Synchronization procedures and light velocity

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Abstract

Although different arguments speak in favour of the anisotropy of the speed of light in the Earth frame, the value of this speed, measured with clocks synchronized by means of the Einstein-Poincaré procedure, is always found invariant and equal to C.

This result can be explained if one realizes that the measurement is affected by systematic errors due to length contraction, clock retardation and the synchronization itself.

Many authors give credit to the slow clock transport procedure because they ignore the absolute motion of the Earth. Taking account of this absolute motion, one can demonstrate that this method of synchronization entails the same difficulties as the Einstein-Poincaré procedure.

I. Measurement of the speed of light with one or two clocks by means of the Einstein-Poincaré procedure.

In order to measure the speed of light, we can use one or two clocks. When we use one clock, the signal is sent from the clock towards a mirror, and, after reflection, comes back to its initial position. So, in this case, what we measure is the average round trip velocity of the light signal.

As we have seen in ref [1], even if one takes for granted the assumptions of Lorentz, which assume the anisotropy of the speed of light in the Earth frame, the theory demonstrates that this average velocity must be (erroneously) found equal to C in any direction of space. It also appears independent of the relative speed of the frame in which it is measured, with respect to the aether frame. (These results follow from the systematic errors entailed by Lorentz contraction, clock retardation and the method of synchronization itself).

So, a priori, it seems justified to use two clocks in order to accurately measure the one way speed of light. For that, we need, beforehand, to synchronize two distant clocks A and B.

According to the Einstein-Poincaré procedure (E.P) we must proceed in two steps. Firstly we send a light signal from clock A to clock B at the instant t_0; after reflection the signal comes back to A at the instant t_1. Secondly we send another signal at the instant t'_0. The clocks will be considered synchronous if when the signal reaches clock B, the display of clock B is:

$$t'_0 + \frac{t_1 - t_0}{2} = t'_0 + \varepsilon$$

ε is the apparent average two way transit time of the signal measured with the retarded clocks of the Earth frame.

But in the E.P procedure it is identified with the one way transit time of light.

As we have seen in ref [1], $\varepsilon = \frac{\ell_0}{C}$ in any direction of space (where ℓ_0 is the length that AB would assume if it were at rest in the aether frame) and since, because of the contraction of the meter stick used to measure it, the distance AB is always found equal to ℓ_0, the speed of light is found equal to C in the same way as when we use one clock.

So, even if the speed of light is given by the formulas $C_1 = -v\cos\theta + \sqrt{C^2 - v^2\sin^2\theta}$ and $C_2 = v\cos\theta + \sqrt{C^2 - v^2\sin^2\theta}$, (see ref [1]) the E.P procedure finds C.

Therefore it appears justified to test another method, i.e the slow clock transport procedure.

II. Measurement of the speed of light by means of the slow clock transport procedure.

Several physicists consider that one can obtain exact appraisal of the speed of light by means of the method of slow clock transport. The procedure consists of synchronizing two clocks A and B at a point O' of the Earth frame, and then of transporting clock B at a distance of A at low speed (v<<C).

Several authors have envisaged the problem in different ways [2-10].

A priori, it seems that, since the transport is very slow, as far as $v \to 0$ the motion would only have entailed a hardly perceptible influence on the time displayed by clock B, and that the two clocks would have remained almost synchronized all the time.

Is it really the case?

- *Point of view of special relativity*

If one considers the assumptions of special relativity as indisputable, then absolute speeds do not mean anything: only relative speeds exist. So, according to these assumptions, the display of clock B will be* :

$$t' = t\sqrt{1 - v^2/C^2} \cong t\left(1 - \frac{1}{2}\frac{v^2}{C^2}\right)$$

where t is the display of clock A. (Note that for convenience we have supposed that the display of the two clocks at the initial instant was $t_0 = 0$).

Once clock B has stopped (at point P), its delay with respect to clock A will remain constant. The synchronism discrepancy between clocks A and B is then to first order.

$$\Delta t = \frac{1}{2}\frac{v^2}{C^2}T$$

where T designates the display of clock A when clock B reaches point P.

So the speed of light will appear to be:

$$\frac{O'P}{T - \Delta t} \cong \frac{O'P}{T} + \frac{O'P\Delta t}{T^2} \qquad (1)$$

Since $v \to 0$ expression (1) reduces to $\frac{O'P}{T}$.

The experimental value of the speed of light obtained by this method is C.

Since the measurements of $O'P$, T and Δt are supposed exact, special relativity concludes that the real value of the speed of light in the Earth frame is C.

So, if one admits the assumptions of special relativity, the measurement of the speed of light by means of the method of slow clock transport is in agreement with the emitted hypotheses. But it does not allow these hypotheses to be verified.

- *Fundamental approach*

Now, it is interesting to verify if the previous results can be obtained with basic hypotheses different from special relativity.

Today, there are some strong arguments in favour of the Lorentz assumptions. According to Lorentz, the speed of light is C exclusively in the aether frame.

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* Note that the value of the speed of light is supposed to be known. So the measurement consists in verifying if the results obtained by this method are in agreement with the premises.

Notice that, if absolute speeds are taken into consideration, then there is no real slow clock transport since the absolute motion of the Earth adds to that of the transported clock. And different arguments, already advanced, demonstrate that the motion of the Earth should not be ignored.

If the method were reliable, it should give a value of the one way speed of light in accordance with the hypotheses emitted.

Let us verify this point. Two cases will be considered successively.

1 – Case where the light ray runs along the direction of motion of the Earth frame with respect to the aether frame.

Consider two inertial systems of coordinates S_0 and S_1. S_0 is at rest in the Cosmic Substratum, and S_1 is firmly linked to the Earth frame. At the initial instant the two frames are coincident. At this very instant, a vehicle, equipped with a clock, starts from the common origin and moves slowly and uniformly along the x axis of frame S_1, towards a point P of this frame. We suppose that the x axis is aligned along the direction of motion of the Earth with respect to the Comic Substratum (see figure 1).

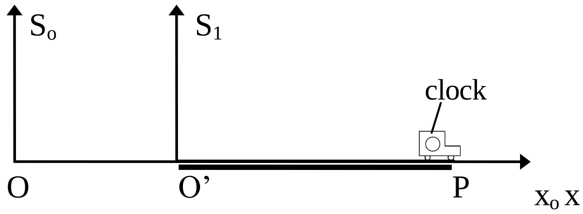

Figure 1

v_{01} is the speed of the Earth with respect to the fundamental frame S_0, v_{02} is the speed of the vehicle with respect to S_0 and v_{12} the speed of the vehicle with respect to S_1.

(Note that, during a short time, the motion of the Earth with respect to the Cosmic Substratum can be considered as rectilinear and uniform. In effect, if this were not the case, the bodies standing on the Earth platform would be submitted to perceptible accelerations).

The duration of the transport should be short enough in order that the orbital and rotational motions of the Earth would not significantly affect the measurement.

When the vehicle reaches point P, it stops. The real time needed to reach point P is given by

$$t_r = \frac{\ell}{v_{02} - v_{01}} = \frac{\ell_0\sqrt{1 - v_{01}^2/C^2}}{v_{02} - v_{01}}$$

where ℓ is the length of O'P (which is contracted because of the motion of the Earth with respect to the Cosmic Substratum); ℓ_0 is the length that O'P would assume if it were at rest in the aether frame; t_r is the real transit time of the vehicle from O' to P. (It is the time that a clock of the aether frame, facing the vehicle at the instant when this one reaches point P, would display).

(Let us bear in mind that real speeds, in the fundamental theory, obey the Galilean law of composition of velocities).

But the clock of the vehicle (B) is slow with regard to that of frame S_0, and will display the time

$$\frac{\ell_0\sqrt{1 - v_{01}^2/C^2}\sqrt{1 - v_{02}^2/C^2}}{v_{02} - v_{01}}$$

Now the clock placed at the origin O' of the Earth system (A) slows down with respect to a clock of frame S_0 facing it. When the vehicle reaches point P, it will display the time:

$$\frac{\ell_0\sqrt{1 - v_{01}^2/C^2}\sqrt{1 - v_{01}^2/C^2}}{v_{02} - v_{01}}$$

(This implies that, for an instantaneous event occurring at point P, all the clocks of the Cosmic Substratum display the same time). So, between clock B and clock A, a synchronism discrepancy exists which is equal to:

$$\frac{\ell_0\sqrt{1-{v_{01}}^2/C^2}}{v_{02}-v_{01}}\left(\sqrt{1-{v_{01}}^2/C^2}-\sqrt{1-{v_{02}}^2/C^2}\right)$$

$$\cong\frac{\ell_0\sqrt{1-{v_{01}}^2/C^2}}{v_{02}-v_{01}}\left(1-\frac{1}{2}{v_{01}}^2/C^2-1+\frac{1}{2}{v_{02}}^2/C^2\right)$$

$$\cong\frac{\ell_0}{2C^2}\sqrt{1-{v_{01}}^2/C^2}\,(v_{02}+v_{01}) \qquad (2)$$

we can see that, once the vehicle has stopped, the discrepancy will remain constant.

- Speed of light

If one takes for granted the postulates of Lorentz, the real time of light transit along the distance ℓ is theoretically

$$\ell_0\frac{\sqrt{1-{v_{01}}^2/C^2}}{C-v_{01}}$$

(Note that we suppose here, a priori, that the speed of light with respect to frame S_1 is $C-v_{01}$. This is intentional since we want here to verify if the results are in agreement with the premises).

Now, as a result of clock retardation, (and without taking account of the lack of synchronism) the display of a clock of frame S_1 placed at point P when the signal reaches this point should be:

$$\frac{\ell_0\sqrt{1-{v_{01}}^2/C^2}\sqrt{1-{v_{01}}^2/C^2}}{C-v_{01}}=\frac{\ell_0}{C-v_{01}}\left(1-{v_{01}}^2/C^2\right)$$

If, in addition, one takes account of the synchronism discrepancy given by formula (2), the apparent (measured) time of light transit will be:

$$\frac{\ell_0}{C-v_{01}}\left(1-{v_{01}}^2/C^2\right)-\frac{\ell_0}{2C^2}\sqrt{1-{v_{01}}^2/C^2}\,(v_{02}+v_{01}) \qquad (3)$$

Ignoring the terms of higher order, expression (3) reduces to

$$\frac{\ell_0}{C}\left(1+\frac{v_{01}-v_{02}}{2C}\right)=\frac{\ell_0}{C}\left(1-\frac{v_{12}}{2C}\right)$$

Now, since the measured length of $O'P$ is always found equal to ℓ_0, the apparent speed of light will be

$$\frac{\ell_0}{\frac{\ell_0}{C}\left(1-\frac{v_{12}}{2C}\right)}=\frac{C}{1-\frac{v_{12}}{2C}}\cong C\left(1+\frac{v_{12}}{2C}\right)=C+\frac{v_{12}}{2}$$

Since v_{12} is taken as little as possible, the apparent speed of light is found equal to C. So, even if the real speed of light is $C-v_{01}$, the method of slow clock transport will (erroneously) find C in the same way as the method of Einstein-Poincaré.

Therefore the two methods can be considered as equivalent.

2– General case

Let us now measure the speed of light along a rod O'B making an angle θ with respect to the x axis of a system of coordinates S_1, firmly linked to the Earth frame (see figure 2).

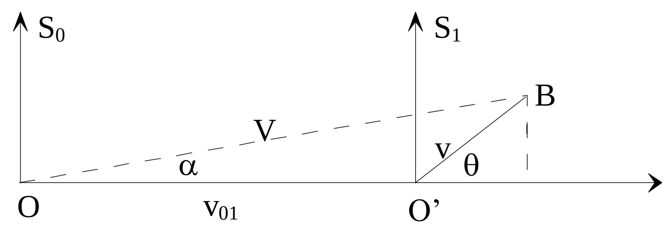

Figure 2

Note that the x axis of S_1 is aligned along the direction of motion of the Earth with respect to the Cosmic Substratum.

(During a short time this motion can be considered as rectilinear and uniform).

(Note also that the rod is in the plane x, y, but obviously, provided that θ remains the same, the following reasoning would be identical in any plane passing by the x_0, x axis).

We can choose a system of coordinates S_0 of the Cosmic Substratum such that, at the initial instant, S_0 and S_1 are coincident. At this very instant, a vehicle leaves the common origin, and moves slowly and uniformly along the rod towards point B.

As we have seen in ref [1] on account of length contraction along the x_0, x axis, the length of the rod is given by

$$\ell = \frac{\ell_0 \sqrt{1 - {v_{01}}^2/C^2}}{\sqrt{1 - {v_{01}}^2 \sin^2\theta / C^2}}$$

where v_{01} is the speed of the Earth with respect to the fundamental frame S_0.

Let us designate as v the real speed of the vehicle with respect to S_1, and V its real speed with respect to S_0 (see figure 2).

The real time needed by the vehicle to reach point B is $\frac{\ell}{v}$, but the apparent time in frame S_1 calculated in taking account of clock retardation is

$$\frac{\ell}{v}\sqrt{1 - {v_{01}}^2/C^2}$$

The apparent time as measured with a clock enclosed inside the vehicle is

$$\frac{\ell}{v}\sqrt{1 - V^2/C^2}$$

So that, the synchronism discrepancy between the apparent time displayed by a clock of frame S_1 placed at point O' and that of the vehicle is

$$\Delta = \frac{\ell}{v}\left(\sqrt{1 - {v_{01}}^2/C^2} - \sqrt{1 - V^2/C^2}\right)$$

we easily verify that

$$V^2 = v^2 \sin^2\theta + (v_{01} + v\cos\theta)^2$$

so $$V = v_{01}\sqrt{\frac{v^2}{{v_{01}}^2} + 1 + \frac{2v}{v_{01}}\cos\theta} \qquad (4)$$

Taking account of the inequality $v << v_{01}$, expression (4) reduces to

$$V \cong v_{01}\left(1 + \frac{v}{v_{01}}\cos\theta\right) = v_{01} + v\cos\theta \quad \text{(see fig 2)}$$

So, to first order, Δ becomes

$$\frac{\ell}{C^2}\left(v_{01}\cos\theta + \frac{1}{2}v\cos^2\theta\right)$$

- Measurement of the speed of light along O'B by means of the slow clock transport procedure.

Let us now suppose that in O' and B stand two clocks which have been (apparently) synchronized by means of the slow clock transport method. In fact there is an error of synchronism equal to Δ.

The real speed of light along the rod from O' to B is (see ref [1]).

$$C_1 = -v_{01}\cos\theta + \sqrt{C^2 - {v_{01}}^2 \sin^2\theta}$$

As a result of clock retardation but without synchronism discrepancy effect, the apparent time needed by the light ray to reach point B should be

$$T_L = \frac{\ell}{C_1}\sqrt{1-{v_{01}}^2/C^2}$$

But we must take account of the synchronism discrepancy, so that the apparent measured transit time of light will be:

$$\frac{\ell}{C_1}\sqrt{1-{v_{01}}^2/C^2} - \Delta \qquad (5)$$

Ignoring the terms of higher order, expression (5) reduces to

$$\ell_0 \frac{\left(1-\frac{v_{01}}{C}\cos\theta - \frac{1}{2}\frac{v}{C}\cos^2\theta\right)}{C\left(1-\frac{v_{01}}{C}\cos\theta\right)}$$

Since the rod O’B is measured with a contracted meter stick, it appears equal to ℓ_0.

The apparent speed of light is then

$$C_{app} = \frac{\ell_0}{T_L - \Delta} = \frac{C\left(1-\frac{v_{01}}{C}\cos\theta\right)}{1-\frac{v_{01}}{C}\cos\theta - \frac{1}{2}\frac{v}{C}\cos^2\theta}$$

since $v \to 0$, $C_{app} \to C$

which is different from its real value C_1 as seen previously.

So, contrary to what is often believed [11], the method of slow clock transport does not permit one to exactly measure the speed of light.

It is approximately equivalent to the method of Einstein-Poincaré, and in the same way as this method, gives, erroneously, the value C for all measurements.

It is interesting to note that, even if the speed of light is not constant, it is found constant when one uses the usual methods of synchronization.

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