has members 3, 10, 137 and $2^{127}+136 \approx 1.7\times 10^{38}$, ending on the last term. It was such an unusual structure, based on the simplest hypothesis on discerning similarity and difference and also using a simple method of aggregation to consolidate each judgement, that many were sure it held some deep truth. It was suggested that 3 described the weak interaction, 10 the strong interaction, 137 the electromagnetic interaction and $\sim 1.7\times 10^{38}$ gravity. The trilogy of papers and this paper uses the ANPA hierarchy to calculate some of the arbitrary parameters of the Standard Model, the recognised theory combining the electromagnetic, the weak and the strong interactions. There are eighteen arbitrary parameters in the Standard Model [Cottingham and Greenwood 1998], where the Model does not predict the value and this is set to agree with experiment. We have already calculated several.

We calculated the fine structure constant in the first paper of the trilogy [Bell 2005], and in the second paper in parts A [Bell and Diaz 2005b], B [Bell and Diaz 2005c] and C [Bell and Diaz 2005d] we calculated the weak coupling constant, which depends on the Weinberg angle. We also showed that we could calculate the mass of the *W*-boson and *Z*-boson without knowing the mass of the Higgs boson or the vacuum expectation value of the Higgs field. We calculated the ratio of the mass of the μ and τ meson to that of the electron, and showed that one angle defined by the Cabibbo-Kobayashi-Maskawa (C-K-M) matrix is zero. We discussed the varying size of orbits in terms of the relevant hierarchy, and a way of representing more than one particle without recourse to a field theory. In the third part of the trilogy, [Bell and Diaz 2005e], we derived the strong interaction coupling constant, we calculated the masses of the π meson and proton relative to that of the electron, and two more parameters associated with the C-K-M matrix. We also continued our discussion of the weak interaction and the interrelation between terrestrial and celestial radiation.

In this paper we put quantum chromodynamics in a form that shows it can participate in the same scaling hierarchies of spins as we have used throughout our account [Bell et al. 2000], [Bell et al. 2004a&b], [Bell and Diaz 2002], [Bell 2004, 2006], [Bell and Diaz 2003], [Bell and Diaz 2004a&b], [Bell 2005], [Bell and Diaz 2005a,b,c,d&e]. We also put the last touches on our study of the scaling hierarchy of spins for the Electro-weak interaction. Finally, we use the fourth term in the ANPA hierarchy to calculate the mass of the electron itself, which necessitates a review of the Galaxy seen as a quantum state. To demonstrate that this is unexceptional we also show that the solar system and the universe can also be seen in this light. All our calculations are then based on the coupling constants and our model of scaling spins alone, except for the dimension-bearing constants, that is, Planck's constant, *h*, the speed of light, *c*, and the gravitational constant, *G*. This means that thirteen of the eighteen arbitrary parameters of the Standard Model depend on only on the four coupling constants, which depend in their turn on the ANPA hierarchy and its derivatives. The quantum gravitational theory, of which general relativity is a classical limit, forms an integral part of all this, which provides evidence for the applicability of this quantum theory.

2. SOLUTIONS TO THE QCD AND WEAK EQUATIONS

2.1 Introducing a Symmetry into QCD

Now we shall show the connection between the Kabbalah model in paper three A and quantum chromodynamics (QCD). We shall find that our remarks also apply to our models of the Electro-weak theory, for the unbroken symmetry we discussed in paper two parts A, B and C. We shall call the Electro-weak theory with unbroken symmetry Quantum Flavour Dynamics or QFD. Attempts have been made to simplify QCD before [Davies and Lepage 2003], [Mackenzie 2005]. We take this one stage further, doing away with the requirement for a computer altogether in certain simple cases. We simplify QCD by overlaying the three dimensional complex space on which the Gell Mann matrices of SU(3) act so that these particularly simple states are singled out for attention. We find that these states account for all the relevant calculations we have made in this trilogy of papers. In detail, these are for the mass of the proton, which involves two up quarks and one down quark, the mass of the $\pi_{\pm}$ meson, which involves one up and one down quark, and parameters of the C-K-M matrix, which involves all flavours of quarks. The main difference between our approach and solving the equations in their original complexity using lattice QCD [Montvayand Münster 1997] is that the quarks appear as concentrations of the wave function of a single particle in the former and as independent particles in the latter.

We use the Kabbalah model described in paper three A of the trilogy, placing it on its usual base composed of the points marked by rubber bands, G_P, G_R and P_R. We assign to the vector between the knot and the upper vertex marked by the purple and green bead the symbol P_G, and similarly for R_P and R_G. We assign to the plane specified by the vectors between the knot and R_P and R_G the symbol (P_G), and similarly for the planes (R_P) and (R_G). We now pick three Kabbalah models and assign the first to $P_G(\cdot)$, the second to $R_P(\cdot)$ and the third to $R_G(\cdot)$. This provides a nine-dimensional space instead of the eight-dimensional one required for the QCD Gell-Mann matrices. We attach the label inside the brackets to the vector in the model orthogonal to the plane rather than the plane itself. We reduce the space to eight dimensions by identifying $R_P(P_G)$ and $R_G(R_P)$. Choosing one vector and one plane in cyclical fashion, we assign these to the generators of SU(3) [Cottingham and Greenwood 1998],

Table I

SU(2) Assignations for SU(3) (first four)

Symbol	**SU(3) Matrix**	**Vector (Plane)**	**SU(2) Matrix**
λ_1	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$P_G\left(P_G\right)$	$i\mathbf{i}_1$
λ_2	$\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$P_G\left(R_P\right)$	$i\mathbf{i}_2$
λ_3	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$P_G\left(R_G\right)$	$i\mathbf{i}_3$
λ_4	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$	$R_P\left(R_P\right)$	$i\mathbf{j}_1$

Table I

SU(2) Assignations for SU(3) (second four)

Symbol	**SU(3) Matrix**	**Beads**	**SU(2) Matrix**
λ_5	$\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$	$R_P\left(R_G\right)$	$i\mathbf{j}_2$
λ_6	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$R_G\left(R_G\right)$	$i\mathbf{k}_1$
λ_7	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$	$R_G\left(P_G\right)$	$i\mathbf{k}_2$
λ_8	$\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$	$\frac{R_P\left(P_G\right)+R_G\left(R_P\right)}{2}$	$1/2\times\left(i\mathbf{j}_3+i\mathbf{k}_3\right)$

Here the $\mathbf{i}_r$ are a first copy of the quaternion matrices [Bell et al. 2000], [Bell and Diaz 2003], the $\mathbf{j}_r$ a second copy and the $\mathbf{k}_r$ a third copy. λ_8 has a different fractional multiplier to that usually quoted, $1/\sqrt{3}$, [Cottingham and Greenwood 1998]. This will change the structure constants, although we could accommodate it by changing the value of G_8, the scalar magnitude assigned to this matrix for gluon field. However, in any case we are going to assign the quaternion matrices in such a way that they fit the limitations of the Kabbalah model, with only three planes distinct rather than the eight required by the SU(3) generators.

To that end, we consider the product,

$$\lambda_8^2 = \frac{1}{4} \times \left(-\mathbf{j}_3^2 - \mathbf{k}_3^2 - 2\mathbf{j}_3\mathbf{k}_3 \right) \tag{2.1.A}$$

This will be one, like *i* times the square of all the individual quaternion matrices, if we set $\mathbf{j}_3 = \mathbf{k}_3$. Since we chose the definitions of the λ_r on a strictly cyclical basis, we have no room for manoeuvre, but must follow suit with,

$$\mathbf{j}_r = \mathbf{k}_r = \mathbf{i}_r \tag{2.1.B}$$

leaving distinct only three combinations, say $P_G\left(P_G\right)$, $P_G\left(R_P\right)$, and $P_G\left(R_G\right)$, identifying the three physically distinct planes of the model. Here, we have overlapped, say, only dimensions two and three of the three-dimensional complex spin space on which the λ_r act, since the first five of the λ_r can be used to generate the other three λ_r matrices [Cottingham and Greenwood 1998].

It may be objected that this has changed SU(3) into SU(2), which does not have the same structure constants. Since these appear in the dynamical equations of QCD for the gluon-gluon current self-interaction, that might mean that we are not replicating QCD at all. The reason we may nevertheless adopt this approach is that, under conditions of great symmetry for the G_μ^r, where *G* is the amplitude of the gluon field, *r* indicates the Gell-

Mann matrix and μ is the spatial index, the structure constants do not appear. We therefore do not mind which group we are using. The questions are, does this condition of symmetry allow a solution? and, if so, does it fit a physical state or states? The answer to both is "yes" as we will show.

If we consider a particular example of the gluon field, say, $\mathbf{G} = \sum_{l=1}^{8} G_{\mu}^{l`} \lambda_r$, where we have dropped the usual factor of an half, the overlapping of spin space means that we must have,

$$G_{\mu}^{1} = G_{\mu}^{4} = G_{\mu}^{6}, \quad G_{\mu}^{2} = G_{\mu}^{5} = G_{\mu}^{7} \quad G_{\mu}^{3} = G_{\mu}^{8}, \qquad (2.1.C)$$

because the spaces that these $G_{\mu}^{l`}$ describe are not physically distinct. We may accomplish this by setting,

$$\begin{aligned} &G_{\mu}^{1} \rightarrow 3G_{\mu}^{1}, \quad G_{\mu}^{2} \rightarrow 3G_{\mu}^{2}, \quad G_{\mu}^{3} \rightarrow 2G_{\mu}^{3}, \\ &G_{\mu}^{4} = G_{\mu}^{6} = G_{\mu}^{5} = G_{\mu}^{7} = G_{\mu}^{8} \rightarrow 0 \end{aligned} \qquad (2.1.D)$$

Because we are considering a suitably symmetrical field, we have been able to reduce the SU(3) character of QCD to its subgroup, SU(2). This is also the group for QFD. Because of this, we may and shall ensure that our ensuing remarks fit this interaction too. In particular, our discussion of gauge invariance following in section 2.2 applies to both. This was omitted in our previous discussion of the Electro-weak theory in paper two A. We shall show that if a field has U(1) gauge invariance, it must automatically have SU(2) gauge invariance also, and vice versa, if it is invariant under all types of rotation. This effectively reduces the group SU(2) to the group U(1), and we shall do exactly that in sections 3, 4 and 5.

2.2 Gauge Invariance

We shall discuss a field with a U(1) symmetry, which is the one enjoyed by the electromagnetic field and also a quantum theory with General Relativity as a classical limit, since this follows the rules of Quantum Electrodynamics (QED) [Bell and Diaz 2002], [Bell 2004]. We will use Cottingham and Greenwood's notation [1998, pages 67 and 104]. We may write for the gauge transformation,

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu \chi(x) \tag{2.2.A}$$

where A_μ is a component of the potential, x stands for all four co-ordinates and χ is an arbitrary function. We introduce the reflector form of the potential,

$$A_\mu \to \underline{\mathbf{A}} = \begin{pmatrix} 0 & \mathbf{A} \\ \mathbf{A}^\ddagger & 0 \end{pmatrix}, \quad \mathbf{A} = A_0\mathbf{i}_0 + A_1\mathbf{i}_1 + A_2\mathbf{i}_2 + A_3\mathbf{i}_3 \tag{2.2.B}$$

where superscript $\ddagger$ stands for quaternion conjugation, $\mathbf{i}_0$ is the unit matrix and the $\mathbf{i}_r$ are the quaternion matrices to a base $-i\sigma_r$, where the σ_r are the Pauli matrices and we may introduce imaginary components. The details are to be found in Bell et al. [2000]. We introduce an expression using $\chi(x)$,

$$\chi \to \underline{\chi} = \begin{pmatrix} 0 & \chi \\ \chi & 0 \end{pmatrix} \tag{2.2.C}$$

Given the properties of quaternions [Bell et al. 2000], and since $\chi^\ddagger = \chi$, we may lift equation (2.2.A) to,

$$\underline{\mathbf{A}}(x) \rightarrow \underline{\mathbf{A}}'(x) = \underline{\mathbf{A}}(x) + \partial_\mu \underline{\boldsymbol{\chi}}(x) \tag{2.2.D}$$

We define two possible rotator matrices [Bell et al. 2000],

$$\mathbf{R}| = \begin{pmatrix} \mathbf{R} & 0 \\ 0 & \mathbf{R} \end{pmatrix}, \quad \mathbf{R}| = \begin{pmatrix} \mathbf{R} & 0 \\ 0 & \mathbf{R}^{\ddagger} \end{pmatrix} \tag{2.2.E}$$

where $\mathbf{R}$ is a quaternion with unit modulus. In this case $\mathbf{R}|$ is the matrix used to obtain a spatial or temporal rotation of the Dirac and Maxwell equations [Bell et al. 2000]. $\mathbf{R}\mathbf{R}^{\ddagger}$ is unity and $\mathbf{R}|\mathbf{R}|^{\ddagger}$ the unit matrix. The right-hand side in equation (2.2.D) continues to be a valid solution of the Maxwell and Dirac equations after a rotation, provided the other variables are suitably transformed, and in that case equation (2.2.D) becomes

$$\underline{\mathbf{A}}(x) \rightarrow \underline{\mathbf{A}}'(x) = \mathbf{R}|\,\underline{\mathbf{A}}(x)\mathbf{R}|^{\ddagger} + \mathbf{R}|\partial_\mu \underline{\boldsymbol{\chi}}(x)\mathbf{R}|^{\ddagger} \tag{2.2.F}$$

Now the upper quaternion element of the reflector solution of this matrix equation reproduces the generalisation of the U(1) symmetry implied in equation (2.2.A) to an SU(2) symmetry or temporal rotation of such, as may be checked [Cottingham and Greenwood 1998 page 104].

The U(1) symmetry in equation (2.2.A) must be accompanied by a change in the wave function of the particle described, if the result is to continue to be a solution of the Dirac and Maxwell equations. This is given by,

$$\psi \rightarrow \psi' = e^{iq\chi}\psi \tag{2.2.G}$$

where q is a real scalar. Here ψ is a column matrix satisfying the Dirac equation in its original form. Indexing the four components by μ, we may write for each one,

$$\psi_\mu \rightarrow \psi'_\mu = e^{iq\chi}\psi_\mu \tag{2.2.H}$$

Since the components of the wave function are linear transformations of the ψ_μ, say ϕ_μ, [Bell et al. 2000], we have also,

$$\phi_\mu \rightarrow \phi'_\mu = e^{iq\chi}\phi_\mu \tag{2.2.I}$$

Defining,

$$\underline{\mathbf{\Phi}} = \begin{pmatrix} 0 & \mathbf{\Phi}^1 \\ \mathbf{\Phi}^2 & 0 \end{pmatrix}, \quad \mathbf{\Phi} = \phi_0^r\mathbf{i}_0 + \phi_1^r\mathbf{i}_1 + \phi_2^r\mathbf{i}_2 + \phi_3^r\mathbf{i}_3 \tag{2.2.J}$$

where $\underline{\mathbf{\Phi}}$ satisfies the Dirac equation, we may lift equation (2.2.I),

$$\underline{\mathbf{\Phi}} \rightarrow \underline{\mathbf{\Phi}'} = e^{iq\chi}\underline{\mathbf{\Phi}} \tag{2.2.K}$$

The right-hand side continues to be a valid solution of the Maxwell and Dirac equations after a rotation, provided the other variables are suitably transformed, and in that case the equation becomes

$$\underline{\mathbf{\Phi}} \rightarrow \underline{\mathbf{\Phi}'} = e^{iq\chi}\mathbf{R}\!\!\mid \underline{\mathbf{\Phi}}\mathbf{R}\!\!\mid^{\ddagger} \tag{2.2.L}$$

However, if $\underline{\mathbf{\Phi}'}$ is a solution of the Dirac equation with four-vector behaviour of both the mass and the wave function of the particle, then so is $e^{iq\chi}\mathbf{R}\!\!\mid \underline{\mathbf{\Phi}}$ [Bell et al. 2000] for scalar behaviour of the mass, permitting us to write,

$$\underline{\mathbf{\Phi}} \rightarrow \underline{\mathbf{\Phi}''} = e^{iq\chi}\mathbf{R}\!\!\mid \underline{\mathbf{\Phi}} \tag{2.2.M}$$

The upper quaternion element of the reflector solution of the matrix equation reproduces the generalisation of the U(1) symmetry implied in equation (2.2.G) to an SU(2) symmetry or a temporal rotation of such, as may be checked [Cottingham and Greenwood 1998 page 105].

Since, given the terrestrial Maxwell and Dirac equations, we may prove that a celestial version is also true, if relativistic invariance is also assumed, [Bell and Diaz 2003], we have demonstrated that the Maxwell and Dirac equations necessarily imply that a SU(2) symmetry and a temporal rotation of an SU(2) symmetry can hold as well as the U(1) one in either the terrestrial or celestial case. Our arguments may be rehearsed backwards, and the converse also holds. If we then reduce our reflectors to a single quaternion, as we do for the Electro-weak interaction, all will be well for spatial rotations, but we shall have to posit rather than derive the gauge behaviour just exhibited.

2.3 A First Simplification of the Equations of QCD

We shall continue to follow Cottingham and Greenwood [1998] throughout the rest of section 2. The quark has a wave function, $\mathbf{q}$,with colour attached. There are three of these colours, usually called red, green and blue. In our case, we have replaced blue with purple. These go with the three by three generators of SU(3). The quark wave function reduces from three components to two, the same number as the components of QFD, when we overlap spaces as we suggested in 2.1. That is

$$\mathbf{q} = \begin{pmatrix} q_r \\ q_g \\ q_p \end{pmatrix} \to \begin{pmatrix} q_0 \\ q_1 \end{pmatrix} \tag{2.3.A}$$

The matrices describing the radiation which is responsible for the interaction of one quark with another, the gluon field, are reduced from eight for the eight generators of SU(3) to three for the three matrices of SU(2), using equation (2.1.D),

$$\mathbf{G}_\mu = G_\mu^1 i\mathbf{i}_1 + G_\mu^2 i\mathbf{i}_2 + G_\mu^3 i\mathbf{i}_3, \qquad (2.3.B)$$
$$\mathbf{G}_\mu = G_\mu^0 \mathbf{i}_0 + G_\mu^1 i\mathbf{i}_1 + G_\mu^2 i\mathbf{i}_2 + G_\mu^3 i\mathbf{i}_3$$

where the second equation corresponds to QFD, which has an extra term of $G_\mu^0 \mathbf{i}_0$, where $\mathbf{i}_0$ is the unit matrix. We will write,

$$\left(i\mathbf{i}_1, i\mathbf{i}_2, i\mathbf{i}_3\right) \rightarrow \mathbf{i}_\xi \qquad (2.3.C)$$
$$\left(\mathbf{i}_0, i\mathbf{i}_1, i\mathbf{i}_2, i\mathbf{i}_3\right) \rightarrow \mathbf{i}_\xi$$

Leaving context to decide whether we mean the first expression on the left, referring to QCD, or the second, referring to QFD. It should be recalled that, in the case of QCD rather than QFD, the matrices, $\mathbf{i}_\xi$, perform the same role as the eight λ_r. We define,

$$\mathbf{G}_{\mu\nu} = \partial_\mu \mathbf{G}_\nu - \partial_\nu \mathbf{G}_\mu + ig\left(\mathbf{G}_\mu \mathbf{G}_\nu - \mathbf{G}_\nu \mathbf{G}_\mu\right) \qquad (2.3.D)$$

where we note that the first two terms on the right also appear in the electromagnetic case, but that the last two terms do not appear, although they do when we consider the Electro-weak interaction.

The first of the field equations of QCD is then,

$$i\gamma^\mu\left(\partial_\mu + ig\mathbf{G}_\mu - m_f\right)\mathbf{q}_f = 0 \qquad (2.3.E)$$

where γ^μ are the Dirac matrices, g is the coupling constant, f indexes the flavours and m_f is the mass of the quark with that flavour. Expanding $\mathbf{G}_\mu$ using equation (2.3.B),

$$\left(i\gamma^{\mu}\partial_{\mu} - ig\gamma^{\mu}G_{\mu}^{\xi}\mathbf{i}_{\xi} - m_f\right)\mathbf{q}_f = 0 \tag{2.3.F}$$

The second of the equations of QCD, describing the gluons, is then,

$$\partial_{\mu}G^{\xi\mu\nu} = j^{\xi\nu} \tag{2.3.G}$$

with j the current. This is defined by,

$$j^{\xi\nu} = g\left[f_{\xi\beta\chi}G_{\mu}^{\beta}G^{\chi\mu\nu} + \sum_f \overline{\mathbf{q}}_f\gamma^{\nu}\mathbf{i}_{\xi}\mathbf{q}_f\right] \tag{2.3.H}$$

where ξ, β, χ range over one to three in the case of QCD and zero to three for QFD. The first term on the right involving the structure constants of SU(3), $f_{\xi\beta\chi}$, does not appear for QED. It does for QFD, but in that case it involves the structure constants of SU(2). The structure constants of SU(2) and SU(3) differ, but our approach will ensure that they do not appear.

2.4 The QCD Particle Equation in a Special Case

We now solve these equations in a special case. We have specialised already in assigning some of the same values to the QCD currents, reducing them to three different values, but we know that more is required. The first term in the QCD or QFD equivalent of the Dirac equation, (2.3.F), is

$$i\gamma^{\mu}\partial_{\mu}\mathbf{q}_f = i\left(\gamma^0\partial_0 + \gamma^1\partial_1 + \gamma^2\partial_2 + \gamma^3\partial_3\right)\begin{pmatrix} q_{0,0} & q_{0,1} \\ q_{1,0} & q_{1,1} \\ q_{2,0} & q_{2,1} \\ q_{3,0} & q_{3,1} \end{pmatrix} \tag{2.4.A}$$

where we have specifically indicated the space-time index (column) and colour index (row) on the wave function, q, dropping the flavour index, f, from q and m. That is because, in this rendition, there will only be a single particle, with each of the quarks being represented as a component of this single wave function. We continue with some more simplifying assumptions about the wave function. We suppose the state of each colour component of each quark is the same for each space-time component of the field and set,

$$q_{\mu,0} = q_\mu p_0, \quad q_{\mu,1} = q_\mu p_1 \tag{2.4.B}$$

This means that we may write the wave function in the form,

$$\mathbf{q} = \begin{pmatrix} q_{0,0} & q_{0,1} \\ q_{1,0} & q_{1,1} \\ q_{2,0} & q_{2,1} \\ q_{3,0} & q_{3,1} \end{pmatrix} = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} \otimes \begin{pmatrix} p_0 \\ p_1 \end{pmatrix} \tag{2.4.C}$$

where the $\otimes$ indicates a Cartesian product.

The second term in equation (2.3.F) is $ig\gamma^\mu G_\mu^\xi \mathbf{i}_\xi \mathbf{q}$, which we may expand,

$$\begin{aligned} & ig\gamma^\mu G_\mu^\xi \mathbf{i}_\xi \mathbf{q} = \\ & ig\left(\gamma^0 G_0^\xi \mathbf{i}_\xi + \gamma^1 G_1^\xi \mathbf{i}_\xi + \gamma^2 G_2^\xi \mathbf{i}_\xi + \gamma^3 G_3^\xi \mathbf{i}_\xi\right)\mathbf{q} = \\ & ig\left(\gamma^0\left(G_0^0 \mathbf{i}_0 + G_0^1 i \mathbf{i}_1 + G_0^2 i \mathbf{i}_2 + G_0^3 i \mathbf{i}_3\right) + \gamma^1\left(G_1^0 \mathbf{i}_0 + \ldots\right) + \ldots\right)\mathbf{q} \end{aligned} \tag{2.4.D}$$

where G_μ^0 is zero for QCD but not in general for QFD. We suppose a colour symmetry for the gluons in which,

$$\begin{aligned} & G_\mu^1 = G_\mu^2 = G_\mu^3 = G_\mu \big/ \sqrt{3}, \quad G_\mu^0 = 0, \\ & G_\mu^0 = G_\mu / 2, \quad G_\mu^1 = G_\mu^2 = G_\mu^3 = G_\mu \big/ 2\sqrt{3} \end{aligned} \tag{2.4.E}$$

where the first two equations apply to QCD and the last two equations apply to QFD, obtaining,

$$ig\gamma^{\mu}G_{\mu}^{\xi}i\mathbf{i}_{\xi}\mathbf{q} = igi\frac{\mathbf{i}_1+\mathbf{i}_2+\mathbf{i}_3}{\sqrt{3}}\left(\gamma^1 G_1+\gamma^2 G_2+\gamma^3 G_3\right)\mathbf{q}$$

$$ig\gamma^{\mu}G_{\mu}^{\xi}\mathbf{i}_{\xi}\mathbf{q} = ig\frac{\mathbf{i}_0+i\left(\left(\mathbf{i}_1+\mathbf{i}_2+\mathbf{i}_3\right)/\sqrt{3}\right)}{2}\left(\gamma^0 G_0+\gamma^1 G_1+\gamma^2 G_2+\gamma^3 G_3\right)\mathbf{q} \tag{2.4.F}$$

with the first equation applying to QCD and the second to QFD. For QCD, this is the point at which we transfer our notion of three components of the potential from the $\mathbf{i}_{\xi}$ matrices to the γ^{μ} matrices. Correspondingly, we set G_0 to zero, which means we are going to fix the frame of the equation. We shall not expand the last term in equation (2.3.F), merely remarking that m_f is scalar. In future references we shall drop the *f*.

We no longer want the $\mathbf{i}_{\xi}$ matrices, so we perform a rotation in SU(2) spin space next [Bell et al. 2000], under which equation (2.3.F) in either its original form, or the simplified form we have just derived, is invariant,

$$i\frac{\mathbf{i}_1+\mathbf{i}_2+\mathbf{i}_3}{\sqrt{3}}\mathbf{q}_f \rightarrow i\left(\mathbf{Q}\frac{\mathbf{i}_1+\mathbf{i}_2+\mathbf{i}_3}{\sqrt{3}}\mathbf{Q}^{\dagger}\right)(\mathbf{Q}\mathbf{q}),$$

$$\frac{\mathbf{i}_0+i\left(\left(\mathbf{i}_1+\mathbf{i}_2+\mathbf{i}_3\right)/\sqrt{3}\right)}{2}\mathbf{q}_f \rightarrow \left(\mathbf{Q}\frac{\mathbf{i}_0+i\left(\left(\mathbf{i}_1+\mathbf{i}_2+\mathbf{i}_3\right)/\sqrt{3}\right)}{2}\mathbf{Q}^{\dagger}\right)(\mathbf{Q}\mathbf{q}), \tag{2.4.G}$$

where the first expression applies to QCD and the second to QFD, such that,

$$\mathbf{Q}i\frac{\mathbf{i}_1+\mathbf{i}_2+\mathbf{i}_3}{\sqrt{3}}\mathbf{Q}^\dagger = i\mathbf{i}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.4.H}$$

$$\mathbf{Q}\frac{\mathbf{i}_0+i\left(\left(\mathbf{i}_1+\mathbf{i}_2+\mathbf{i}_3\right)/\sqrt{3}\right)}{2}\mathbf{Q}^\dagger = \frac{i\mathbf{i}_1+\mathbf{i}_0}{2} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

where **Q** is a quaternion with unit modulus and the first equation applies to QCD and the second to QFD. We then find,

$$\mathbf{Qq} = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} \otimes \mathbf{Q}\begin{pmatrix} p_0 \\ p_1 \end{pmatrix} = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} \otimes \begin{pmatrix} p_0' \\ p_1' \end{pmatrix}, \tag{2.4.I}$$

for both QCD and QFD. We set,

$$\tilde{\mathbf{q}} = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} \tag{2.4.J}$$

We multiply out the terms for QCD and QFD respectively using equation (2.4.H),

$$i\mathbf{i}_1\begin{pmatrix} p_0' \\ p_1' \end{pmatrix} = \begin{pmatrix} p_1' \\ p_0' \end{pmatrix}, \quad \frac{1}{2}\left(\mathbf{i}_0 + i\mathbf{i}_1\right)\begin{pmatrix} p_0' \\ p_1' \end{pmatrix} = \frac{1}{2}\begin{pmatrix} p_0' + p_1' \\ p_0' + p_1' \end{pmatrix} \tag{2.4.K}$$

and find that the simplified equation (2.3.F) drops into two halves. We choose another simplification, supposing that the wave function of each is the same whatever the colour and get, for both QCD and QFD, a wave function,

$$p_0'\tilde{\mathbf{q}} \otimes \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.4.L}$$

Each half of the original equation (2.3.F) is now the same and reads,

$$\left(i\gamma^{\mu}\partial_{\mu} - ig\gamma^{\mu}G_{\mu} - m\right)p_0'\tilde{\mathbf{q}} = 0 \tag{2.4.M}$$

This leaves us with two identical copies of the Dirac equation with a potential of G_{μ}. For QCD, G_0 is zero, but given the form of the Dirac equation, we know immediately how to make it relativistic: allow G_0 to take on other values.

2.5 The QCD Radiation Equation in the Special Case

We turn to the equations governing the gluons, (2.3.D), (2.3.G) and (2.3.H). Given our simplifications of the last section, equation (2.3.D) becomes

$$\begin{aligned}
&\mathbf{G}_{\mu\nu} = \partial_{\mu}\mathbf{G}_{\nu} - \partial_{\nu}\mathbf{G}_{\mu} + ig\left(G_{\mu}i\mathbf{i}_1 \times G_{\nu}i\mathbf{i}_1 - G_{\nu}i\mathbf{i}_1 \times G_{\mu}i\mathbf{i}_1\right) \\
&\mathbf{G}_{\mu\nu} = \partial_{\mu}\mathbf{G}_{\nu} - \partial_{\nu}\mathbf{G}_{\mu} \\
&+igG_{\mu}\left(\mathbf{i}_0 + i\mathbf{i}_1\right)\times G_{\nu}\left(\mathbf{i}_0 + i\mathbf{i}_1\right) - G_{\nu}\left(\mathbf{i}_0 + i\mathbf{i}_1\right)\times G_{\mu}\left(\mathbf{i}_0 + i\mathbf{i}_1\right)
\end{aligned} \tag{2.5.A}$$

where the former equation refers to QCD and the latter to QFD, from equations (2.3.B), (2.4.E) and (2.4.H), and we have left out the normalising factors for brevity. This leaves finally,

$$\begin{aligned}
&\mathbf{G}_{\mu\nu} = \partial_{\mu}G_{\nu}i\mathbf{i}_1 - \partial_{\nu}G_{\mu}i\mathbf{i}_1, \\
&\mathbf{G}_{\mu\nu} = \partial_{\mu}G_{\nu}\left(\mathbf{i}_0 + i\mathbf{i}_1\right) - \partial_{\nu}G_{\mu}\left(\mathbf{i}_0 + i\mathbf{i}_1\right)
\end{aligned} \tag{2.5.B}$$

where the first applies to QCD, the second to QFD. The matrices do not contribute to these expressions and equating elements,

$$G_{\mu\nu} = \partial_{\mu}G_{\nu} - \partial_{\nu}G_{\mu} \tag{2.5.C}$$

which is what we find for the like electromagnetic expression. We expect, therefore, that the gluons will behave like photons.

For the current we obtain from equation (2.3.5),

$$\partial_\mu G^{0\mu\nu} = j^{0\nu}, \quad \partial_\mu G^{1\mu\nu} = j^{1\nu} \tag{2.5.D}$$

where the second equation applies to QCD and both equations apply to QFD. Equation (2.3.H) becomes,

$$j^{1\nu} = g\left[f_{1\beta\chi} G^\beta_\mu G^{\chi\mu\nu} + \bar{\mathbf{q}}\gamma^\nu \mathbf{i}_1 \mathbf{q} \right], \tag{2.5.E}$$

$$j^{0\nu} = g\left[f_{0\beta\chi} G^\beta_\mu G^{\chi\mu\nu} + \frac{1}{2}\bar{\mathbf{q}}\gamma^\nu \mathbf{i}_0 \mathbf{q} \right],$$

$$j^{1\nu} = g\left[f_{1\beta\chi} G^\beta_\mu G^{\chi\mu\nu} + \frac{1}{2}\bar{\mathbf{q}}\gamma^\nu \mathbf{i}_1 \mathbf{q} \right]$$

where the first equation applies to QCD and the second two to QFD. Since the structure constants depend on the commutator and only $\mathbf{i}_1$, in the case of QCD, and $\mathbf{i}_1$ and $\mathbf{i}_0$ in the case of QFD are left, they make no contribution, and,

$$j^{1\nu} = g\bar{\mathbf{q}}\gamma^\nu i\mathbf{i}_1\mathbf{q}, \quad j^{0\nu} = g\bar{\mathbf{q}}\gamma^\nu \frac{\mathbf{i}_0}{2}\mathbf{q}, \quad j^{1\nu} = \frac{1}{2} g\bar{\mathbf{q}}\gamma^\nu i\frac{\mathbf{i}_1}{2}\mathbf{q} \tag{2.5.F}$$

The first equation applies to QCD and the last two equations apply to QFD. As well as the summation occasioned by the matrix γ^ν, there is also a summation associated with $i\mathbf{i}_1$ and $\mathbf{i}_0$, as most clearly appears in equation (2.4.I). Performing the second summation,

$$j^{1\nu} = 2g\bar{\tilde{\mathbf{q}}}p_0'\gamma^\nu\tilde{\mathbf{q}}p_0', \quad j^{0\nu} = g\bar{\tilde{\mathbf{q}}}p_0'\gamma^\nu\tilde{\mathbf{q}}p_0', \quad j^{1\nu} = g\bar{\tilde{\mathbf{q}}}p_0'\gamma^\nu\tilde{\mathbf{q}}p_0' \tag{2.5.G}$$

from equations (2.4.I) and (2.4.L), where the first equation applies for QCD and the second two for QFD. Remembering that all colours behave in the same way, we add the two currents for QFD, ending with a total current for both QCD and QFD,

$$j^{\nu} = 2g\bar{\tilde{\mathbf{q}}}p_0'\gamma^{\nu}\tilde{\mathbf{q}}p_0' \tag{2.5.H}$$

This current belongs to the total for both copies of the Dirac equation (2.4.M), leaving the current for each,

$$j_q^{\nu} = g\bar{\tilde{\mathbf{q}}}\gamma^{\nu}\tilde{\mathbf{q}} \tag{2.5.I}$$

if we define,

$$p_0' = 1 \tag{2.5.J}$$

We have recovered the Dirac current. We have also recovered the Dirac equation,

$$\left(i\gamma^{\mu}\partial_{\mu} - ig\gamma^{\mu}G_{\mu} - m\right)\tilde{\mathbf{q}} = 0 \tag{2.5.K}$$

from equation (2.4.M), the definition for $\mathbf{G}_{\mu\nu}$,

$$G_{\mu\nu} = \partial_{\mu}G_{\nu} - \partial_{\nu}G_{\mu} \tag{2.5.L}$$

from equation (2.5.C), and Maxwell's equations, (2.5.D),

$$\partial_{\mu}G^{\mu\nu} = j^{\nu} \tag{2.5.M}$$

that apply for QED. This is true whether it is the unbroken Electro-weak theory or QCD that we are investigating.

We have succeeded with the task of simplifying and solving QCD and QFD for a class of simple solutions. By themselves these results can be used to justify our calculations of the masses of the μ, τ and π mesons, the proton and our model for calculating the elements of the C-K-M matrix. We may even query whether the extreme complications of QCD, which make it so hard to apply, are indeed necessary to the theory, as do Davies and Lepage [Mackenzie 2005]. We conjecture that our theory above may yield solutions for other states showing slightly less symmetry than the one we have pursued. One could for example choose a class of solutions that led to the Maxwell and Dirac equations with a generalised potential, or more than one instance of the equations, with differing potentials. A wave function describing two or three particles, although not necessarily quarks as we have known them hitherto, could then be proposed as a possible solution. We leave the matter there.

3. THE SOLAR SYSTEM, GALAXY AND COSMOS

3.1 The Solar System

We have only calculated the mass of the electron in terms of an unknown, m_p, in paper 2B of this trilogy. However, there is one more member of the ANPA hierarchy which we have not yet used, the fourth member,

$$\xi = 2^{127} + 136 = 1.701411835 \times 10^{38} \tag{3.1.A}$$

Although we suppose that this too must be corrected to become scaling, the correction is so small that it is negligible. We suppose that $1/\xi$ functions as a coupling constant like all the other members, but, if so, its magnitude leads us to suppose it refers to the macrocosm, rather than the microcosm we have dealt with hitherto. Moving up by degrees, we start by providing evidence that the solar system not only obeys Bohr's first equation, as follows from the theory of General Relativity [Bell and Diaz 2002], but may also be considered to be in a quantum state. We have already remarked that bodies in the solar system may be viewed as in approximate quantum states in many cases [Bell 2004], and we were not the first to notice this [Spolter 1993]. However, we cannot leave the matter with an aside on Spolter because of the difficulties with her book. She cites the Titus-Bode empirical law that relates the radius of planetary orbits to a simple arithmetic recipe: start with zero, continue with three, double this, then double each successive number, and end by adding four to each number so produced and then dividing the result by ten. This number describes the orbital radius of the planets in astronomical units. The relation is approximate, but quite marked. Spolter produces an arbitrary formula, which allegedly does the same job,

$$r_S = \frac{31.946 \times 1.71^n \times 10^9}{1.49597870 \times 10^{11}} \tag{3.1.B}$$

where r_S is the radius, n is an integer quantum number starting with one, 10^9 turns the radius into metres and the denominator turns the result into astronomical units. The difficulty is that this formula produces worse results than does the original Titus-Bode recipe. The recipe in its original form is difficult to explain, and so we produce another formula that can be explained more easily and that is arguably as good as the Titus-Bode recipe. It is

$$r_B = 0.3 \times 2^{n-3} + 0.4 \tag{3.1.C}$$

In the appendix we present a program that will perform the calculation and can easily be amended to include the satellites of the planets, for which Spolter has like claims, but which we have not examined. We shall compare all the estimates of planetary orbital radii with the actual radii. The reader may then judge for himself. We obtain,

Table II

The Guises of Our Sun

Q.S. No.	Spolter's Formula	Titus-Bode	Q.B. No.	Our Formula	Spolter's Value	Nine Planets
1	0.365	0.4	1	0.475	0.39	0.38
-	-	-	2	0.55	-	-
2	0.624	0.7	3	0.7	0.72	0.72
3	1.067	1.0	4	1.0	1.00	1.00
4	1.825	1.6	5	1.6	1.52	1.52
5	3.122	2.8	6	2.8	2.77	2-4
6	5.339	5.2	7	5.2	5.2	5.20
7	9.129	10	8	10	9.53	9.54
8	15.61	19.6	9	19.6	19.2	19.22
9	26.69	38.8	(9.5)	(27.55)	30.1	30.06
10	45.65	77.2	10	38.8	39.8	39.5

The orbital radius of the planets is given in the order, Mercury, Venus, Earth, Mars, Asteroids, Jupiter, Saturn, Uranus, Neptune, and Pluto. The first column is the quantum number assigned by Spolter, the second column gives the value of the orbital radius calculated from Spolter's formula, equation (3.1.B), the third column gives the radius according to the original Titus-Bode relation, the fourth column gives the quantum number we assign, the fourth column gives the value of the radius calculated by us using equation (3.1.C), the sixth column gives the actual radius quoted by Spolter, and the seventh column gives the radius on the University of Arizona's Nine Planets web site [University of Arizona].

We can explain our formula in equation (3.1.C) using table 2 derived by Bell and Diaz [2002] showing the scaling Bohr orbit hierarchy for the copyshell, in the current case the solar system. Here the value of *m*, the integer winding number, starts at some arbitrary chosen number and increases by a factor of $\sqrt{2}$ at each succeeding level. Levels with an integer power of two describe a particle, while the alternate levels describe a tachyon, or vice versa. Moving between two adjacent levels corresponds to moving from an "inside" to an "outside", or vice versa, as we explain in full in the next section [Bell and Diaz 2002], [Bell 2004]. The planets would then be different incarnations of a single "sun" particle orbiting the source, which is the sun. If we suppose the mass of the sun is m_s, then

$$r_B = \frac{m^2}{n^2} m_s \tag{3.1.D}$$

where *n* is the integer excitation level of the "sun atom" and does not vary with the level in the hierarchy [Bell and Diaz 2002]. We see that we cannot directly compare our data with m_s, the known mass of the sun converted into a length in the usual way, because we do not know *n*. We would need to add in a constant, in our case 0.4, if the level of the vacuum energy for the "sun atom" was different from the level we experienced on Earth [Bell et al. 2004b], [Bell and Diaz 2004a].

3.2 Modified Newtonian Dynamics - MOND

We shall now increase in size yet again and consider the Galaxy. Our Galaxy, the Milky Way, may also be regarded as a quantum state or superposition of states. Thus we would be viewing the matter in the Galaxy as forming the wave function of a quantum state, or superpositions of such, part of a hierarchy of the sort we have explored before in papers one, two, three A in this trilogy and as we found for the solar system. To assert this, we would have to demonstrate how the Milky Way obeys the equations of QED with mass in place of the electric charge [Bell and Diaz 2002], [Bell 2004]. If it obeys the theory of General Relativity, this will suffice, since this theory is a classical limit of a quantum theory that obeys Bohr's equations and hence the equations of QED [Bell et al. 2004a&b]. However, the Galaxy does not appear to obey Newton's equation, which is the limit of General Relativity for the low velocities that apply here. It only obeys General Relativity if one includes a large amount of dark matter, of an unknown nature. This explanation is seriously considered, as shown, for example, by the U.K. Particle Physics and Astronomy Research Council funding, until 2008, a project to detect dark matter at the University of Sheffield, with Dr A. M. Green as the principle investigator [PPARC 2005].

As an alternative, one may assume that the Galaxy does not obey General Relativity, but obeys the theory of Modified Newtonian Dynamics (MOND) [Famaey and Binney 2005], [Sanders and McGaugh 2002]. This theory assumes that Newtonian's equation holds up to some distance from the centre, providing a centrifugal force whose strength varies inversely with the square of the distance. Then there is then a gradual changeover to an inverse distance law. We will show that this theory is consistent with the assumption that the equations of QED hold. First, we consider a simplified version in which the changeover from an inverse distance-squared law to an inverse distance law is sharp rather than gradual. A gradual changeover is found experimentally, and so, second, we will consider why the changeover is actually gradual.

We will assume the system can be regarded as two-dimensional, a reasonable first simplification for the Milky Way. There are then two immediate ways of viewing a system obeying Bohr's equations in the region inside the Bohr radius. We explored the first in relation to the Bohr model of one electrically charged particle circling a stationary charged source. Here we described a two-dimensional space, *M*, [Bell et al. 2004a]. We could embed *M* in a three-dimensional Euclidean space as the surface of a cylinder. Distance measured along the axis is proportional to the original varying radius and the circumference of a circle on the curved surface of the cylinder is equal to the fixed Bohr radius multiplied by 2π. The particle orbits round this circle with a constant velocity whatever the original radius.

The second way explored corresponds to the space L_0 inside a thin, hollow sphere of self-gravitating matter of radius equal to the Bohr radius [Bell and Diaz 2002], [Bell 2004]. From the point of view of General Relativity, the space inside is Euclidean and flat. If we identify this space too as behaving like *M*, a circular orbit round the centre of the sphere will have the same constant velocity whatever the radius. However, we tacitly assumed that here the radius varies with distance from the centre as it normally would in a Euclidean space with the usual topology. If the velocity of a spherical orbit round the centre is then to remain constant whatever the radius, there must appear to be an acceleration towards the centre proportional to this radius.

Thus the constant orbital velocity seen in the two cases is assigned a different cause in each. In one neither the circumference of the orbit nor the acceleration nor the velocity of the orbiting particle vary and in the other both the circumference and the acceleration vary with the radius in such a way as to keep the velocity constant. We suppose that these are two ways of viewing the same phenomenon.

Outside the Bohr radius, the inverse square law for the force describes events both for the electromagnetic [Bell et al. 2004a], and the gravitational interaction [Bell and Diaz 2002], [Bell 2004]. However, we went on to show that thin spheres of self-gravitating matter with a common centre could be nested, since we could also describe the gap between two such spheres as either curved with gravity present or flat, depending on our point of view. In the former case we would regard our position as "outside" and in the latter as "inside". This description held for gravity, but we also showed that it must apply for electromagnetism too, since this can be derived from the gravitational theory in a way described by Bell and Diaz [2004a] and in the first paper in this trilogy. We will not pursue the electromagnetic version here, as our current quest is gravitational in nature.

We do suggest that there may be a sequence of such spheres centred on the centre of the Milky Way, with the space between each two being seen as "inside" and "outside" alternately. This might account for the success of the MOND hypothesis in explaining the region where the orbital velocity is constant when we exclude the presence of dark matter. However, it would not account for the transition region found, where the velocity is intermediate between what would be calculated for a Newtonian theory and what MOND would dictate asymptotically. We suggest that whether a region should be counted as "inside" or "outside" may vary close to a changeover point from one type to another. Here the two possible states may both exist as a superposition. In a superposition in the microcosm, a particle will have a certain probability of being in one state or another. In the macrocosm however, with particles as large as stars, which possess internal structure, such a body may have a velocity intermediate between two, in our current case, or, more generally, *n* possible states,

$$v = p_1 v_1 + p_2 v_2 + p_3 v_3 + \ldots + p_n v_n, \quad \sum_{i=1}^{n} p_i = 1 \tag{3.2.A}$$

where v is the actual velocity, p_i is the probability that the star belongs to the ith state, which would have a definite velocity of v_i. p_i may vary with the distance from the centre. We suggest that small subsystems within the star, small enough for us to want to apply quantum mechanics, are in a superposition of the possibilities, but if the wave function of each were to collapse to a definite state, the probability of the subsystem having the velocity v_i is proportional to p_i. Other similar but more complicated forms of calculation may also be possible.

We will now estimate the Bohr radius of the Galaxy, seen as a single quantum state embodying both an "inside" and an "outside", in ball-park figures. The basic relation on which MOND rests is that when $a \ll a_0$, where a is the acceleration of a massive object towards the centre of attraction, the acceleration is given by,

$$\frac{a^2}{a_0} = \frac{MG}{r^2} \tag{3.2.B}$$

where M is the mass of the attracting body, r the distance of matter feeling the acceleration from the centre of attraction and a_0 is constant [Milgrom 2001]. When $a \gg a_0$, it is given by General Relativity, which in this region is well approximated by Newtonian gravity [Bell and Diaz 2002], with,

$$a = \frac{MG}{r^2} \qquad (3.2.C)$$

Bohr's equations describe motion in a circle, where,

$$v^2 = ar \qquad (3.2.D)$$

with v as the velocity of the circling body. If we use in addition equation (3.2.C), we obtain the first of Bohr's equations,

$$v^2 = \frac{MG}{r} \qquad (3.2.E)$$

If instead we consider the region where equation (3.2.B) holds, we obtain from equation (3.2.D),

$$v^2 = \sqrt{MGa_0} \qquad (3.2.F)$$

with v constant. This is the condition required for us to interpret this region as a space of type M or L_0.

We will take the asymptotic value of the velocity, v, of the Galaxy from one of the models provided by Famaey and Binney [2005], 175 km s^{-1}, and the value of the constant acceleration, a_0, from the same source, about 1.2×10^{-13} km s^{-2}. Assuming the Bohr radius is the one where both equations (3.2.B) and (3.2.C) hold, we calculate the Bohr radius of this quantum state of the Milky Way in kilometres from equation (3.2.D),

$$r = \frac{v^2}{a_0} = 2.55 \times 10^{17} \qquad (3.2.G)$$

By contrast, our sun is about 2.47×10^{17} km from the centre of the Milky Way and travelling at a speed of about 200 km s^{-1}.

3.3 The Mass of the Proton

From the considerations we have advanced already, the Milky Way must have at least one region in which General Relativity holds near the centre, an "outside", and another further out in which the space is of type *M*, an "inside". The Bohr radius will fall between these two regions. Supposing that we must have at least another space regarded as "inside" near the centre, it is clear that more than one sphere is involved. However, we shall find that we must include other divisions between "inside" and "outside", if we want to find the fourth member of the ANPA hierarchy. We shall find that this new Bohr radius is well outside what we think of as the Milky Way proper. Here, we suppose, we are close to the limits of our island universe, which at this distance we model as a proton, thinly distributed over the surface of a sphere and held together by gravitational attraction, like the proton we are already familiar with from paper 3A in this trilogy. We suppose this is a limit of space as we know it, in which it is equally likely that the proton is anywhere on the sphere, since there are no differentiating factors. We will then calculate the coupling constant and radius of this "atom," on the assumption that the Planck constant that applies is the usual electromagnetic one, *h*, and that the coupling is gravitational, with the usual gravitational one, *G*.

It is significant if h for this gravitational atom is the same as the electromagnetic h. For the solar system, for example, the constant with the role of Planck's constant may have a different value, and in general this varies with the level in the hierarchy that applies, as we showed in paper 2C. The coincidence of value implies, therefore, that the two, the macrocosmic and the microcosmic proton, are at one and the same level. We suggest that this is where size becomes relative allowing the two to coincide. In practical terms, we cease to be able to calculate longer scaling intervals. It must be the case then that no hierarchy can be formed of which this is a member at a lower level, as found by Parker-Rhodes [1981] and Bastin and Kilmister [1995] for the ANPA hierarchy. We require two conditions to inhibit the hierarchy we derived for gravity for the copyshell [Bell and Diaz 2002] and stop it applying to the Galaxy. Firstly, the velocity of rotation for a matching flat and curved space must be the same, rather than differing by a factor of two, as we calculated it must. We therefore have to be prepared to regard the surface of the thin sphere on which the "cosmic" proton dwells as flat. Secondly, we must have the same value for the mass of the notional orbiting "particle" and stationary "source" [Bell and Diaz 2002] participating in the interaction, rather than the former being twice the latter. This requires us to disregard even the first order of small quantities in calculating the force due to the gravitational self-interaction. The source and particle are identically the proton itself. In that case the force follows,

$$F \propto \frac{m_P^2}{2} \rightarrow m_P^2 \qquad (3.3.A)$$

where m_P is the mass of the proton. We may phrase this differently by saying that the distinction between self and other is lost. In the language of ANPA, they are indistinguishable.

Solving Bohr's equations [Bell and Diaz 2002] for the gravitational bond, we obtain for the inverse of the coupling constant,

$$\xi = \frac{\hbar c}{m_P^2 G} \tag{3.3.B}$$

We are not the first to point out the relevance of this equation [Parker-Rhodes 1981], [Bastin and Kilmister 1995]. Retrieving the required values from CODATA 2002 using the m kg s system of units,

$$\begin{aligned} m_P &= 1.67262171 \pm 0.00000029 \times 10^{-27} \\ \hbar &= 1.05457168 \pm 0.00000018 \times 10^{-34} \\ c &= 299792458 \\ \sqrt{\frac{\hbar c}{G}} &= 2.17645 \pm 0.00016 \times 10^{-8} \end{aligned} \tag{3.3.C}$$

we calculate the observed value of the inverse of the coupling constant from equation (3.3.B),

$$\xi^{\odot} = 1.693175322 \times 10^{38} \tag{3.3.D}$$

Looking at the ξ predicted by the ANPA hierarchy in equation (3.1.A), we see that two differ by about eight in the fourth figure, so the model is only accurate to this extent.

We calculate the Bohr radius associated with this coupling constant. Bohr's second equation gives for this,

$$r = \frac{\hbar\xi\sqrt{1-(1/\xi)^2}}{cm_P} \tag{3.3.E}$$

where we shall ignore the square root. From equations (3.3.C) and (3.3.D),

$$r = 3.56089856 \times 10^{19} \tag{3.3.F}$$

in kilometres, well outside the visible Galaxy with radius, say, 5.7×10^{17} km, and the critical radius for MOND region, at 2.55×10^{17} km from equation (3.2.G), which justifies our assertion that the associated velocity may be very small.

Equation (3.3.B) enables us to find the mass of the proton in terms of the fundamental dimension-bearing constants and the fourth member of the ANPA hierarchy. However, we have already found the mass of the proton in terms of the first three members of the ANPA hierarchy and the mass of the electron in papers 2B and 3A. We can therefore eliminate the mass of the proton and find the mass of the electron in terms of the four members of the ANPA hierarchy, the coupling constants for the weak, strong, electromagnetic and gravitational interactions, and the dimension bearing constants. Since we found all the other masses, those of the μ, τ and the π mesons, together with the W and Z bosons, in terms of the mass of the electron in the previous papers in the trilogy, we can find all these too in terms of the coupling constants and the dimension bearing constants.

We would want point out that it is not likely that we can go further than this for the physical parameters, given the existing state of physics. The ANPA combinatorial hierarchy predicts the coupling constants of the weak, strong, electromagnetic and gravitational forces from what is, essentially, a mathematical model of man's psychology, so we may take these as derived rather than postulated. Plank's constant, *h*, turns the units of mass into a pure number, *m*, [Bell and Diaz 2002]. The gravitational constant, *G*, and the speed of light, *c*, then turn the units of length into a pure number, using the usual formulation of General Relativity [Martin 1995]. The speed of light, *c*, given the numerical equivalent of the units of length, turns the units of time into a pure number in the usual way, and finally, given all these, Bohr's equations for the electromagnetic interaction [Bell and Diaz 2002] turns the units of charge, *e*, into a pure number.

3.4 Cosmology

So much for the Galaxy and other galaxies [Famaey and Binney 2005], [Sanders and McGaugh 2002], which can indeed be seen to be functioning much like island universes, with the other galaxies behaving like our own. This leaves the unishell still outstanding [Bell and Diaz 2002]. The generally accepted method of discussing our universe as a whole is not to regard it as the Galaxy, but to find a solution of the classical equations of General Relativity describing a homogeneous mass. It is possible that, just as QED can form a better picture of an electromagnetic atom than can Maxwell's classical theory, a quantum solution for our universe might describe aspects not so fully allowed for by a classical one. The unishell was derived as a model of our space itself, and it would not depart from the spirit of the classical model given by General Relativity to view the unishell as the universe. In that case, if we are to continue to pursue the quantum theory of gravity we have hitherto, our universe would be circular, spherical or hyper-spherical, depending on the current problem of interest, and, since we may rotate the temporal into the spatial and vice versa, it would not be possible to say whether time flowed along the radius or formed part of the surface. If we placed time along the radius, as time passed our universe would expand or contract, but, if we placed time as forming part of the surface, our universe would rotate. It might look as though it would not be possible to hold both points of view, but it would, if the laws of physics were scaling, as we have found. Then there would be no such thing as an absolute size, only a size in a limited set of scales from the particular point of view of an observer inside his universe. Our universe itself would have no size, and we could equally well assume that either the radius or some great circle on the surface was temporal. Since the other consequences of assuming the universe expands are so very well-known, we shall explore the consequences of assuming it rotates.

As remarked by Milgrom [2001], MOND leads to a strange coincidence,

$$a_0 \sim cH_0 \tag{3.4.A}$$

where H_0 is the Hubble constant used to describe an expanding universe. This means that a body accelerating at a_0 from will approach the speed of light in the lifetime, $T = 1/H_0$, of the universe. This speed may be seen to apply to the flow of time with appropriate co-ordinates. Such a body will be orbiting at a speed of the order *c* round our universe if we switch from an expanding to a spinning model. This fits the unishell relatively well, since it has a rotational velocity of $c/\sqrt{2}$. The level one higher in the unishell hierarchy has a speed of *c* exactly. We could then tentatively identify a_0 as the acceleration experienced by our spinning universe due to the centrifugal force. Each of the galaxies has a copy of this, with a much smaller rotational velocity and radius.

The wavelengths of the lines in the spectrum of a body in the universe are longer in proportion to the distance from the observer on Earth, the redshift [Martin 1995]. This is explained as a Doppler effect due to the velocity of recession of bodies as the universe expands. We may, however, explain it by supposing that time for a body lies along the radius joining the centre to the body, with the surface spatial. In that case, the angle, ζ, between the direction of time for the body and the observer would be proportional to the distance, *d*, between them. For small distances,

$$R\zeta = R\sin\xi = R\tan\varsigma = d \qquad (3.4.B)$$

where *R* is the radius of our universe, and so, when we consider light from a distant body, it is seen red-shifted because the difference between our own velocity and that of the object appears to be proportional to *d* [Bell and Diaz 2003]. It might be objected that this picks out a particular direction as temporal rather than allowing the temporal direction to rotate. However, if we imagine the universe to be a sphere and swing the triangle with vertices given by the body, the observer and the centre of the universe about a fixed axis, the line between the body and observer, we may rotate the triangle so that it lies orthogonal to a radius, and the temporal directions for the two along with it. In this instance it may form part of the surface for our universe if we remember that such bound "atoms" may also be polyhedral, as we discussed in paper 2C.

Finally, the equation of circular motion applies to the unishell hierarchy,

$$a_0 R = c^2 \tag{3.4.C}$$

Applying this to the universe, not necessarily using the same definition of the temporal direction as we did for the Galaxy, and using equation (3.4.A), permits us to estimate the radius of the universe. It is of the order,

$$R \sim \frac{c}{H_0} \tag{3.4.D}$$

and, from this view, $1/H_0$ would be the period. Calculating $1/H_0$ from the speed of light given in equations (3.3.C) and our previous value of a_0, we obtain about 2.5×10^{18} seconds. Then R is of the order of 7.5×10^{23} km a factor of about twenty thousand greater than the radius of the cosmic proton that we discussed in section 3.3. Granted the uncertainties in the calculation we have just made, the two could well coincide.

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Appendix

Program to Calculate Orbital Radii of the Planets

The program below calculates the values required for table II. It is in R.T. Russell's BBC Basic, which is to be found at the following web address: http://www.rtrussell.co.uk/.

```
10 REM SPOLTER
   20 REM 10th September 2005
   30 REM: Copyright (C) 2005 by S B M Bell
   40 REM
   50 REM Radius of orbit is RRADIUS(N%) where N% is the
quantum number.
   60 REM TRRADIUS is RRADIUS calculated from Titus Bode's
law
   70 DIM
TRRADIUS(11),RRADIUS(11),MRRADIUS(11),MASS(11),SRRAD$(11),B
RRAD(11)
   80
   90 REM Mercury, Venus, Earth, Mars, Asteroids, Jupiter,
Saturn, Uranus, Neptune, Pluto
  100 DATA 0.4,0.7,1.0,1.6,2.8,5.2,10.0,19.6,38.8,77.2
  110 REM Titus-Bode radius in astronomical units
  120 DATA 0.39,0.72,1.0,1.52,2.77,5.20,9.53,19.2,30.1,39.8
```

```
  130 REM Observed radii in astronomical units
  140 DATA
0.33022,4.8690,5.9742,0.64191,0.001273,1898.8,568.54,86.625
,102.78,0.015
  150 REM Masses of the planets
  160
  170 REM Read Titus-Bode radius
  180 FOR N%=1 TO 10
  190   READ TRRADIUS(N%)
  200   TRRADIUS(N%)=TRRADIUS(N%)
  210 NEXT
  220
  230 REM Read Actual radius
  240 FOR N%=1 TO 10
  250   READ RRADIUS(N%)
  260 NEXT
  270
  280 REM Not used.
  290 FOR N%=1 TO 10
  300   READ MASS(N%)
  310 NEXT
  320
  330 PRINT"Comparison of radii. Planet order is:"
  340 PRINT"Mercury, Venus, Earth, Mars, Asteroids,
Jupiter, Saturn, Uranus, Neptune, Pluto."'
  350 PRINTTAB(0);"Q no"TAB(10)"Spolter's
formular"TAB(30)"Titus-Bode radius"TAB(50)"Bell's
formula"TAB(70)"Actual radius"'
  360
  370 FOR N%=1 TO 10
  380   N=N%
  390   SRRAD=31.946*(1.71)^N*10^9/1.49597870/10^11
  400   REM Calculate the radius from Spolter's formular
  410   SRRAD$(N%)=LEFT$(STR$(SRRAD),5)
  420   REM Chop off after 4 digits
  430   BRRAD(N%)=0.3*2^(N-3)+0.4
  440   REM Calculate value from Bell's formular
  450
PRINTTAB(0)STR$(N);TAB(10)SRRAD$(N%);TAB(30)TRRADIUS(N%);TA
B(50)STR$(BRRAD(N%));TAB(70)STR$(RRADIUS(N%))
  460   REM Print out values
  470 NEXT
  480
  490 PRINT'"There is no planet for Bell's formular for the
value "STR$(BRRAD(2))"."
  500 PRINT"There is no Bell's formular value for planet
number 9, Neptune, radius "STR$(RRADIUS(9))"."
```

```
510
520 END
```