Twins: never the twain shall part

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Abstract

It is shown that asymmetric ageing arises in Special Relativity from comparing unlike entities, namely perceived intervals with proper-intervals. When an obscurity in the hypothesis set for this theory is eliminated with the *Ansatz* that proper-length is a global invariant, then it is seen that there is no twin paradox and that proper-time can serve as the independent variable for point particle mechanics in Special Relativity. The resulting conceptual modifications should be most useful for understanding various cosmic scale phenomena.

1   Introduction

A most persistent conundrum of modern Physics is that derived from the perceived absurdity of the dependence of time and length spans on their *relative* motion with respect to an the observer; i.e., time-dilation and Lorentz-Fitzgerald contraction. While such concerns are frequently written off as due to fixation with outmoded reasoning, in fact there are absolutely rigorous arguments supporting skepticism in this matter, which, we hold, must have patently observable manifestations at a cosmic scale. It is our purpose herein to argue, however, that in this matter Special Relativity contains the seeds of its own salvation.

The twin paradox, for example, was recognized by Einstein himself in his seminal article on special relativity, wherein he opined that Lorentz transformations yield asymmetric ageing between various inertial frames.[1] This feature was thereafter anthropomorphized by Langevin, who applied the principle of asymmetric ageing to the now fabled example involving twins, one of whom makes a round trip while the other stays put.[2] As is very well known, conventional analysis shows that the traveling twin returns home to meet his sibling, now much older than himself. In so far, however, as *kinematically* seen, both twins experienced a symmetric relationship, an asymmetric ageing difference is paradoxical. Langevin resolved this paradox by calling on the fact that the kinematical symmetry is broken by the *dynamical* fact that only the traveler experienced acceleration. Almost immediately, however, von Laue observed that the effect was independent of the acceleration, as the ageing effect could be extended simply by extending the length of the trip without altering the accelerations involved; that is, the *dynamical* aspects of the trip must be irrelevant![3] It can not be both ways however—such a conflict, i.e., a new paradox, or more precisely: an antimony, can not

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be ascribed to failure to appreciate rigor. Ninety years, hundreds of books and thousands of articles later, despite excursions prompted by all manner of considerations, this matter still stands conceptually at the point von Laue left it. [1]

2 Proper-length

Analysis leading to the twin paradox, apparently, is not based on careful consideration of the determination of the distance to the turn-around point (herein for brevity called the *pylon*) of the traveling twin. This *lacuna* in reasoning is a consequence of the confusing circularity inherent in special relativity in which time intervals and spatial extensions are interrelated because most critical definitions are formulated with reference to light. The situation is further complicated by the fact that interactions between objects, and the means of observing these very same objects, are all mediated by electromagnetic signals, light.

The final result of this ambiguity is, in our view, that an optimum set of basic notions needed to close conceptual loops is missing. One fundamental *Anzatz* of Special Relativity, as Einstein formulated it, is that the equations describing basic physical laws should be the same in all frames; i.e., invariant. This is a statement, as it were, about the 'grammar' of physical relationships. By itself it is not useful. It is also necessary to have a vocabulary to which this grammar applies in order to make meaningful statements. In this simile, physical objects are words, the *onta* about which the theory describes regularities in behavior. Let us now take the prototype of this onta to be 'meter sticks.' Without an explicit statement of invariance of the "onta" governed by basic equations, the invariance of equations alone is, in a certain sense, useless. In the end, the theory should tell us how these meter sticks interrelate via light signals when in motion with respect to each other.

Consider a frame F_o in which there is a collection of identical meter sticks, each given a label M_j. Now let us suppose that these meter sticks are distributed to other inertial frames F_j. Consider now as a fundamental *Ansatz*, in addition to Einstein's two, (that, 1. all fundamental equations are invariant from frame to to frame and that, 2. the speed of light is a global invariant), that *onta*, i.e., the meter sticks, are likewise global invariants. That is, for an observer in each frame, his meter stick, when measured in his frame with respect to events at the ends of his meter stick that are simultaneous to him, can be made to coincide with an interval on a pure space-like axis in his frame from 0 to $1m$. In other words, proper-lengths are invariants. An immediate consequence of this additional definition is to remove Einstein's procedure to determine simultaneity as an independent input into the theory, i.e., it becomes redundant. In other words, it becomes true as a consequence of Einstein's two main assumptions and this new third one that in each frame; i.e., clocks are so coordinated that for a two-way light pulse, it is true that:

$$t_2 = t_0 + \frac{1}{2}(t_1 - t_0), \tag{1}$$

where t_2 is the time of reflection off an 'un-calibrated' worldline given that t_0 is the time

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of emission, and t_1 the time of return of the pulse at a 'standard' worldline. A moments reflection convinces one that as a result of this alternate *Ansatz*, 'simultaneous' events in any frame must be those on a worldline parallel to the pure space-like axis.

Einstein's synchronization procedure, as an independent input in addition to his two fundamental principles, builds in an essential circularity. All events which are to be 'measured' are identified as the sources of light pulses. Then, time intervals are used to measure distance between these events. Such 'distance' is then what is used to define velocity, not to mention to verify the constancy of the speed of light. This could be the ultimate source of the pervasive conceptual turbidity evident throughout the history of Special Relativity.

Simultaneous events in one frame are not simultaneous in another. Thus, if the two particular events used in one frame to measure the length of a meter stick, say, correspond to the sources of pulses propagated to another frame where the arrival times of these same two pulses are then used in this second frame to calculate the length of a meter stick in the first frame, the result will be foreshortened by the Lorentz-Fitzgerald contraction factor. In this paradigm this effect is clearly a matter of space-time perspective. While it is a "real" effect, it is of an 'epistemological' nature; it has to do just with what an observer in the second frame will 'know' about the length of the meter stick in the first frame as he can discern it via light signals. The results of such measurements-at-a-distance do not speak to the ontological (we assert, invariant) character of moving objects. In other words, if the moving meter stick is measured by using the two events which in the moving frame were simultaneous as if they are points in the stationary frame, the result exhibits contraction. On the other hand, if the two points in the moving frame are transformed ("pulled back") first to the corresponding points in the stationary frame, then the proper-length will be invariant; as is to be expected because it is the norm of a Lorentz vector and Lorentz transformations were constructed in the first place to leave such norms invariant!

For analysis of the 'twin paradox,' these definitions are crucial. They have to do with the proper-length of the trip made by the traveling twin. The question here is: how far does the traveling twin actually go; i.e., how far is it from the terminus to the pylon? On the time scale of the imagined trip, the worldlines of the terminus and pylon are parallel, and if viewed in the rest frame of either, straight. If we imagine the terminus to be on Earth, then the distance, "D," to the pylon, a selected star, can be obtained from standard astronomy references. In the traveler's frame, this distance is one that the traveler will experience directly, that is, he will not measure it on the basis of signals from the earth's frame, but will, so to speak, pace it off in person, in other words: measure it in *his* frame. This distance, then, will not be one 'perceived' by the traveler using his eyes which capture light signals, but rather, a direct experience of the separation of the terminus and pylon. Such a spacial extention is conceptually equivalent to a meter stick. It should be an invariant. It should extend from his frame origin (assuming origins coincide at the start of the trip) to the point on his spacial axis, the abscissa, with the exact numerical value D.

Thus, a Minkowski chart for the fixed twin will show the worldline of the terminus congruent with the time axis, and the worldline of the pylon parallel to it crossing the abscissa at D. If now a Minkowski chart for the traveler is superimposed on top of that for the fixed twin, then its axes are squeezed together in the well know way. Also,

however, the scale is changed by the factor $\gamma = \left(1 - (v/c)^2\right)^{-1/2}$, which means that on this chart, the worldline for the pylon, although still parallel to that of the terminus, can not be congruent to that of the pylon worldline on the superimposed fixed chart. It will be displaced outward along with the filar marks on the moving frame axis by this scale factor. (This scale anisotropy has been ignored by others, taking it into account is what distinguishes our approach.) The consequence of this displacement is that the intersection of the traveler's worldline with the worldline of the pylon is found to be further out on the traveler's worldline than usually thought; i.e., the proper-time taken by the traveler to reach the turn-around is seen to be greater than heretofore estimated. In fact, it is equal to the proper-time of the stay-at-home as he himself computes it for the time taken by the traveler to reach the turn-around point. Thus, when the whole trip is completed, both twins agree that they have experienced equal portions of proper-time since the start of the trip. Their *reports* to each other via light signals while underway on the passage of time, in the usual way do not agree, however. But they are such that the final totals at the end of the trip do agree.

These ideas are depicted graphically in Figure 1.

Alternately, this same conclusion also follows directly from the fact that given *any* proper-time dependant vector in Minkowski space, $\mathbf{x}$, for which the four velocity $\mathbf{v}$ is then $d\mathbf{x}/d\tau$, it is a tautology that four-velocity norm equals: c; i.e., $\mathbf{v} \cdot \mathbf{v} = c^2$. (Although this equation is to be found in textbooks, see e.g.:([4] p. 100), its full consequences are inexplicably ignored.) By differentiating this then by τ, it is immediately clear that the four-acceleration is always orthogonal to the four-velocity. This, in turn, permits one to write:

$$c\,d\tau_j = (d\mathbf{x_j} \cdot d\mathbf{x_j})^{1/2} = (d\mathbf{x_i} \cdot d\mathbf{x_i})^{1/2} = c\,d\tau_i, \qquad (2)$$

which establishes the identity of proper-time intervals for distinct particles. This is a kinematical argument; the dynamical parallel would be based on

$$(\mathbf{F}_j \cdot \mathbf{p}_j) = \dot{m}_0 \, m_0 c^2 = 0, \qquad (3)$$

which holds for all forces that do not alter rest mass.

The basic character of proper-time is crucial for the formulation of dynamics consistent with Special Relativity. The arguments presented above obviously remain true when reduced to infinitesimals, thereby enabling the piece-wise composition of an arbitrary time-like trajectory in Minkowski space. They provide a substantiation of a resolution proposed in [5] of a perplexing problem in (special) relativistic mechanics derived from the oft surmised lack of coordination among individual proper-times for interacting particles.The considerations in this note constitute a didactical elaboration of arguments therein where it was observed that the differential of arc length in Minkowski space is an invariant under Lorentz transformation. That is, the differential of arc-length is the pure time-like unit vector in the instantaneous rest frame along the orbit of the k-th particle at point p, and is related to the differential of arc-length expressed in the instantaneous rest frame of the differential of arc-length at any location p' on that or any other arc j by a suitably constructed Lorentz transformation: $\mathcal{L}(p, p', k, j)$:

$$dx_k|_p = \mathcal{L}(p, p', k, j) dx_j|_{p'} \qquad (4)$$

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It follows, that the arc-length is an invariant as:

$$(dx_k|_p \cdot dx_k|_p)^{1/2} = (dx_j|_{p'} \mathcal{L}^T \cdot \mathcal{L} dx_j|_p)^{1/2} = (dx_j|_{p'} \cdot dx_j|_{p'})^{1/2}. \qquad (5)$$

This permits setting all such differential arc-lengths equal to a common expression:

$$c\, d\tau = (dx_j \cdot dx_j)^{1/2}, \qquad (6)$$

which can be rewritten as:

$$d\tau = \gamma_j^{-1} dt_j \quad \forall j, \qquad (7)$$

where γ then is the usual Lorentz scale factor.

The utility of this structure for a theory of point particle mechanics in Special Relativity is exploited in [5].[2] An inspection of the interaction derived in that formulation reveals that it is governed by a system of equations coupled via the Lorentz Force Law as mediated by Liénard-Wiechert potentials. This force indeed does not alter rest mass, thereby not being in conflict with a single parameter dynamical law.

3 Conflict with experiments

All standard works on Special Relativity cite experiments attesting to the "reality" of time dilation and the consequent ageing discrepancy. How are they to be understood in view of the above considerations? First, note that to date, no experiment meets the conditions leading to the twin-paradox. Certain experiments, those involving muon decays, for example, are described by linear transformations but are not round trips. "Clocks-around-the-world" experiments did involve round trips, but not linear (acceleration free) motion. Further, note that time dilation is 'real' in the sense that it actually occurs with respect to signals. It is an effect attendant to 'perspective' in space-time. Thus, all physical effects resulting from the 'appearance' (i.e., the way in which light signals transmit information or momentum-energy) will be modified by the perspective. So, any test of time dilation which involves a report from, or the interaction between objects in relative motion, will exhibit phenomena resulting from relative positions and times of emitter and receiver; i.e., space-time perspective.

Some experiments, however, seem exempt from the effects of perspective. The two customary examples are the muon decay curve in the atmosphere, and the transport of atomic 'clocks-around-the-world.' Here the situation is less clear. Each of these experiments, however, is afflicted with features that seem to allow contest.[6]

Muon decay, for example, largely seems to ignore possible cross-section dependence of the target particles in the atmosphere on the velocity of the cosmic projectile as well as secondary production. The clocks-around-the-world experiment has been strongly criticized for its data reduction techniques. Without access to the details of

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[2]In [5] a so-called case II was considered in which the worldline of one particle should puncture the future light-cone of the other. We now see that this case does not occur; indeed its presumed existence results from the same error leading to asymmetric ageing, i.e., failure to take the anisotropism of Minkowski space correctly into account.

these experiments and their subsequent (largely unaccessible) data analysis, an outsider is not in position to do critical reanalysis; nevertheless, there is sufficient information in the literature to reasonably justify considering conclusions drawn on their basis as disputable. Of seemingly direct relevance moreover, the existence of time delay effects for transported clocks has been questioned. Experience with contemporary communication technology seems to present numerous practical reasons to doubt the conventional understanding of time delay effects for transported clocks.[7]

In addition, experimental attempts to verify length contraction have been met uniformly with null results, completely in accord with our contention. Among the experiments mentioned in ([6] p. 105), an attempt by Phipps to observe the Ehrenfest effect—Fitzgerald contraction of the circumference of a disk as a consequence of high tangential velocity due to rotation—gave unambiguous null results, for example.[8] The lack of evidence of systematic radial dependence of evolutionary processes in galaxies, e.g., element abundance or star species, can be taken as cosmic scale confirmation of Phipps' result.

Likewise, an experiment attempting to observe the stresses in a bar rotating in a translating frame, also obtained null results.[9]

In fact it appears that routine experience communicating on an interplanetary scale provides further macroscopic evidence that proper-time intervals are Lorentz invariants. Consider, for example, a radar pulse sent from the Earth to Mars, say. When the pulse returns, it will exhibit at least geometric distortion (due to $1/r^2$ dispersion). Now, if the logic of asymmetric ageing is valid, then in the frame of the pulse, no time has transpired while in the frame of the radar station, several minutes have passed. This, in turn, means that in the pulse's frame, the distortion occurred instantly, which is not in accord with any solution of Maxwell's Equation — in crass violation of Einstein's Principle that the fundamental equations should be identical in all frames. On the other hand, if proper-time intervals are invariants, then the time development of the distortions to the pulse can proceed in its frame too in a fully local and deterministic way, as a solution to Maxwell's Equations.

4 A proposed test

Crucial to a test of this formulation is that the aging of 'twins' be compared directly rather than via reports conveyed between frames. Because customary experiments rely on signals sent from the moving to the fixed frame one way or another, it is not possible to exclude 'space-time' perspective effects.

Perhaps this can be overcome. Consider a variation of a Pound-Rebka experiment employing a material with an element whose nucleus is naturally unstable. Let a sample of this material be divided and then hold half at a high temperature and half at a low temperature long enough such that the calculated time dilation of the more rapidly moving atoms of the heated half is great enough to yield a detectable difference in decay products. The ratios of decay products then should be compared finally in the same frame, i.e., at the same temperature. An experiment of this structure would not be dependant on the transmission of signals from frame to frame but simply internally tally the total passage of eigen time in terms of decay half-lives in each frame for subsequent

comparison. (Note: this scheme can be considered only conceptual inspiration. In fact, the shape of decay curves conceals differences in the accumulation of proper aging.)

5   Conclusions

The modifications proposed herein seem to us to be of a very uncertain degree of originality. Einstein himself actually asserted that logically proper-length must be invariant.([1], §2) He also clearly identified the difference with 'perceived' length as observed between moving frames. Nonetheless, he remained seemingly one sentence short of observing that these "definitions" result, then, in a requirement to take into account the change in scale when considering the location of the worldline of the pylon. Perhaps this is simply a consequence of not having the benefit of Minkowski charts, introduced initially some three years later [10], to assist in formation of his reasoning on this issue. In the mean time a long, vigorous but very discontinuous debate has taken place on the true nature of the aberration of coordinate intervals; i.e., are they "real" or just artifacts of perception? As often as not, this debate was only implicitly engaged. For a more complete discussion of this point, see, for example, Czerniawski in [11]. In any case, as a formal logical structure, Special Relativity admits wide latitude in the definitions.[12] At the same time, there is confusing circularity because of the dual role played by electromagnetic signals (as both standard and caliper—judge and jury, as it were). Nonetheless, the logical latitude is constrained; the structure, to be valid physics, must mimic nature.

Ideally, a final and full understanding of all ramifications of consistently taking the anisotropy of Minkowski space into account awaits a full reinterpretation of the details surrounding accomplished experiments and numerical verification of single-parameter relativistic mechanics by simulation and comparison with empirical results. In addition, there are numerous conceptualization issues, such as those pertaining to the Sagnac Effect and others discussed in, say, [11], that await deeper reconciliation. In short, however, it seems that aberration of perceptions, i.e., space-time perspective, is an incontestable effect, but that ontologically significant quantities must space-time invariants. Replacing Einstein's synchronization procedure with the assumption that proper-intervals are Lorentz' invariants preempts the error leading to the infamous paradoxes of Special Relativity; which in this context are seen not to be the fault of the theory, but of misconstrual. Given the extensive history of Special Relativity, however, reconciling this matter in historical context is an immense undertaking of which this note can be only an initial step.

Our fundamental point is that when all the fundamental *Ansätze* — including the synchronization procedure — are taken in account simultaneously in a self consistent manner, that none of the infamous paradoxes arise. It is, however, advantageous to replace the synchronization procedure as a fundamental input to the theory by the stipulation that proper-length is a Lorentz invariant. This change prevents overlooking the anisotropy of Minkowski space, a crucial aspect in comprehending those situations in which paradox has so often arisen. Contradiction results from the comparison of perceived intervals as modified by space-time perspective with proper-intervals. Resolving such contradictions most directly facilitates understanding several cosmic scale

phenomena.

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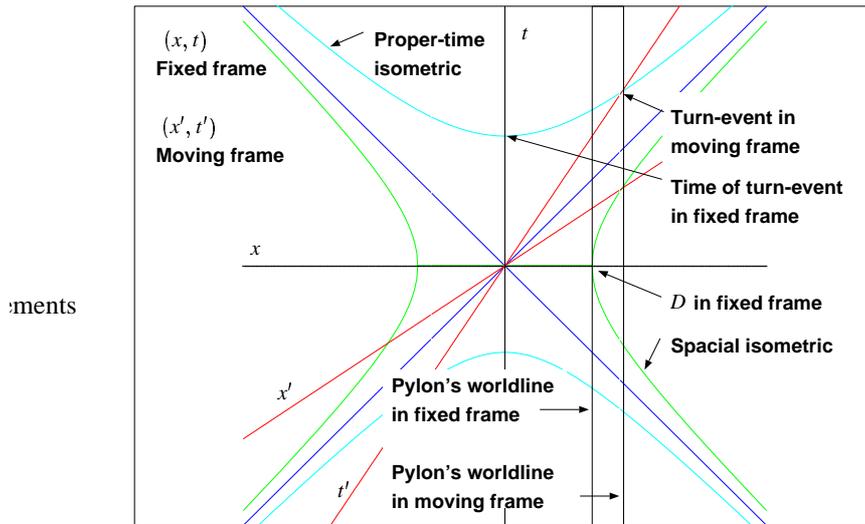
Minkowski Charts for Relative Motion

Figure 1: This figure is comprised of two Minkowski charts superimposed on top of each other. The worldline of the Pylon in the fixed frame chart passes through the point 'D' on the x-axis. The corresponding point on the x'-axis, the traveler's axis, is found by sliding up the proper-length isocline to the intersection with the x'-axis. The worldline of the pylon passes through this point on the prime chart. The intersection of the Pylon's worldline with the t'-axis is the point on the traveler's chart representing the 'turn-around' event. The proper-time of the turn-around event in the fixed frame is found by sliding down that proper-time isocline which passes through the turn-around event to its intersection with the t-axis. It is clear that this value is identical with the time assigned by the fixed twin himself to the turn-around event as it may be projected horizontally over to the intersection of the Pylon's worldline in his (fixed) frame with the time axis of the traveler. Apparent asymmetric ageing arises by using, incorrectly, that proper-time isocline which passes through the intersection of the traveler's and the pylon's fixed frame worldlines.