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ABSTRACT

Doppler shift in the spherical wave: The game with the "very small" and the "very big"

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The observation of moving light sources is associated with important physical effects like aberration, Doppler shift, photographic and radar detection. If the derivation of the formula that describes the aberration of light is free of simplifying assumptions, the formula that describes the nonlongitudinal Doppler effect is the result of particular assumptions concerning the distance between the source and the receiver or the period of the involved light signals.

It is worth to quote Einstein[1]: *"If an observer is moving with a constant velocity* v=β *c relatively to an* infinitely *distant source of light of frequency* ν, *in such a way that the connecting line source-observer makes an angle* θ *with the velocity of the observer referred to a system of coordinates which is at rest relatively to the source of light, the frequency* ν' *of the light received by the observer is given by the equation:*

$$\nu' = \nu \frac{1-\cos\theta}{\sqrt{1-\beta^2}} \qquad (1)$$

This is the Doppler effect principle for any velocity whatever…".

The purpose of our paper is to present an exact derivation of the formula which describes the nonlongitudinal Doppler shift, free of simplifying assumptions.

As it is well known, a Doppler shift experiment involves an electromagnetic wave and two inertial observers, R and R', in relative motion, who measure the period of the oscillations occurring in the wave, obtaining the values *T* and *T'* respectively. The Doppler shift formula provides a relationship between *T* and *T'*, *D*= *T'*/*T*, *D* representing a Doppler factor, which depends on the character of the wave, that is, either plane or spherical.

Relativists make a net distinction between the longitudinal Doppler effect (LD), when the receiver moves along a direction that coincides with the direction of a ray in the wave propagation, and the non-longitudinal Doppler effect (NLD), when the directions mentioned above do not coincide.

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French[2] presents a derivation of a formula that describes the NLD effect. The source of light is located on a satellite, the receiver being located on the Earth. The proposed formula holds only in the case when the satellite travels a very small distance during one cycle of its transmitter signals.

Peres[3] derives the Doppler shift formula for the case when the receiver, located on an aircraft, moves at a given altitude, parallel to the Earth's surface and measures the period of a light source in a state of rest on Earth. He presents the Doppler factor as:

$$D = \frac{dT'}{dT} \tag{2}$$

warning the potential user that it holds only in the case of infinitesimal periods. In a purely photonic approach, an energy-momentum four vector is associated with the photon, both energy and momentum being expressed as a function of the photon's frequency, ν in the rest frame of the source and ν' in the rest frame of the receiver, respectively. The transformation equations of the corresponding components of the four vector lead to a Doppler factor:

$$D = \frac{\nu}{\nu'} \tag{3}$$

which surprisingly has the same algebraic structure as Eqs. (1) and (2)[3].

Consider that observer R' moves in a spherical wave emitted by a pointlike source. If the distance traveled by R', during the reception of two successive zeros of the wave, is "very small", then we can consider that during a period, the angle θ' under which the successive zeros are received does not change. Such a situation is favored by "very large" source-receiver distance (plane wave approximation), by "very small" periods (very small period approximation), or by low values of the relative velocity between source and receiver.

In this paper the original derivation of an exact formula describing the non-longitudinal Doppler shift is presented, leading to a Doppler factor *D* free of simplifying assumptions:

$$T_r' = \gamma\, T_e \frac{B}{A + \sqrt{A^2 - B}} \tag{4}$$

Two factors $A = f(m, \theta')$ and $B = g(m)$ will be used, θ' being measured by observer R' when he receives the second zero. The dimensionless parameter $m = \frac{r}{\lambda}$ depends on $\lambda = cT_e$, i.e. on the wavelength of the wave in the rest frame of the source.

Astronomers are interested in the particular case when the dimensionless parameter $m = \frac{r}{\lambda}$ is "very big" (plane wave approximation). We show that with increasing values of *m*, *D* increases too, tending toward a saturation value that corresponds to the value of *D* furnished by the formula that holds in the case of the plane or very high frequency approximation. The diagrams enable us to evaluate the error committed working with the approximate formula in the range of *m* values where the exact and the approximate formula give different values.

In order to visualize the influence of the relative velocity on the Doppler factor *D*, its variation with *m* for a constant value $\theta' = \pi/2$ and for different values of the relative velocity $v = \beta c$ will be analyzed, illustrating the fact that high values of β favor the discrepancy between the results given by the two formulae.

The plane wave (very high frequencies) approximation is associated with the concept of locality in the period's measurement by uniformly moving receivers, considering that the moving receiver can receive two successive light signals (zero of the wave) being located at the same point in space. That is best reflected by the approximate formula that is source-observer distance independent, supposing that the observer receives the two successive zeros under the same angle θ' between his velocity and the connecting line source-observer. We say that under such conditions locality in the period measurement by an uniformly moving observer takes place. Locality is favored by big values of the parameter *m*, but also by small values of the relative velocity v. The analysis of results obtained with our exact derivation of the Doppler factor will provide a serious argument to inquire about when using the approximate formula (1) it is suitable to mention the limits in which it holds.

References

[1] A. Einstein, "Zur Elektrodynamik bewegter Körper ", Ann. Phys. **17**, 891-921 (1905)
[2] A.P. French, "Special Relativity ", Nelson, 1968, p.139
[3] A. Peres, "Relativistic telemetry", Am.J.Phys. **55**, 516-519 (1987)
[4] V.A.Ugarov, "Special Theory of Relativity" Mir Publishers Moscow, 1979, p.261