## ON SOME INTRINSIC ATTRIBUTES OF MOTION AND PROPAGATION

Samuel H. Talbert (c/o Prof. Stavros Tavoularis, Department of Mechanical Engineering, University of Ottawa, 161 Louis Pasteur, Ottawa, Ontario K1N 6N5, Canada) e-mail: talbert.samy@ic.gc.ca

**Abstract**

We show the existence of some attributes of motion and propagation that are "intrinsic" in the sense that, for defining and calculating their values, neither the classical time-and-space nor the relativistic timespace manifolds need be invoked. The intrinsic kinematics of motion and light propagation is analyzed with systems of rulers in collinear motion. Since "space" and "time" are always extrinsic – they affect the system under consideration but are not affected by it – we could assign intrinsic physical meaning to neither the velocity of material bodies nor that of light, and thus do not use these in this paper. And yet, the results of our intrinsic kinematical analysis when compared with classical results show the same notable differences usually attributed to relativistic effects. This suggests to us that the three fundamental elements of Special Relativity: (1) the constancy of light velocity in all inertial frames, (2) the procedure to synchronize the inertial frames' clocks, and (3) the Lorentz invariance of the transformations between inertial frames, although clearly *sufficient* for the purpose of explaining the well known expected deviations from classical kinematics, are strictly speaking, not *necessary*.

## 1. INTRODUCTION

Linear bodies in constant collinear motion, such as two or more rulers sliding at constant speeds along each other, are simple mechanical systems seemingly ideal for studying the properties of motion. As it is well known, a classical description of how a light signal propagates in these systems would, by necessity, have to prescribe a different constant velocity of light with respect to each of the rulers in the system in order to retain a measure of time that is absolute and common to all rulers of the system. To overcome this problem, a relativistic description would then (i) replace the classical time with a relativistic time, relative and local to each ruler; (ii) specify along each ruler of the system a procedure for time synchronization that makes the velocity of light have the same constant values for all the rulers; and (iii) prescribe the Lorentz transformations between any pair of the system's rulers, viewed as inertial frames. It is the conclusion of this paper that the three steps just listed, though *sufficient* for the intended kinematical description, are (strictly speaking) not *necessary* since the intrinsic kinematical approach described here provides a physical interpretation of the relevant facts without invoking the geometry of either the classical space-and-time or its relativistic spacetime counterpart.

In the time-honored tradition of explaining the new by the old, consider for a moment the difference between locating points on a surface using a pair of Gaussian intrinsic coordinates and using three of the much older Descartes' Cartesian coordinates. The former dispenses with the three-dimensional space that supposedly contains the surface. In doing so, it disentangles the properties of the surface itself from those of the embedding space. To quote Bell [1] "Latitude and longitude on the earth are instances of these intrinsic, 'natural' coordinates; it would be most awkward to have to do all our navigation with reference to three mutually perpendicular axes drawn through the center of the Earth, as would be required for Cartesian sailing."

In the same Gaussian spirit, an intrinsic kinematical analysis disentangles the intrinsic properties of physical circumstances under scrutiny from those of any *embedding* space, which classical mechanics views "as container of all material objects [2]," by dispensing with such an embedding space. Dispensing with the embedding space does not automatically imply that we adopt here the alternative view of a *relational* space, which Leibnitz, Mach and the relationists that follow them conceived "as positional quality of the world of material objects," to quote Einstein's description of it [2], because this would mean accepting "distances" between moving objects as intrinsic, which we do not do here. There is a good reason for that rejection, and we explain it briefly below.

It is not possible to <u>intrinsically</u> measure distances between moving bodies since, as it is well known, such measurement requires a procedure for synchronizing distant clocks, that is, for "spreading time through space," as Bridgman so aptly describes it [3]. And, according to Reichenbach-Grünbaum thesis of the conventionality of simultaneity [4], the simultaneity of distant events can be legitimately fixed in different manners. Thus any clock synchronization procedure is clearly extrinsic to the physical situations where the supposedly synchronized clocks are to be used to ensure that all changing distances between moving bodies are measured at the *same time*.

With the last paragraph, we already passed from discussing why space cannot be intrinsic to hinting that time also could not be intrinsic. There are many reasons why this is indeed true, as briefly explained next.

At least two distinct and different notions of time are used in mechanics: The Newtonian time of classical mechanics is universal and absolute, that is, the same everywhere in space, irrespective of the choice of an inertial frame of reference. In contrast, the Einsteinian time of relativistic mechanics is local and relative, that is, the simultaneity of distant events is a matter of convention, regardless whether we measure it with clocks on the same or different inertial frames. Thus, for example, events that are regarded as simultaneous in some inertial frame on the basis of some given procedure for spreading time over space are not regarded as simultaneous when the same procedure is applied in some other inertial frame.

But there are other views of time aside from the mechanical ones. Raju [5] starts with the Newtonian and Einsteinian time and then continues to list the thermodynamic time, the electromagnetic time, the quantum-mechanical time, the cosmological time, and the mundane time. Note that these are distinct concepts of time, not just different choices of time scales, as exemplified by such “times” as: solar time, civic time, standard time, ephemeris time, sidereal time.

The distinctions between the different concepts of time listed by Raju are important, though they all introduce a time that, like the classical time, is extrinsic to the systems under consideration since it clearly affects the system under consideration but is unaffected by any its elements. We briefly illustrate below the thermodynamic and the electromagnetic time concepts (see Raju’s well-research monograph [5] for more details on these and the other time concepts).

The thermodynamic time is asymmetric and is thus unlike either the absolute or the relative times of mechanics, which are both symmetric. The entropy law furnishes the thermodynamic time with an arrow, so to speak, a direction, that goes from the past to the future, the latter distinguishing itself from the former through an increase in the entropy of closed systems.

The electromagnetic time is the time of the Maxwell’s field equations. Even though this time, like the mechanical times, is symmetric, its symmetry is entirely unlike that of the mechanical times. With the mechanical times, there is an accepted meaning to time reversal – it simply returns all systems to previous configurations, along the same “path,” now traversed in the reverse order. This is not so for the electromagnetic time. Maxwell field equations admit retarded and advanced solutions, which are both needed in practical applications such as radiation of electromagnetic waves. The advanced solutions describe electromagnetic waves that travel in the future and are thus acceptable as they imply the possibility of detecting now some radiation emitted well in the past. In contrast, the retarded potentials come with the implication that we should be able to detect now radiation emitted well in the future!

That a multitude of extrinsic time concepts is by necessity involved when, for example, studying the motions of hot plasma immersed in an electromagnetic field, raises the question of how to reconcile the different time concepts so as to have a sound logical foundation for such studies. But the everyday practice of researching such motions is seemingly unaffected by the ambiguous time concept that is actually used, and thus perhaps no time concept is ever needed. A system under consideration might always possess among its many variables one or more variables that could be indifferently selected to play the role of the independent variable against which to measure changes in all the other system variables.

We would not engage here in the still open debates over the history and meaning of space, time and spacetime issues highlighted above [6-12]. Instead, we wish to explore the possibility of analyzing mechanical systems without making any reference to time and space, thus neatly avoid having to take sides in these disputes.

We did not find even in recent textbooks on classical or relativistic mechanics (see for example [13-15]) any mention of such possibility. The few attempts at relational theories that we are aware of [11, 16] use "time" as the independent variable of motion in order not to lose the meaning they attach to mutual distances between moving bodies. In contrast, the view that such possibility exists is reinforced by the many practical examples where the independent time variable eventually drops out of calculations.

With the above in mind, we coined the expression "intrinsic attributes of motion and propagation" to reflect those aspects of the physical situation that could be fully analyzed without invoking any concepts of space and time whatsoever.

Thus as defined, intrinsic attributes of motion and propagation are properties unaffected by either the classical time-and-space manifold, or by its relativistic timespace counterpart. But do such properties exist? Do they have physically significant consequences?

It is the purpose of this paper to show that such properties exist and have significant physical consequences. The presentation here is limited to the kinematics of collinearly moving bodies and propagating signals such as sound and light, but we also have arrived at the same conclusion while further researching the intrinsic properties of motion and propagation in systems with bodies in coplanar motion.[1]

## 2. TWO-RULER SYSTEMS

Consider a system consisting of two rulers that could be made to slide side by side. The markings on each rule provide a coordinate to locate places on it. The physically interesting events in such a system consist of *encounters* where a place on one ruler is positioned just across a place on the other ruler. Each event is fully specified by two coordinates, one for each of the two places present at the encounter.

The dimensionality of the events manifold is an intrinsic property of the system. Its physical significance is that it allows to intrinsically distinguish between *motion* and *rest* in such a system. We say that the rulers of the system are at *rest* when its events manifold is one-dimensional since in such case, from knowing the coordinate of any one place present at an event, we know the coordinate of the other place present at the event. When this is not the case, we say that the rulers are in *motion* since a two-dimensional manifold of events materializes only with rulers that slide on each other.

All systems of two rulers are intrinsically similar since there are no dimensionless parameters that could distinguish between them. (A pair-wise comparison between different two-ruler systems involves of course a system of more than two rulers, for which the intrinsic elements discussed so far are insufficient.) The distinction ordinarily introduced through the relative velocity between the rulers has no intrinsic basis. Results are devoid of physical meaning when they depend on the numerical value of such velocity, which uses an extrinsic arbitrary measure of time duration.

## 3. LIGHT PROPAGATION IN TWO-RULER SYSTEMS

Consider a system with two rulers that slide on each other. To preserve the physical symmetry inherent in such a situation, we would arrange the rulers side by side with the markings on each ruler increasing in the direction of their relative motion, as illustrated.

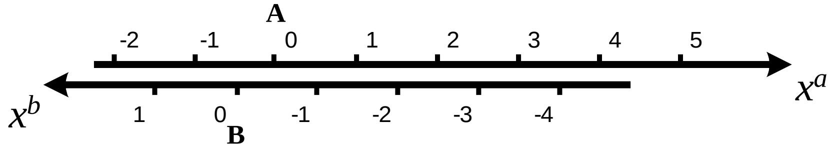


**Fig. 1.** Rulers **A** and **B** with their coordinate axes

[1] Manuscript in preparation: *Intrinsic Mechanics of Motion and Propagation, Part 3 – Kinematics of coplanar motion and propagation,* Technical Report TR-IM/P3, University of Ottawa, September 2002)

Say sparks are ignited at various two-place encounters, the *i*-th signal being generated at the event $E_i,\ i = 1, 2, \ldots$, which is an encounter of two places that carry appropriate electrical charges. The light from these sparks would then be detected at some other encounters. Say the light from the *i*-th signal is detected at a series of encounters $\overset{\rightarrow}{D}_{ij},\ j = 1, 2, \ldots$ with positive $x^a$-coordinates.

The coordinates of the two places present at each detection event could be recorded during the experiment and the records analyzed thereafter. They would produce one of the following two mutually exclusive sets of results:

1. Within the degree of accuracy provided by the experimental procedure and its measuring instruments, the recorded results would fit the following relation:

$$\frac{x^a(\overset{\rightarrow}{D}_{1j}) - x^a(E_1)}{x^b(\overset{\rightarrow}{D}_{1j}) - x^b(E_1)} = \frac{x^a(\overset{\rightarrow}{D}_{2j}) - x^a(E_2)}{x^b(\overset{\rightarrow}{D}_{2j}) - x^b(E_2)} = \ldots \text{ regardless of } j \tag{1}$$

2. The results would not fit the above relation.

In the first case, it would be appropriate to say that the two rulers are approaching (separating) at a constant *rate*.

In the second case, it would be appropriate to say that the approach (separation) rate between the two rulers is variable. This case presents no particular interest to us at this time and would not be discussed any further.

Note that the above relation is intrinsic to the two-rulers system and the signal under consideration – the relation is unchanged by changes of scale or origin of markings on the rulers.

Assuming we found the above ratio constant, the experiment could then be repeated, but instead of detections at encounters with a positive $x^a$-coordinate, we could record the series of detections $\overset{\uparrow}{D}_{ij},\ j = 1, 2, \ldots$ with positive $x^b$-coordinate. Again, one of two mutually exclusive situations would arise under these circumstances:

1. Within the degree of accuracy provided by the experimental procedure and its measuring instruments, the recorded results would fit the following relation:

$$\frac{x^a(\overset{\uparrow}{D}_{1j}) - x^a(E_1)}{x^b(\overset{\uparrow}{D}_{1j}) - x^b(E_1)} = \frac{x^a(\overset{\uparrow}{D}_{2j}) - x^a(E_2)}{x^b(\overset{\uparrow}{D}_{2j}) - x^b(E_2)} \ldots \text{ regardless of } j \tag{2}$$

2. The results would not fit the above relation.

Only the former situation interests us. There, we could further distinguish between the following two mutually exclusive cases:

1.1. The constant ratios of the two series of experiments are reciprocal, that is:

$$\left[\frac{x^a(\overset{\rightarrow}{D}_{1j}) - x^a(E_1)}{x^b(\overset{\rightarrow}{D}_{1j}) - x^b(E_1)}\right]\left[\frac{x^a(\overset{\uparrow}{D}_{1j}) - x^a(E_1)}{x^b(\overset{\uparrow}{D}_{1j}) - x^b(E_1)}\right] = \left[\frac{x^a(\overset{\rightarrow}{D}_{2j}) - x^a(E_2)}{x^b(\overset{\rightarrow}{D}_{2j}) - x^b(E_2)}\right]\left[\frac{x^a(\overset{\uparrow}{D}_{2j}) - x^a(E_2)}{x^b(\overset{\uparrow}{D}_{2j}) - x^b(E_2)}\right] = \ldots = 1 \tag{3}$$

1.2. The constant ratios of the two series of experiments are not reciprocal.

All light signals detected with the two-rulers system must retain the intrinsic symmetry of the system and are thus likely to fall in the first category above. This is because, in the absence of a transmitting medium, the light signal brings nothing to the system to intrinsically break the existing symmetry between its rulers. Clearly, without us assigning different names (**A** and **B**) to the rulers, the two series of tests with light signals would have been intrinsically indistinguishable. (No use in trying to distinguish between the *top* and the *bottom* ruler since this merely replaces a naming convention based on the letter of the alphabet, with one based on the top-down directions of the paper we write on!)

In contrast, the presence of a medium that, for example, transmits a sound signal, breaks the symmetry of the system, and the experiments are then likely to produce results that fall in the second category above.

If we assume that a light signal behaves intrinsically the same way regardless of the system where it propagates, the sequence of encounters that detect the light provides an intrinsic element capable of distinguishing among different two-ruler systems.

Say we denote $k^*_{ab}$ the magnitude of the (always negative) constant ratio found in the first series of experiments, and call any two-ruler system that manifests such constant ratio a $k^*_{ab}$ *-reference base*. Reference bases are then distinguished by the value of their $k^*$-constant. Thus a bi-directional light emitted at the encounter $\mathsf{E}$ of a $k^*_{ab}$ -reference base system A&B, would be detected at all encounters $\mathsf{D}$ attended by places on the two rulers whose coordinates meet the following relations:

$$x^a(\mathsf{D}) - x^a(\mathsf{E}) + k^*_{ab}[x^b(\mathsf{D}) - x^b(\mathsf{E})] = 0 \quad \text{when} \quad x^a(\mathsf{D}) - x^a(\mathsf{E}) \geq 0 \tag{4}$$

$$k^*_{ab}[x^a(\mathsf{D}) - x^a(\mathsf{E})] + x^b(\mathsf{D}) - x^b(\mathsf{E}) = 0 \quad \text{when} \quad x^b(\mathsf{D}) - x^b(\mathsf{E}) \geq 0 \tag{5}$$

We expect the experiments to produce results such that $0 < k^*_{ab} \leq 1$ since $k^*_{ab} = 1$ when the two rulers of the system are at rest with respect to each other (that is, the events manifold is one-dimensional), and, for $k^*_{ab} = 0$ to be true, one ruler has to co-move with the light signal, which is physically unattainable.

The following quantitative assessment of the relations *faster* and *slower* that apply to the rate at which two rulers approach (separate) is thus a significant intrinsic property of light propagation in two-ruler systems:

- Larger and larger $k^*$-values, eventually bound by the attainable maximum value of 1, indicate slower and slower approach (separation) between the rulers and includes the case when the rulers are rest relative to each other.

- Smaller and smaller $k^*$-values, eventually bound by the (unattainable for material bodies) limiting value of 0, indicate faster and faster approach (separation) between the rulers, but excludes the case of the rulers keeping up with the light signals.

## 4. THREE-RULER SYSTEMS

The presence of the third ruler **C** brings the need to consider events that are *three-place* (rather than two-place) encounters attended by one place from each ruler. The manifold of identifiable events is now three-dimensional, but the manifold of realizable events could still be one-dimensional or two-dimensional, never three-dimensional. In general, we could assume that $\mathsf{O}(0,0,0)$ is a three-place realizable encounter.

The symmetry present in a two-ruler system is lost in a three-ruler system. In Fig. 3, the three rulers are shown pair-wise, side-by-side, as laid at rest on each other. Although in each of the three possible pairs, the rulers are equipped with coordinates in the exact manner earlier indicated, thus retaining pair-wise symmetry, to do so, the

**B**-ruler's extension has to be equipped with two coordinates of opposite orientations ( $\bar{x}^b \equiv -x^b$ ), thus setting it apart from the other two rulers and breaking the symmetry between them.

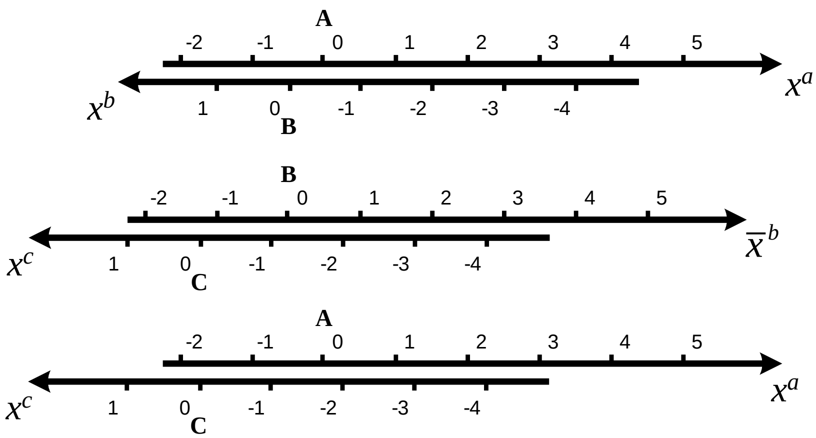


**Figure 3.** Coordinates of the rulers, laid pair-wise at rest on each other.

This is not merely a matter of coordinates convention. Its physical significance comes from the fact that at each encounter, a place that attends it has one of two mutually exclusive relations with the other two places involved. For example, the following relations could exist between the rulers' origins present at the event $Q(0,0,0)$ :

1. An origin could have the other two origins approaching/separating from the opposite sides. Exactly one of the three origins has this property. We denote the rulers so that this origin is that of the **B**-ruler.
2. An origin could have the other two origins approaching/separating from the same side. Two of the three origins have this property. These are the origins on the rulers denoted **A** and **C** here; for symmetry reasons, these labels are interchangeable.

In a three-ruler system, each of the three origins is present at encounters with places on the other two rulers. Were we to set a recording device on one of the origins, we could record sequences of pairs of coordinates of places on the other rulers as they pass by the origin where the device is located. Then, at any encounter of the sequence, we would have the ratio of the differential changes of coordinates from an encounter to a neighboring encounter in the sequence of encounters attended by the origin where the device is located. This ratio is thus an intrinsic element of such systems. Since there are three origins, we have three such *change* ratios, which could differ among the origins and, for the same origin, could differ from encounter to encounter:

$$k^a_{bc} \equiv (d\bar{x}^b/dx^c)\big|_{dx^a=0} < 0\,, \quad \text{for the encounters attended by } \mathbf{A}\text{-ruler's origin,} \tag{6a}$$

$$k^b_{ca} \equiv (dx^c/dx^a)\big|_{dx^b=0} > 0\,, \quad \text{for those attended by } \mathbf{B}\text{-ruler's origin, and} \tag{6b}$$

$$k^c_{ab} \equiv (dx^a/dx^b)\big|_{dx^c=0} < 0\,, \quad \text{for the ones where } \mathbf{C}\text{-ruler's origin is present} \tag{6c}$$

(The above relations, chosen for the convenience of what follows, break the symmetry between **A** and **C** since $dx^a$ is placed at the numerator of $k^c_{ab}$ but $dx^c$ is at the denominator of $k^a_{bc}$ ; this does not limit their generality.)

All our further discussion focuses on systems with constant change ratios, that is, systems such that each of the rulers' origins records the same constant $k$ -value at all the encounters that it attends, though possibly – and most likely - different origins would record different $k$ -values. Our immediate objective is to find for such systems, their manifold of realizable encounters.

At any realizable encounter, the coordinates of two of the three places that attend it determine the coordinate of the third place present. Thus, for example, say $x^c = x^c(x^a, x^b)$. Then after differentiation, we have on account of the constant ratios above:

$$dx^c = \frac{\partial x^c}{\partial x^a} dx^a + \frac{\partial x^c}{\partial x^b} dx^b \quad \text{with its integrated counterpart} \quad x^c = k_{ca}^b x^a - \frac{1}{k_{bc}^a} x^b \tag{7a}$$

This intrinsic relation must therefore be met both classically and relativistically. Equivalently,

$$dx^a = \frac{\partial x^a}{\partial x^b} dx^b + \frac{\partial x^a}{\partial x^c} dx^c \quad \text{with} \quad x^a = k_{ab}^c x^b + \frac{1}{k_{ca}^b} x^c \tag{7b}$$

$$dx^b = \frac{\partial x^b}{\partial x^a} dx^a + \frac{\partial x^b}{\partial x^c} dx^c \quad \text{with} \quad x^b = \frac{1}{k_{ab}^c} x^a - k_{bc}^a x^c \tag{7c}$$

Furthermore, since the partial derivatives meet $\frac{\partial x^a}{\partial x^b}\frac{\partial x^b}{\partial x^c}\frac{\partial x^c}{\partial x^a} = -1$, the constant ratios above are not independent:

$$k_{bc}^a k_{ca}^b k_{ab}^c = 1 \tag{8}$$

The comparable relations of classical mechanics are arrived at using a common time for the three rules and the Galilean transformation between the coordinates of two inertial frames in relative motion:

$$x^a = -\frac{v_{ac}}{v_{bc}} x^b + \frac{v_{ab}}{v_{bc}} x^c, \qquad x^b = -\frac{v_{bc}}{v_{ac}} x^a + \frac{v_{ab}}{v_{ac}} x^c, \qquad x^c = \frac{v_{bc}}{v_{ab}} x^a + \frac{v_{ac}}{v_{ab}} x^b \tag{9}$$

For (7) and (9) to represent the same physical condition for all coordinate values, we must necessarily have: $\frac{v_{ab}}{v_{ac}} = -k_{bc}^a$, $\frac{v_{bc}}{v_{ab}} = k_{ca}^b$, and $\frac{v_{ac}}{v_{bc}} = -k_{ab}^c$. Note however the following substantial differences between the classical and the intrinsic results.

Classical mechanics associates a numerical value to the approach (separation) speed between a pair of bodies in relative motion. But we have associated a numerical value to three, not two, rulers in relative motion. Using only the numerical *k*-values associated with three rulers in relative motion, there is no unique way to attribute values to the classical time rates of change of the pair-wise distances between bodies in relative motion. In other words, the *k*-values under-determine the relative approach (separation) speeds between the two rulers, though the speed ratios are fully determined.

The classical law of addition of collinear velocities: $v_{ac} = v_{ab} + v_{bc}$ is equivalent to: $\frac{v_{ab}}{v_{ac}} + \frac{v_{bc}}{v_{ac}} = 1$. But for this to be so, we must have $-k_{bc}^a - k_{ab}^{c\,-1} = 1$. Later we show that this cannot be so, but even without a formal proof it is evident that if such a relation were true, it would reduce the number of intrinsically independent $k$-values from two to just one. There is no intrinsic reason why the three-ruler system should have one rather than two independent $\lambda$-values.

The classical law of addition of relative velocities is thus not compatible with the intrinsic kinematics of a system of three bodies in collinear motion. The incompatibility is traced back to two basic aspects of classical mechanics:

(i) the introduction of an extrinsic absolute time, and (ii) the use of algebraic relations involving coordinates of two places on bodies that are in relative motion, which goes beyond what ordinary geometry would allow.

## 5. LIGHT PROPAGATION IN A THREE-RULER SYSTEM

We reconsider now the earlier physical set-up of three rulers: **A**, **B** and **C**, sliding along each other as shown in Fig. 3. Say that repeated experiments with this set-up and propagating light have shown that:

1. A&B is a reference base with $k^*_{ab}$.
2. **C**-ruler's origin attends all pair-wise encounters of A&B that meet $x^a - k^c_{ab} x^b = 0$, with constant $k^c_{ab}$.

We show next that under these two conditions, it is possible to assume without fear of contradictions that:

1. A&C and B&C are also valid reference bases, with $k^*_{ac}$ and $k^*_{bc}$, respectively.
2. The other two $k$-values: $k^a_{bc}$ and $k^b_{ac}$, of three-ruler system A&B&C are constant and calculable in terms of the known $k^*_{ab}$ and $k^c_{ab}$.

## 6. RELATIONS BETWEEN $k^*$-VALUES IN RELATED REFERENCE BASES

The rulers in Fig. 3 are so marked that the origin of the third ruler **C** is present at the encounter $\mathcal{O}(0,0)$ in A&B. Let a light signal be generated at $\mathcal{O}(0,0)$ and subsequently detected at some two-place encounter $\mathcal{D}(x^a, x^b)$ of A&B, with $x^a(\mathcal{D}) > 0$. This means that the place $x^a(\mathcal{D})$ on the **A**-ruler and the place $x^b(\mathcal{D})$ on the **B**-ruler detect the signal at their encounter, and since A&B is a reference base, we have from (4):

$$x^a(\mathcal{D}) + k^*_{ab}\; x^b(\mathcal{D}) = 0 \tag{10}$$

Denote by $x^c(\mathcal{D})$ the coordinate on the **C**-ruler of the place present at $\mathcal{D}(x^a, x^b)$. We would then say that the event $\mathcal{D}$, which is realized by the three-place encounter $\mathcal{D}(x^a, x^b, x^c)$, is identified in the A&B reference system through the two-place encounter $\mathcal{D}(x^a, x^b)$. Since the signal emitted at the event $\mathcal{O}(0,0,0)$ is detected at event $\mathcal{D}$, the place $x^c(\mathcal{D})$ also detects it.

Alternatively, the same event $\mathcal{D}$ could be identified through the two-place encounter $\mathcal{D}(\bar{x}^b = -x^b, x^c)$ of B&C, so this two-place encounter detects the signal. And if B&C is a valid reference base, then we must also have:

$$\bar{x}^b(\mathcal{D}) + k^*_{bc}\; x^c(\mathcal{D}) = 0 \tag{11}$$

The above two relations then lead to:

$$x^a(\mathcal{D}) + k^*_{ab} k^*_{bc}\; x^c(\mathcal{D}) = 0 \tag{12}$$

Now, for reasons similar to the ones just described, if A&C is a valid reference base, the signal is also detected at the two-place encounter $\mathcal{D}(x^a, x^c)$; hence, we must also have:

$$x^a(\mathcal{D}) + k^*_{ac}\; x^c(\mathcal{D}) = 0 \tag{13}$$

But since our choice of detection event $\mathcal{D}$ was arbitrary, we would conclude from the above that the $k^*$-values of the pair-wise reference bases of a three-ruler system are related as follows:

$$k^*_{ac} = k^*_{ab}\, k^*_{bc} \tag{14}$$

The symmetry of the situation ensures that similar considerations are valid for a detection event with negative first coordinate in A&B. We are then lead to the same conclusion, and to the same relation between the $k^*$-values.

## 7. A FURTHER LOOK AT THE PROPAGATING LIGHT SIGNAL IN THREE-RULER SYSTEMS

As earlier discussed, with a system of three rulers, we have a three-dimensional manifold of <u>identifiable</u> three-place encounters that contains a two-dimensional manifold of <u>realizable</u> three-place encounters, namely, the events manifold of this system.

The emission of the signal at $\mathcal{O}(0,0,0)$ is by our choice a realized three-place encounter that detects the signal, and thus belongs to the events manifold.

Considerations similar to the ones used earlier with respect to a specific encounter $\mathcal{D}(x^a, x^b)$ that detects a signal in A&B, and to its counterparts $\mathcal{D}(x^a, x^c)$ and $\mathcal{D}(\bar{x}^b, x^c)$ that detect the signal in A&C and B&C, respectively, can be extended to all such triples of two-place detection encounters. In each case, three two-place encounters such as $\mathcal{D}(x^a, x^b)$, $\mathcal{D}(x^a, x^c)$ and $\mathcal{D}(\bar{x}^b, x^c)$ could be identified and each of theses two-place encounters could be used to reference one and the same realized event, a three-place encounter that detects the signal. A sequence of such three-place encounters that detect the <u>same</u> emitted signal is said to trace a *ray* on the events manifold.

In the three-dimensional manifold of the conceivable three-places encounters, a light signal that is emitted at $\mathcal{O}(0,0,0)$ would be detected at a series of events $\vec{\mathcal{D}}$ with a positive first coordinate $x^a$. These detections trace a ray that is to be found at the intersection of the following three planes of the manifold:

$$\begin{cases} x^a(\vec{\mathcal{D}}) + k^*_{ab}\; x^b(\vec{\mathcal{D}}) = 0 \\ \bar{x}^b(\vec{\mathcal{D}}) + k^*_{bc}\; x^c(\vec{\mathcal{D}}) = 0 \\ x^a(\vec{\mathcal{D}}) + k^*_{ac}\; x^c(\vec{\mathcal{D}}) = 0 \end{cases} \tag{15}$$

The system of homogeneous equations (15) has non-zero solutions only when its determinant vanishes, that is, when $k^*_{ac} = k^*_{ab}\, k^*_{bc}$, which we have found previously to be true. Any solution of (15) is a realizable three-place encounter that detects the light signal, an event $\vec{\mathcal{D}}$.

Because of the fundamental coordinate-symmetry involved, the emission of a light signal at $\mathcal{O}(0,0,0)$ would also be detected at a series of events $\overleftarrow{\mathcal{D}}$ with negative $x^a$ values. The detections on this ray must then meet:

$$\begin{cases} k^*_{ab}\; x^a(\overleftarrow{\mathcal{D}}) + x^b(\overleftarrow{\mathcal{D}}) = 0 \\ k^*_{bc}\; \bar{x}^b(\overleftarrow{\mathcal{D}}) + x^c(\overleftarrow{\mathcal{D}}) = 0 \\ k^*_{ac}\; x^a(\overleftarrow{\mathcal{D}}) + x^c(\overleftarrow{\mathcal{D}}) = 0 \end{cases} \tag{16}$$

Any solution of (16) is a realizable three-place encounter that detects the light signal, the event $\overleftarrow{\mathcal{D}}$.

In the three-dimensional manifold of conceivable three-place encounters, the events manifold is precisely the plane that contains the two rays of the solutions to (15) and (16). It can then be specified as a linear combination of the two ray vectors $\overrightarrow{OD}$ and $\overleftarrow{OD}$, since the events manifold plane contains these vectors.

From the above equations of the two rays involved, we get the vectors $\overrightarrow{OD}$ and $\overleftarrow{OD}$ as directional derivatives along the two light signal rays involved, that is:

$$\overrightarrow{OD}: \begin{cases} \dfrac{dx^a}{dl} = -k^*_{ac} \\ \dfrac{d\overline{x}^b}{dl} = -k^*_{bc} \\ \dfrac{dx^c}{dl} = 1 \end{cases} \quad \text{and} \quad \overleftarrow{OD}: \begin{cases} \dfrac{dx^a}{dl} = -\dfrac{1}{k^*_{ac}} \\ \dfrac{d\overline{x}^b}{dl} = -\dfrac{1}{k^*_{bc}} \\ \dfrac{dx^c}{dl} = 1 \end{cases} \tag{17}$$

As explained, a vector $OP$ that specifies some realizable three-place encounter $P$, though not necessarily one that detects the signal emitted at $O(0,0,0)$, would be a linear combination of the two vectors $\overrightarrow{OD}$ and $\overleftarrow{OD}$:

$$OP: \begin{cases} \dfrac{dx^a}{dl} = -h_1 k^*_{ac} - h_2 \dfrac{1}{k^*_{ac}} \\ \dfrac{d\overline{x}^b}{dl} = -h_1 k^*_{bc} - h_2 \dfrac{1}{k^*_{bc}} \\ \dfrac{dx^c}{dl} = h_1 + h_2 \end{cases} \quad \text{and, by integration,} \quad OP: \begin{cases} x^a = -k^*_{ac} z^1 - \dfrac{1}{k^*_{ac}} z^2 \\ \overline{x}^b = -k^*_{bc} z^1 - \dfrac{1}{k^*_{bc}} z^2 \\ x^c = z^1 + z^2 \end{cases} \tag{18}$$

The two new variables $z^1$ and $z^2$, which are obtained from the product of the constants $h$ and the parameter $l$, are thus the Gaussian coordinates of three-places encounters $P$ in a three-rulers system. When these Gaussian variables are eliminated from the above equations, we obtain the counterparts of (7). For example, we list below the counterpart of (7a):

$$x^c = \frac{k^*_{ab}(1 - k^{*\,2}_{bc})}{k^*_{bc}(1 - k^{*\,2}_{ab})} x^a + \frac{k^*_{ab}(1 - k^{*\,2}_{ac})}{k^*_{ac}(1 - k^{*\,2}_{ab})} x^b \tag{19}$$

The events manifold of a three-ruler system thus consists of those three-place encounters whose coordinates meet the above relation.

## 8. RELATIONS BETWEEN THE $k$-CONSTANTS AND THE $k^*$-CONSTANTS

For (7) and (19) to be both valid for three-place encounters that belong to the same manifold of events, we must have the following identities:

$$k^a_{bc} = -\frac{k^*_{ac}(1 - k^{*\,2}_{ab})}{k^*_{ab}(1 - k^{*\,2}_{ac})} \le 0, \qquad k^b_{ca} = \frac{k^*_{ab}(1 - k^{*\,2}_{bc})}{k^*_{bc}(1 - k^{*\,2}_{ab})} \ge 0, \qquad k^c_{ab} = -\frac{k^*_{bc}(1 - k^{*\,2}_{ac})}{k^*_{ac}(1 - k^{*\,2}_{bc})} \le 0 \tag{20}$$

We could substitute these values in the relation mandated by the classical addition of velocities $-k_{bc}^{a}-k_{ab}^{c\ -1}=1$, to conclude that it requires: $k_{ab}^{*}+k_{bc}^{*}-k_{ab}^{*}k_{bc}^{*}=1$. But the left-hand side of the latter relation is a monotonically increasing function in each $k^{*}$-constant that attains its maximum value 1 when the values of both $k^{*}$-constants are exactly 1, and thus:

$$-k_{bc}^{a}-k_{ab}^{c\ -1}\leq 1 \tag{21}$$

This is the formal basis of our earlier conclusion that the classical rule for the addition of velocities cannot be met when the rulers are sliding on each other.

## 9. INTRINSIC KINEMATICS OF LORENTZ-FIZGERALD CONTRACTION

We analyze next the intrinsic kinematics of the *Gedanken* experiment envisaged by Einstein to show that one needs no clocks to "observe" the Lorentz-Fitzgerald contraction of moving rods. The experiment is often cited in the literature on Special Relativity and is elaborated in Miller's authoritative text [17] on the emergence and early interpretation of Einstein's Special Relativity, with full reference to Einstein's original text.

Two rods that have the same length while at rest on each other are set in motion to move in opposite directions and symmetric with respect to a common frame of reference so that their opposite ends meet (front end of one encounters the back end of the other, and vice-versa) and the location of the two resulting encounters could be marked on the common frame. In such case, Einstein reasoned, the distance between the marks on the common frame would always be smaller than the length of the rods, regardless of the magnitude of the rods' velocity relative to this common frame.

In Fig. 3, say we placed a unit length rod with its back end at the origin of the **A**-ruler, and its front end at $x^{a}=1$. We also placed another unit length rod with the front end at the origin of the **C**-ruler and its back end at $x^{c}=-1$. (The front and the back ends are being reckoned in the respective directions of motion of the two rods relative to the **B**-ruler.) We then set the rulers in sliding motion as earlier discussed.

When the **A**- and **C**-rulers slide on each other, symmetric with respect to the **B**-ruler, the back and front ends of the rods meet and mark the place on the **B**-ruler that is present at the their encounter. The experimental outcome of interest to us thus consists of the two marks left on the **B**-ruler. We designate $L_{b}$ the distance between these marks. This distance Einstein predicted to be less than the length of the rods regardless of their relative velocities with respect to the common frame, here the **B**-ruler.

The experiment is thus such that one could in principle obtain an experimental record of the predicted contraction. (We prefer to interpret it as a *kinematical projection* since, unlike the contraction predicted by Special Relativity, which involves <u>two</u> frames of reference in relative motion, the set-up of Einstein's experiment requires <u>three</u> such frames.) Surprisingly, however, there is still a widely held view that length contraction, unlike time dilation, is not observable, a view restated in a recent textbook [18]:

> Whereas time dilatation is readily observed experimentally, the corresponding spatial effect, called Lorentz contraction, is not… Nevertheless an appreciation of the effect is often helpful in understanding the content of a relativistic calculation.

To calculate $L_{b}$, we first apply twice the earlier result (7c) to find the location of the marks on the **B**-ruler. One mark is left at $x^{b}=0$ when the back end of the rod on the **A**-ruler meets the front end of the rod on the **C**-ruler. One further mark is left at $x^{b}=k_{ab}^{c}+k_{bc}^{a\ -1}$ when the front of the rod on the **A**-ruler meets the back of the rod on the **C**-ruler. The length $L_b$ between these marks is thus $L_b = -\left(k_{ab}^{c} + k_{bc}^{a\,-1}\right)$. But from (21) this means that $L_b \leq 1$ as predicted by Einstein.

Having succeeded to prove intrinsically Einstein's assertion that the distance between the marks is at most equal to the length of the rods, the equality being attained only when both rods are resting, not sliding, along a common frame of reference, we inquire next into what other consequences of relativistic kinematics could be intrinsically derived. Clearly, all consequences that do not involve the extrinsic time could potentially have intrinsic physical significance. We show next that this is in fact so.

## 10. INTRINSIC LORENTZ TRANSFORMATIONS

The following introduction of a pair of generalized coordinates $(\chi^a, \chi^b)$ preserves the required symmetry between the coordinates of a reference base. Through these generalized coordinates, we mimic the use of the product $ct$ in Special Relativity, but without attempting to assign a physical meaning either to each factor separately or to their joint product:

$$\begin{cases} \chi^a = \dfrac{1}{\beta_{ab}}\left(-x^a - \dfrac{1}{\gamma_{ab}} x^b\right) \\ \chi^b = \dfrac{1}{\beta_{ab}}\left(-\dfrac{1}{\gamma_{ab}} x^a - x^b\right) \end{cases} \text{ and its inverse } \begin{cases} x^a = \dfrac{1}{\beta_{ab}}\left(-\chi^a + \dfrac{1}{\gamma_{ab}} \chi^b\right) \\ x^b = \dfrac{1}{\beta_{ab}}\left(\dfrac{1}{\gamma_{ab}} \chi^a - \chi^b\right) \end{cases} \tag{22}$$

These equations of definition use a yet <u>unspecified</u> parameter $\beta_{ab}$ and the familiar shorthand $\gamma_{ab} \equiv 1\big/\sqrt{1-\beta_{ab}^{2}}$ .

Any of the pairs $(x^a, x^b)$ or $(\chi^a, \chi^b)$ could identify encounters in a reference system A&B. But so could the cross-combinations $(x^a, \chi^a)$ and $(\bar{x}^b, \chi^b)$. The above equations of definition show that the cross-combinations are related by a Lorentz transformation:

$$\begin{cases} \bar{x}^b = \gamma_{ab}(x^a + \beta_{ab}\chi^a) \\ \chi^b = \gamma_{ab}(\beta_{ab} x^a + \chi^a) \end{cases} \text{ and its inverse } \begin{cases} x^a = \gamma_{ab}(\bar{x}^b - \beta_{ab}\chi^b) \\ \chi^a = \gamma_{ab}(-\beta_{ab}\bar{x}^b + \chi^b) \end{cases} \tag{23}$$

It is remarkable that the Lorentz transformations (23), unlike those of Special Relativity, embody only the intrinsic kinematics of the system under consideration and are thus devoid of any assumptions about light propagation and clock synchronization. We refer to them as the *intrinsic* Lorentz transformations.

Similar generalized coordinates could be introduced in the reference bases B&C and in A&C (with $\bar{x}^c = -x^c$). We then obtain relations similar to (23):

$$\begin{cases} \bar{x}^c = \gamma_{ac}(x^a + \beta_{ac}\chi^a) \\ \chi^c = \gamma_{ac}(\beta_{ac} x^a + \chi^a) \end{cases} \text{ and } \begin{cases} \bar{x}^c = \gamma_{bc}(\bar{x}^b + \beta_{bc}\chi^b) \\ \chi^c = \gamma_{bc}(\beta_{bc}\bar{x}^b + \chi^b) \end{cases} \text{, with their respective inverses} \tag{24}$$

The three parameters $\beta_{ab}$, $\beta_{bc}$ and $\beta_{ac}$ of the above intrinsic Lorentz transformations are conceptually unrelated to the $\beta$ -parameters of Special Relativity, though they provide the necessary theoretical basis for deriving all the intrinsic attributes of motion and propagation that are usually obtained relativistically.

For example, (23) and (24) are compatible (that is, the <u>same</u> generalized coordinates $x^a$, $x^b$ and $x^c$ could be used) only when the yet to be specified intrinsic parameters $\beta_{ab}$, $\beta_{bc}$, and $\beta_{ac}$ meet some restrictions. To find these, we substitute in the first set of equations (24) the inverse relations of (23), and then compare the result with the second set of equations (24). The following necessary relation between the parameters $\beta$ obtains:

$$\beta_{ac} = \frac{\beta_{ab} + \beta_{bc}}{1 + \beta_{ab}\beta_{bc}} \quad \text{and, conversely} \quad \beta_{bc} = \frac{\beta_{ac} - \beta_{ab}}{1 - \beta_{ab}\beta_{ac}} \tag{25}$$

Though the relation is the one familiar from relativistic kinematics, it would be meaningless to call it <u>here</u> the law of “addition of velocities” since we use no extrinsic concept of velocity.

Further analysis of the three-ruler systems leads to the counterparts of (7); we list below the counterpart of (7a):

$$x^c = \frac{\beta_{bc}\gamma_{bc}}{\beta_{ab}\gamma_{ab}} x^a + \frac{\beta_{ac}\gamma_{ac}}{\beta_{ab}\gamma_{ab}} x^b \tag{26}$$

For (19) and (26) to represent the same physical condition at all coordinate values, we must have:

$$\frac{k^*_{ab}\beta_{ab}\gamma_{ab}}{(1-k^{*\,2}_{ab})} = \frac{k^*_{bc}\beta_{bc}\gamma_{bc}}{(1-k^{*\,2}_{bc})} = \frac{k^*_{ac}\beta_{ac}\gamma_{ac}}{(1-k^{*\,2}_{ac})} \tag{27}$$

Equations (27), under the conditions imposed by (25), yield a one-to-one relation between the earlier defined constants of $k^*$-reference bases and the respective $\beta$-parameters of the intrinsic Lorentz transformations:

$$k^* = \sqrt{\frac{1-\beta}{1+\beta}} \quad \text{and its inverse} \quad \beta = \frac{1-k^{*2}}{1+k^{*2}}, \text{ with } \gamma = \frac{1+k^{*2}}{2k^*} \tag{28}$$

(Appropriate identical pairs of subscripts are implied.)

## 11. CONCLUDING REMARKS

Relations between the coordinates of realized three-place encounters in simple systems of rulers in collinear motion could be derived not only by classical or relativistic approaches, but also intrinsically. Our intrinsic analysis here yields two major conclusions:

(1) It is possible to correlate the results of the classical and intrinsic kinematical analyses, and when this is done, it becomes clear that the classical law of addition of velocity is not compatible with the intrinsic kinematics of a three-ruler system.

Since classical kinematics uses the extrinsic concepts of space, time and velocity whereas the intrinsic approach does not do so, we conclude (invoking Ocham’s razor, of course) that the classical addition of velocities is not sustainable even by classical standards, that is, even without invoking the assumptions of Special Relativity.

(2) It is possible to correlate the results of the relativistic and intrinsic kinematical analyses, and when this is done, it shows that the relativistic addition of velocities is compatible with the intrinsic kinematics of a three-ruler system. Furthermore, earlier indications are that all the kinematical results of Special Relativity that do not specifically invoke the relativistic time might have intrinsic physical meaning.

The above last statement notwithstanding, the relativistic and intrinsic kinematics of systems of three bodies in collinear motion differ substantially in at least the following aspects:

1. Intrinsic kinematics makes no assumptions about space and time. In contrast, assumptions about clock synchronization and light propagation play an essential role in Special Relativity and no relativistic analyses are possible without them.
2. The introduction of generalized coordinates in intrinsic kinematics is strictly a matter of choice; solutions are arrived at without having to use such generalized coordinates. The intrinsic Lorentz transformations are thus not necessarily needed, though they apply to the generalized coordinates earlier discussed. In contrast, Lorentz transformations of spacetime coordinates are essential to relativistic kinematics.
3. The generalized variables introduced by intrinsic kinematics mimic the product $ct$ of Special Relativity but the values of these generalized variables cannot be meaningfully and intrinsically factored into a constant velocity of light and a variable that is physically meaningful and related to the frame-dependent time of relativistic kinematics.
4. Both relativistic and intrinsic analysis uses some $b$ -parameters. The former does it by necessity - the latter, on an optional basis. But only relativistic kinematics, not the intrinsic one, assigns a particular extrinsic meaning to these parameters.

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