Essays Upon Special Relativity

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ABSTRACT: First we shall show when the time-measure is transferred in practice from one system to another then Lorentz-transformation occurs by the CR-measurement. Then we shall study and solve the so-called clock paradox. We shall: show if we interpret Lorentz-transformation correctly then no antinomy occurs since according to relativity there can be only symmetrical correlations between two translatoric systems, but L (Lorentz-transformation) is asymmetric. The occurence of the paradox is unavoidable because we tried to force symmetry on asymmetry. We shall see that the importance of everything is much deeper than just correcting a miscalculation.

THE TRANSFERING OF TIME-MEASURE AS A GENERATOR OF LORENTZ-TRANSFORMATIONS

The L-measure is determined best by those two axioms of relativity theory worked out by Constantin Caratheodory and (independently) Hans Reichenbach which follow Hilbert's geometric train of thought.Their measuring instrument is only one static central clock, which is considered as absolute "etalon" (e.g. a completely homogeneous steel-ball rotating in absolute space freely from every kind of influence). The measurement happens by measuring of time of line sight of light impulse (radar measurement) reflected forward and backwards on the basis of the fact that in this case the light velocity is constant in every translatoric system. The result is the relativistic co-ordinates required by L (the Lorentz-transformation). This method of measuring or the definition of measurement will be called CR-measurement in the following.

Take two Euclidean planes S_1 and S_2, which coincide in the beginning. Let x_1 , y_1, z_1 and x_2, y_2, z_2 the axes of the co-ordinate systems belonging to S_1 and S_2 respectively. Axes x_1 and x_2 coincide, axes y_1 and y_2, axes z_1 and z_2 are parallel, and the distance between axes y_1 and y_2 is l_0 in the given moment. The two planes move at constant speed v as compared with each other in the direction of x_1. Two observers work in S_1 and S_2 and to make a comparison between their results the observers need to know mutually the value relations between the results. For this purpose observer M(x_1,y_1) imparts his unit of time to observer M(x_2,y_2). There is one clock in the place $x_1 = 0$,$y_1 = 0$, $z_1 = 0$ and it is the most expedient to suppose that the clock emits electric radiation (electromagnetic radiation) with constant time of oscillation. If S_1 and S_2 are at relative rest and their y-axes coincide then the light signal which transmits time runs the distance $y_{1\rightarrow}y_2$ (= l_0) and it transmits the interval δt_1 being equivalent to the radiated one to y_2.

If system S_2 moves at constant speed v as compared with S_1 in the mentioned direction while its axes remain parallel to their original direction then even in this case both observers can recognize the moment of time when their y-axes coincide. Now it is decisively important that in the case of their relative motion the two observers can follow exactly the same measuring procedure they followed in the case of their being in relative rest. This fact is obvious since they are about to start to study their systems just now and they have not known the quantitative values of either l_0 or v yet. So M(x_1,y_1) transmits the light signal with frequency δt_1 at transmitting y_1 again in the moment when $x_2 = x_1 = 0$. But the point of the axis y_2 where M(x_2,y_2) receives the signal during the time of line sight comes out of place and when the light reaches the axis x_2 then $x_2 = vt_2$. Here $t_2 = 1/c$ so when $c = 1$ then $x_2 = -v$. Consequently the angle of incidence α of the light signal will not be 0 and the ratio of the growth of δt_1 will be $\delta t_1/\cos\alpha$. But

$$\frac{1}{\cos\alpha} = \frac{1}{(1^2 - v^2 1^2)^{1/2}} = \beta$$

and this implies that

$$\delta t_2 = \beta\delta t_1 \qquad (1)$$

the Lorentz -transformation if we suppose that $x_1 = 0$. By this means the rate of M(x_2, y_2)'s clock is given directly while the position of the pointer of the clock can be determined by the radar measurement of the distance l when the transmission of time happens. We realize at once that transversal Doppler effect occurs in connection with $t_2 = \beta t_1$ and Bradley-aberration occurs in the change of angle α. And we will see that the CR-measurement, which uses the point-coincidence of light signals can transmit the exact time-measure only if it corresponds to the L-connection.

THE CLOCK PARADOX

The bibliography of clock-paradox could be written in many volumes. Already in 1906 Einstein emphasized the clock paradox in his basic publication but Longevin was the first to call attention to its paradoxical character. Since then the most famous physicists: Einstein, Lorentz, von Laue, Pauli, Weyl, Eddington, Petzoldt, Knopf, Reichenbach, etc... dealt with its solution without getting an acceptable result. Now it has a central position in the popular scientific literature, especially since the cosmic expeditions came up. Here it is stated seriously that it can be used as a possibility proved by Einstein's authority to extend human life unlimitedly.

The change of direction of motion is necessary for most attempts in solving the problem: we will see as follows that the clock paradox is independent of the change of direction. We emphasize the mentioned fact again that Lorentz already realized in 1913 that the problem could not be solved by means of relativity.

Suppose that M(x_1,y_1)'s clock becomes inaccessible or unusable for a time for a reason. In order for the continuity of the observations to be ensured M(x_1,y_1) has to do nothing but take back his time-measure again from M(x_2,y_2) where the continuity of time did not break off. You need not be a convinced relativist in order to realize that the above procedure must be exactly the same as the procedure in the case of the previous transmission. In other words: the subscripts of x_1, x_2 and y_1,y_2 must be commuted in a symmetrical way. From the point of view of the relativists the theorem is very important which says that none of the systems with subscripts 1 and 2 are distinguished, so their correlation is symmetrical, their subscripts can be commuted. The result is the following:

In the direction $1 \rightarrow 2$: $\delta t_2 = \beta\delta t_1$
in the direction $2 \rightarrow 1$: $\delta t_1 = \beta\delta t_2$

and this implies that for any t the clock paradox is

$$\frac{t_1}{t_2} = \frac{t_2}{t_1}. \qquad (2)$$

This should be a natural and observable phenomenon since the two observers can make a comparison between the rates of their clocks later by setting their clocks directly against each other.

First of all we must point out two important facts:

1. The change of direction of v is not a "sine qua non" condition of the occurence of clock paradox. The determinant relative speed of S_1 and S_2 is constant during the transmission and the taking back per definitionem.

2. The only parameter or factor to appear in the formal representation is $\pm v$.

The ordinary representation - so first of all the Longevin-representation becomes very descriptive by the fact that the clocks can be set against each other at the same point because of the changing of the directions of their motions, and the following important question arises: Which one of the two "clocks" will be late as compared with the other? The application of the Fermi-theorem makes the problem more obvious. Consequently L can also be applied in case of a moving point on a circule and it is very important that every inertia force, first of all the centrifugal force gets out of computation. Take two points moving on a unidimensional circule at speeds $+\omega$ and $-\omega$ respectively. These points always cover on each other at the two ends of a certain diameter and the clock moving with one of the points will always be late as concerned with the other. But the question "Which is the one and which is the other?" is a flagrant inherent contradiction: we can not answer without giving up postulated symmetry.

THE SOLUTION OF THE CLOCK PARADOX

Let L_{ip} denote the transforming from system S_i into S_p by L, and similarly let L_{pk} denote the transforming from S_p into S_k. L is transitive so there exists a transformation $L_{ik} = L_{ip}L_{pk}$. So the following relation - a double connection, shows the above transmission of time measure from S1 to S2

$$L_{12}L_{21} = L_{11}$$

the resultant of which is the unit transformation L_{11}. So there is no antinomy in case of correct interpretation.

It will be clear if we show it explicitly way: The general form of the solution is $L_{12}L_{23} = L_{13}$, the parameters v_{12} and v_{23} are usually different:

$$x_3 = (1+v_{12}v_{23})\beta_{12}\beta_{23}x_1-(v_{12}+v_{23})\beta_{12}\beta_{23}t_1$$

$$t_3 = (1+v_{12}v_{23})\beta_{12}\beta_{23}t_1-(v_{12}+v_{23})\beta_{12}\beta_{23}x_1$$

So $L_{12}L_{23} = L_{13}$. Since in our case x_1=0:

$$t_3 = (1+v_{12}v_{23})\beta_{12}\beta_{23}t_1 \qquad (3)$$

If we put here $(1+v_{12}v_{23})\beta_{12}\beta_{23} = \beta_{13}$, so if we interpret $L_{12}L_{23}$ as L_{13} with one parameter we immediately realize that in this case the parameter of L is:

$$v_{13} = \frac{v_{12} + v_{23}}{v_{12}v_{23} + 1}$$

that is the L-composition of v_{12} and v_{23}. So in the above $L_{12}L_{21}$ L_{21} is the inversion of L_{12} that is $v_{21} = -v_{12}$. If we substitute this result into (3) we get:

$$t_3 = (1-v^2_{12})\beta^2_{13}t_1$$

Since

$$\beta^2_{12} = \frac{1}{1-v^2_{12}}$$

this immediately implies that

$$t_3 = t_1 \tag{4}$$

So there is no question of paradox in the case of correct computation.

THE CONSEQUENCES OF THE ABOVE RESULTS

The importance of everything is much deeper than just correcting a miscalculation. The above solution of the paradox regardless of its convincing consistence is only a formal solution, therefore we have the following problem:

We gave a practical method of transmitting time above: the relativistic CR-transmission from S_1 to S_2. The question is the following: Will the paradox exist if we take back the transmitted time-measure from S_2 to S_1 exactly in the same way as we did it in the case of the transmission from S_1 to S_2? Our first impression is that the paradox will exist since we have to commute only the subscripts. But we will have some doubts at once: The method of transmission is the application of CR-axioms. So if we insist on the fact that the transmission causes paradox then the CR-axioms must include inherent contradiction. But this does not hold. In these axioms, which are based mainly on Hilbert-axioms we can not find inherent contradiction even if we carry on the most observant research work. The surprising solution of this "para-paradox" is that the application of the CR-measurement in two directions does not cause the paradox but its mentioned solution. The line of sight are equal to the running time t of the ray of light because c = 1. We realize that $t_2 = \beta_{12}t_1$ as stated above. The transmission happens from x_2 to x_3 and $x_2 = -v_{12}t_2 = -\beta_{12}v_{12}t_1$ so

$$t_3 = \beta_{23}(t_2 - v_{23}x_2) = \beta_{12}\beta_{23}(1+v_{12}v_{23})t_1$$

in accordance with (3). On the other hand $x_3 = v_{13}t_3$, but $v_{13} = \dfrac{v_{12}+v_{23}}{1+v_{12}v_{23}}$ and this implies that $x_3 = -\beta_{12}\beta_{23}(v_{12}+v_{23})t_1$, it is the value of co-ordinate x_3 of L_{13}-connection when $x_1 = 0$.

According to axiom III of item 3 (Szőcs 1995, pp. 176-180, Szőcs 1996, pp. 253-258) the reversal of the sign of parameter v and the symmetry of reflection cause the inversion of L. If we imagine geometrical picture of the phenomenon then the part under the line of reflection of the configuration (the front of electromagnetic waves) line means the transmission, the part above the line means the taking back. The wave fronts f_1, f_2 doing the transmission reach this line in two different points: the interval of which in the direction of x is $v\delta t_1$, but in the case of taking back the wave fronts f'_1, f'_2 reach the line at the same point x'_1.

Now the nature of the mathematical mistake made in the course of the formation of the paradox is obvious. According to relativity there can be only symmetrical correlations between two translatoric systems, but L is asymmetric. The occurence of the paradox is unavoidable because we tried to force symmetry on asymmetry as we insisted on a dogma instead of pure mathematics. On the other hand we refer to the paper (Szőcs 1997, pp. 217-220) where the Doppler effect shows a descriptive variety of the paradox.

If we continue the chain of two members $L_{12}L_{23}$ the result is as follows:

1. $t_2 = \beta_{12}t_1$
2. $t_3 = \beta_{12}\beta_{23}(1+v_{12}v_{23})t_1$
3. $t_4 = \beta_{12}\beta_{23}\beta_{34}(1+v_{12}v_{23}+v_{12}v_{24}+v_{23}v_{34})$

....

We can apply the equation $\beta_{12}t_1 = t_2$ when commuting t_3,t_4, etc... and we can determine v_{12} using the result. So e.g. 1. and 2. imply

$$v_{12} = \frac{1}{v_{23}}\left(\frac{t_3}{t_2}\sqrt{1-v_{23}^2} - 1\right) \tag{5}$$

and similarly 1. and 3. imply

$$v_{12} = \frac{t_4\sqrt{1-v_{23}^2}\cdot\sqrt{1-v_{34}^2}}{t_2(v_{23}+v_{34})} - \frac{1}{v_{24}}$$

In a transitive L-chain there always exists a distinguished system (Szőcs 1995, pp. 176-180, Szőcs 1996, pp. 253-258) without any inversions because $x_1 = 0$. This holds for the above expressions 1., 2., and 3.

All the speeds v_{ik} being on the right side of (5) are relative speeds between two systems S_i and S_k, but v_{12} is the absolute speed relating to $x_{00} = 0$, which is the initial point of the L-chain. We realize that the expressions can be applied not only in the case of the above subscripts 1,2 but also in the case of every subscripts 1,n (n = 2, 3, 4, ...) because of the L-transitivity.

Let S_k and S_l be kinds of L-connected translatoric systems. The local time-measures t_i/t_k of these systems occuring in (5) and the speed v_{ik} of the two systems are all quantities measurable inside of the two systems S_k, S_l and there is no need for any physical transmission of effect by means of a third system S determined by v_{in} in our case. But the pure mathematical connection uniquely apodiktice states that such a system exists.

Without the possibility of avoiding this postulate Knopf and Reichenbach state that the physical system is the matter-energy filling up the space and the classics call it ether. It is obvious however we may call it that its existence is a logico-deductive result, which is true in so far as $2 \times 2 = 4$ is true.

It seems that only one thing is meaningless in connection with the clock paradox. It is very hard to believe that after the discovery of the paradox nobody realized the simple solution of the paradox during 87 years. This immediately implies the paradox mentioned in the beginning of this section, which is not so paradoxical since where the inherent contradiction is?

And the absolute motion as a result of relativity theory was perhaps the phantom that deters people obtaining this result from continuing research. But it is not to be questioned that the problem becomes romantically interesting just at this point and this is the reason that incites us to continue the work with might and main.

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