Planck Mass Plasma Analog of General Relativity And Elementary Particle Physics

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Abstract

For a condensed matter analog of general relativity and elementary particle physics we make the following assumptions: 1. The fundamental group is SU2. 2. The equations describing the model can only contain the Planck length, mass, and time as free parameters. With SU2 isomorph to SO3 space should be three-dimensional, and with the two-valuedness of SU2 there should be negative besides positive Planck masses. We then define as a Planck mass plasma an equal number of positive and negative Planck mass particles, with each Planck length volume filled in the average by one Planck mass, and with the Planck mass particles interacting by the Planck force over a Planck length. To keep the Planck mass plasma stable fixes the sign of the Planck force. The model leads to quantum mechanics as a consequence of the Zitterbewegung caused by the negative Planck masses, and to a spectrum of quasiparticles, both bosonic and fermionic. In particular it leads to gravitons and photons as the symmetric and antisymmetric wave mode of vortex sponge (resp. vortex lattice). Special and general relativity are here a dynamic symmetry through true physical deformations, as in the pre-Einstein theory of relativity by Lorentz and Poincaré.

There has been a growing interest in condensed matter physics analogs of general relativity and elementary physics [1, 2, 3, 4, 5], but the present trend towards a unified theory of elementary physics is to assume that there are higher dimensions, for example with a total of 10 space – and one time dimension. All these theories have a very large group. Instead, it may be more plausible that the fundamental group of nature is small, and if nature works like a computer with a binary number system, the fundamental group should be SU2. With SU2 isomorph to SO3, the rotation group in R3, then would make it understandable why ordinary space is three-dimensional [6]. With the reduction of physics to equations containing as free parameters only the Planck mass, length, and time, the two-valuedness of SU2 demands the existence of negative besides positive Planck masses, suggesting a "Planck mass plasma" as a possible condensed matter analog of the vacuum, consisting of an equal number of positive and negative Planck mass particles, $\pm m_p$, with each Planck length volume $\pm r_p^3$, filled in the average by a Planck mass, and with the Planck mass particles interacting over a Planck length r_p by the Planck force $m_p c^2 / r_p = c^4 / G$ (G Newton's constant). The remaining freedom in the sign of the Planck force can be chosen in a unique way to keep the Planck mass plasma stable, whereby Planck mass particles of equal sign repel and those of opposite sign attract each other, in analogy to a real plasma where charged particles of equal sign repel and those of opposite sign attract each other. And as in a real plasma the range of the force is short, But whereas in a real plasma this range is the Debye-length, it is in a Planck mass plasma the Planck length. There though is an important difference between a real and a Planck mass plasma: While in a real plasma both energy and momentum are conserved during the collision of a positively with a negatively charged particle, in the Planck mass plasma the momentum fluctuates by $\Delta p = m_p c$ during the collision of a positive with a negative Planck mass particle. Because this momentum fluctuation occurs over a Planck length $\Delta q = r_p$, Heisenberg's uncertainty relation $\Delta p \Delta q = m_p r_p c = \hbar$ emerges at the most fundamental level. As a result of the momentum fluctuation the Planck mass particles undergo a "Zitterbewegung"-diffusion process with the diffusion velocity

$$v_D = -(r_p/2)\, c\nabla n / n \tag{1}$$

where n is the particle number density,in the equilibrium $n = 1/2r_p^3$, for the positive or negative Planck mass particles. The kinetic energy of this diffusion process is

$$(m_p/2)\, v_D^2 = (m_p/8)\, r_p^2 c^2 (\nabla n/n)^2 = (\hbar^2/8m_p)(\nabla n/n)^2 \tag{2}$$

It was shown by Fenyes [7], adding (2) to the Lagrangian of a compressible frictionless fluid one obtains the Schrödinger equation with Hamilton's principle. However, as Heisenberg [8] has pointed out in the absence of a physical explanation for the diffusion process the derivation of Schrödinger's equation remains just some mathematics, but in the Planck mass plasma this diffusion process has its cause in the hidden existence of negative masses.
With

$$v = \frac{\hbar}{m_p}\nabla S \tag{2}$$

where S is the Hamilton action, the Lagrangian, including the kinetic energy from this diffusion process, is:

$$L = n\left[\hbar\frac{\partial S}{\partial t} + \frac{\hbar^2}{2m_p}(\nabla S)^2 + U + \frac{\hbar^2}{8m_p}\left(\frac{\nabla n}{n}\right)^2\right] \tag{3}$$

Variation of (3) with regard to S and n results in

$$\frac{\partial n}{\partial t} + \frac{\hbar}{m_p}\nabla \bullet (n\nabla S) = 0 \tag{4}$$

and

$$\hbar\frac{\partial S}{\partial t} + U + \frac{\hbar}{2m_p}(\nabla S)^2 + \frac{\hbar^2}{2m_p}\;\frac{\nabla^2\sqrt{n}}{\sqrt{n}} = 0 \tag{5}$$

With the Madelung transformation

$$\left.\begin{aligned}\psi &= \sqrt{n}\; e^{iS}\\ \psi^* &= \sqrt{n}\; e^{-iS}\end{aligned}\right\} \tag{6}$$

(4) and (5) are obtained from the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m_p} \nabla^2 \psi + U\psi \tag{7}$$

where

$$U = 2\hbar c r_p^2 \left[\psi_+^* \psi_+ - \psi_-^* \psi_-\right] \tag{8}$$

is the average potential of all Planck mass particles (both positive and negative) acting on one Planck mass particle.

Replacing $\psi_\pm^*, \psi_\pm$ with the operators $\psi_{\pm,}^\dagger \psi_\pm$, obeying the canonical commutation relations

$$\left.\begin{aligned} &\left[\psi_\pm(\underline{r})\psi_\pm^\dagger(\underline{r}')\right] = \delta(\underline{r} - \underline{r}') \\ &\left[\psi_\pm(\underline{r})\psi_\pm(\underline{r}')\right] = \left[\psi_\pm^\dagger(\underline{r})\psi_\pm^\dagger(\underline{r}')\right] = 0 \end{aligned}\right\} \tag{9}$$

one obtains the nonrelativistic Heisenberg-type operator field equation

$$i\hbar \frac{\partial \psi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_p} \nabla^2 \psi_\pm \pm 2\hbar c r_p^2 \left(\psi_\pm^\dagger \psi_\pm - \psi_\mp^\dagger \psi_\mp\right)\psi_\pm \tag{10}$$

which we use as a model of the Planck mass plasma. Making the Hartree-Fock approximation

$$\left.\begin{aligned} &\left\langle \psi_\pm^\dagger \psi_\pm \psi_\pm \right\rangle \cong 2\varphi^* \varphi^2 \\ &\left\langle \psi_\mp^\dagger \psi_\mp \psi_\pm \right\rangle \cong \varphi_\mp^* \varphi_\mp \varphi_\pm \end{aligned}\right\} \tag{11}$$

eq. (10) is replaced by a nonlinear Schrödinger equation for a two-component positive-negative mass superfluid:

$$i\hbar \frac{\partial \varphi_\pm}{\partial t} = \pm \frac{\hbar^2}{2m_p} \nabla^2 \varphi_\pm \pm 2\hbar c v_p{}^2 \left[2\varphi_\pm^* \varphi_\pm - \varphi_\mp^* \varphi_\mp\right]\varphi_\pm \tag{12}$$

With the Madelung transformation

$$\left.\begin{aligned} &n_\pm = \varphi_\pm^* \varphi_\pm \\ &n_\pm v_\pm = \mp \frac{i\hbar}{2m_p} \left[\varphi_\pm^* \nabla \varphi_\pm - \varphi_\pm \nabla \varphi_\pm^*\right] \end{aligned}\right\} \tag{13}$$

one obtains from (12) an Euler and continuity equation

$$\left.\begin{aligned}&\frac{\partial \underline{v}_\pm}{\partial t}+\left(\underline{v}_\pm \bullet \nabla\right)\underline{v}_\pm = -\frac{1}{m_p}\nabla\left(U_\pm + Q_\pm\right)\\ &\frac{\partial n_\pm}{\partial t}+\nabla \bullet \left(n_\pm \underline{v}_\pm\right)=0\end{aligned}\right\} \tag{14}$$

where

$$\left.\begin{aligned}&U_\pm = 2m_p c^2 r_p^3\left(2n_\pm - n_\mp\right)\\ &Q_\pm = -\frac{\hbar^2}{2m_p}\;\frac{\nabla^2\sqrt{n_\pm}}{\sqrt{n_\pm}}\end{aligned}\right\} \tag{15}$$

For small amplitude disturbances, large compared to the Planck length, the quantum potential $Q_\pm$ can be neglected. In this limit there are two longitudinal waves, one propagating with c and the other with $\sqrt{3}c$. The first, where the positive and negative Planck mass particles oscillate in phase, is the analog of the ion acoustic wave, and the second, where they are out of phase by 180°, is analogous to the electron plasma oscillations. In addition there are two steady state potential vortex solutions, one where the positive and negative Planck mass particles are co-rotating and the other one where they are counter-rotating. As it has been emphasized by Heisenberg [9] such a doubling of states is a requirement for any viable theory of elementary particles, if it shall describe both the electron and neutrino states. Here however, we want to show that the Planck mass plasma can simulate electromagnetic and gravitational waves, demonstrating that a unification of general relativity with quantum mechanics does not require more than three space dimension, contrary to the claims made by string theory.

With an equal number of positive and negative Planck mass particles, a vortex sponge (resp. vortex lattice) can, without the expenditure of energy, be created out of the Planck mass plasma by spontaneous symmetry breaking. It is then easy to show that this vortex sponge has two kinds of transverse waves, one simulating Maxwell's electromagnetic and the other one Einstein's gravitational wave. For electromagnetic waves this was already shown by Thomson [10].

Let $\underline{v} = \{v_x, v_y, v_z\}$, with div $\underline{v} = 0$,be the undisturbed velocity of the vortex sponge and $\underline{u} = \{u_x, u_y, u_z\}$, with div $\underline{u} = 0$, a small superimposed velocity disturbance.

1a. With the x-component of the equation of motion (ρ, p, density and pressure)

$$\frac{\partial u_x}{\partial t} = -\left(v_x + u_x\right)\frac{\partial\left(v_x+u_x\right)}{\partial x} - \left(v_y + u_y\right)\frac{\partial\left(v_x+u_x\right)}{\partial y} - \left(v_z + u_z\right)\frac{\partial\left(v_x+u_x\right)}{\partial z} - \frac{1}{\rho}\,\frac{dp}{dx} \tag{16}$$

and the continuity equation div $\underline{v} = 0$, resp.

$$v_x \frac{\partial v_x}{\partial x} + v_x \frac{\partial v_y}{\partial y} + v_x \frac{\partial v_z}{\partial z} = 0 \tag{17}$$

one obtains by subtracting (17) from (16) and taking the y-z average

$$\frac{\partial u_x}{\partial t} = -\frac{\partial\left(\overline{v_y v_x}\right)}{\partial y} - \frac{\partial\left(\overline{v_z v_x}\right)}{\partial z} \tag{18}$$

and likewise equations for u_y and u_z. From the condition div $\underline{u} = 0$ one has

$$\overline{v_i v_k} = -\overline{v_k v_i} \tag{19}$$

1b. Taking the x-component of the equation of motion multiplying it by v_y and taking the y-z average, then taking the y-component multiplied by v_x and taking the x-z average, finally subtracting the first from the second equation one finds that

$$\frac{\partial}{\partial t}\left(\overline{v_x v_y}\right) = -v^2\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) \tag{20}$$

where $v^2 = \overline{v_x^2} = \overline{v_y^2} = \overline{v_2^2}$ is the square of the microvelocity in the vortex sponge. Putting $\phi_z = -\overline{v_x v_y} / 2v^2$, one sees that (20) is the z-component of

$$\frac{\partial \phi}{\partial t} = \frac{1}{2} \text{curl } \underline{u} \tag{21}$$

Eq. (18) together with the corresponding eqs. for u_y and u_z takes the form

$$\frac{\partial \underline{u}}{\partial t} = -2v^2 \text{curl} \underline{\phi} \tag{22}$$

Eliminating $\underline{\phi}$ from (21) and (22) leads to the wave equation

$$-\frac{1}{v^2} \frac{\partial^2 \underline{u}}{\partial t^2} + \nabla^2 \underline{u} = 0 \tag{23}$$

The potential vortices of the vortex sponge are quantized according to $m_p r v_\phi = \hbar$ ($m_p r_p c = \hbar$) whereby

$$v_\phi = c\frac{r_p}{r} \qquad r > r_p$$
$$= 0 \qquad r < r_p \tag{24}$$

In the limit $r \to r_p$, one has v=c, whereby (21) and (22) become identical to Maxwell's vacuum equations by putting $\underline{u} = E$ and $\phi = -(1/2c)\underline{H}$.

2a. With the average taken over x, y, and z, the x-component of the equation of motion (16) becomes

$$\frac{\partial u_x}{\partial t} = -\frac{\partial \overline{v_x^2}}{\partial x} - \frac{\partial\left(\overline{v_x v_y}\right)}{\partial y} - \frac{\partial\left(\overline{v_x v_z}\right)}{\partial z} \tag{25}$$

and likewise eqs. for u_y and u_z. In combination with div $\underline{u}$ = 0 this implies that

$$\frac{\partial^2}{\partial x_i \partial x_k}\left(\overline{v_i v_k}\right) = 0 \tag{26}$$

whereby (25) becomes

$$\frac{\partial u_k}{\partial t} = -\frac{\partial}{\partial x_i}\left(\overline{v_i v_k}\right) \tag{27}$$

2b. Multiplying the v_i component of the equation of motion with v_k, and vice verse the v_k component with v_i, adding both and taking the average, one has

$$\frac{\partial}{\partial t}\left(\overline{v_i v_k}\right) = -v^2\left(\frac{\partial u_i}{\partial x_k} - \frac{\partial u_k}{\partial x_i}\right) \tag{28}$$

From (27) one has

$$\frac{\partial^2 u_k}{\partial t^2} = -\frac{\partial^2}{\partial t \partial x_i}\left(\overline{v_i v_k}\right) \tag{29}$$

and from (28) with div $\underline{u} = 0$

$$\frac{\partial^2}{\partial x_i \partial t}\left(\overline{v_i v_k}\right) = -v^2\left(\frac{\partial}{\partial x_k}\frac{\partial u_i}{\partial x_i} + \frac{\partial^2 u_k}{\partial x_i{}^2}\right) = -v^2\frac{\partial^2 u_k}{\partial x_i{}^2} \tag{30}$$

Eliminating $\left(\overline{v_i v_k}\right)$ from (29) and (30), and as before putting $v^2=c^2$, one has

$$\frac{\partial^2 u_k}{\partial t^2} = c^2 \frac{\partial^2 u_k}{\partial {x_i}^2} \tag{31}$$

We compare this solution with a plane gravitational wave into the x_1-direction, for which in the linearized limit the line element is[11]

$$ds^2 = ds_0^2 + h_{22}dx_2^2 + 2h_{23}dx_2dx_3 + h_{33}dx_3^2 \tag{32}$$

where

$$\left.\begin{array}{l} h_{22} = -h_{33} = f(t - x/c) \\ h_{23} = g(t - x/c) \end{array}\right\} \tag{33}$$

The deformation of an elastic body can as well be described by a line element [12]

$$ds^2 = ds_0^2 + 2\varepsilon_{ik}dx_i dx_k \tag{34}$$

where

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial \varepsilon_i}{\partial x_k} + \frac{\partial \varepsilon_k}{\partial x_i}\right) \tag{35}$$

and where $\underline{\varepsilon} = \{\varepsilon_x, \varepsilon_y, \varepsilon_z\}$ is the displacement vector, from which the velocity disturbance is obtained by:

$$\underline{u} = \frac{\partial \varepsilon}{\partial t} \tag{36}$$

For a transverse wave propagating into the x-direction, $\varepsilon_x=\varepsilon_1=0$, with the condition div $\underline{\varepsilon} = 0$ leading to

$$\frac{\partial \varepsilon_2}{\partial x_2} + \frac{\partial \varepsilon_3}{\partial x_3} = \varepsilon_{22} + \varepsilon_{33} = 0 \tag{37}$$

The identity with a gravitational wave is established putting $2\varepsilon_{ik} = h_{ik}$.

Fig. 1 shows the deformation of a ring vortex lattice for both waves.

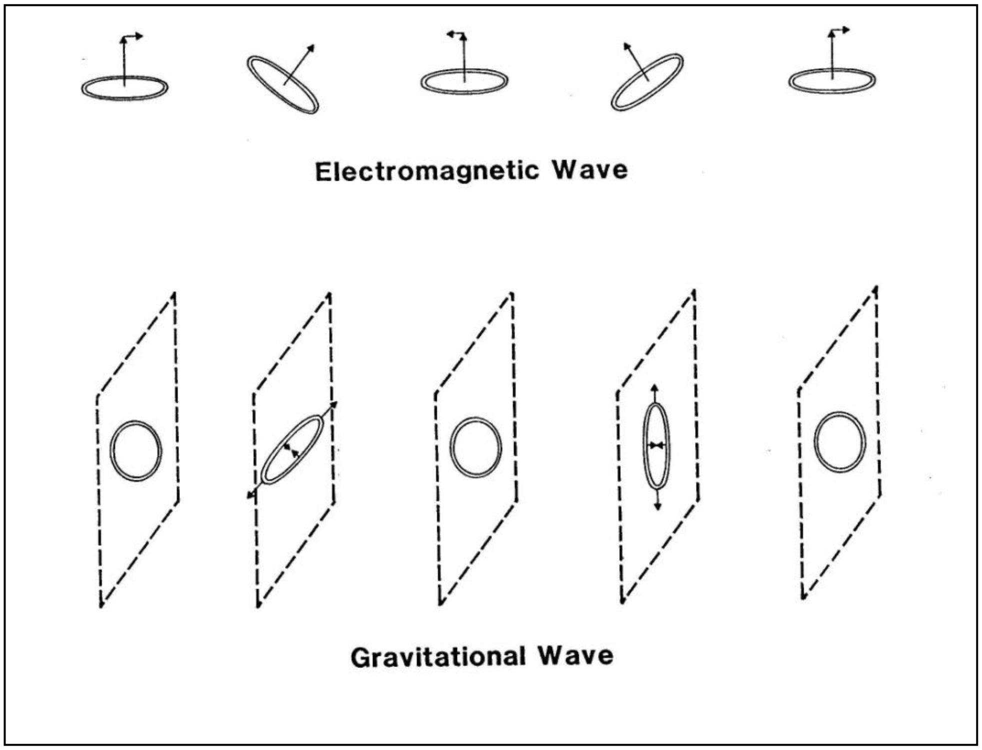

Fig. 1. Deformation of the vortex lattice for an electromagnetic and a gravitational wave.

In the Planck mass plasma model the zero point fluctuations of Planck mass particles bound in filaments of quantized vortices have a kinetic energy density of the order $\varepsilon_k \sim m_p c^2 / r_p^3$, which by order of magnitude is the same as the gravitational field energy density g^2, where $g = \sqrt{G} m_p / r_p^2$, is the Newtonian gravitational field a Planck mass particle would have at $r = r_p$. This explains the phenomenon of charge to result from fluctuations of Planck mass particles bound in quantized vortices, with the fluctuations setting up a field of virtual phonons around the vortices.

Einstein's nonlinear gravitational field equations can here be obtained in flat space-time, by coupling the gravitational field to the energy momentum tensor of matter and the gravitational field, as it was shown by Gupta [13] with the DeDonder gauge.

Lorentz invariance finally, is established as follows: For objects, which are here excitonic quasipartides of the Planck mass plasma, the attractive forces transmitted by virtual bosons and obeying Maxwell-type wave equations are balanced by the repulsive quantum forces. With both forces Lorentz invariant under a uniform Lorentz contraction and Einstein time dilation, Lorentz invariance is here a dynamic symmetry, as in the pre-Einstein theory of relativity by Lorentz and Poincaré, but only valid for energies small compared to the Planck energy.

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