QUANTA OF ACTION AND RELATIVITY THEORY

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1. HISTORICAL INTRODUCTION

The notion of the physical quantity called action was introduced and developed by Pierre Louis Moreau de Maupertuis (1698-1759), Leonhard Euler (1707-1783), Giuseppe Luigi Lagrange (1736-1813) and William Rowan Hamilton (1805-1865).

In the physical processes in which the motion of physical bodies (e.g, of elementary particles) is involved we are dealing with a transport of momentum $m\upsilon$ and energy E along a certain path Δl and during certain time Δt. The physical quantity that charcterizes such a motion is just called action

$$m\upsilon\,\Delta l = action \qquad\qquad E\Delta t = action$$

a. Planck's constant

In 1899 M. Planck [1], when solving the problems connected with the radiation of a black body, regarded atoms as little electromagnetic oscillators which when emitting light, perform a work W (equal to the emitted energy E) during the time equal to one period T of the oscillator. In such a way he discovered that in the acts of emission and absorption of electromagnetic energy we are dealing with a constant portion of action that he denominated „elementary quantum of action"

$$WT = ET = h$$

Where T is not only the period of the oscillator but also the period of the emitted radiation.

Planck's constant $h = 6{,}626176 \cdot 10^{-34}$ J ·s

As well known Planck's constant served as a basis for quantiusation in Quantum mechanics. It changed radically our image of the microscopic world i.e.of the world of elementary particles.

b. Action and Relativity Theory

According to the Italian historian of physics Umberto Forti [12] the physical quantity called action became in Relativity Theory more natural because the tramsmited energy and momentum are there presented by the four vector of energy and momentum and the path and time interval are presented by the space-time interval.

c. Einstein's quanta of electromagnetic energy and the second quantum of action $h_e = e^2/c < h$

In 1905 Einstein introduced his theory of the the quanta of electromagnetic energy [2] which he developed in 1909 [3]. According to him, the electromagnetic radiation is quantized not only in the acts of emission and absorption but also when propagating. Light itself is made up of energy quanta $E = hv$ and therefore we can write

$$ET = mc\lambda = h$$

Where m is the relativistic mass of the quantum and λ the wave-lenght of the radiation.

In 1909, the basic problem for Einstein was to find a modification of the two fundamental theories, particle mechanics and the Maxwell-Lorentz electrodynamics, which would account for the introduced light quanta. In the paper of 1909 [3], he suggested that one clue was given in the dimentional equivalence of Planck's radiation constant h and the quantity $h_e = e^2/c$. Now the elementary charge e itself is a stranger in Maxwell-Lorentz electrodynamics, since this theory would allow a continously varing charge and since one has to assume unknown forces holding the electron together; that is, the structure of a charged particle is unintelligible within Maxwell-Lorentz theory. According to Einstein,

it seems to follow from the relation $h \sim e^2/c$ that the same modification of the theory which will contain the elementary charge e as one of its consequences, will also contain as a consequence the quantum structure of radiation"[3]

Einstein confessed, however, that he has *not yet succeeded in finding a system of equations* [3] which he could *see as appropriate for constructing the elementary electric charge and the light quanta* [3]. Einstein indicated that the second quanta of action h_e is less than Planck's

$$h_e = e^2/c = 7 \times 10^{-30} cgs$$
$$h = 6 \times 10^{-27} cgs$$

d. The quantisation of action in Bohr' model of hydrogen atom

In 1913, N. Bohr, in his model of hydrogen atom, assumed that action which characterizes the motion of the electron moving around the proton (momentum $m\upsilon$ of the electron multiplied by its orbit $2\pi r$) is quantized and is equal to the Planck's elementary quantum of action or its integer multiple

$$m\upsilon\, 2\pi r = hn$$

where $2\pi r$ is the lenght of the electron's orbit and $n = 1,2,3....$
interactions

e. Planck recognizes the importance of the second quantum of action

In 1916, Planck, when studying Sommerfeld's spectral theory, indicated that the latter does not take into consideration the constant $h_e = e^2/c$. According to Planck this constant, although it is about 1000 times smaller than his constant h has *a practical meaning in spectral measurments* [4]

f. Schrödinger's indication of the second quantum of action

In 1923, E. Schrödinger [5] when studying (in the frame-work of H. Weyl world) the problem of the orbits of an electron about a nucleus, indicated that in order to resolve this problem one must introduce a new constant which has the dimensions of action. Since he made several approximate calculations he was not able to give the exact value of this constant. He presented, however, the constant $h_e = e^2/c$ as the candidate which might be the sought-for constant because it was very close to the quantity that he received in his calculations. Schrödinger emphasized also that the new constant must be considered as an universal constant, and he expressed his opinion that the constant $h_e = e^2/c$ and the constant h are interrelated. He showed that the fine-structure constant α is essentially the ratio of h_e to h

$$\alpha_e = 2\pi e^2/hc = 2\pi (e^2/c)/h = h_e/\hbar$$

g. Eddigton's indication of the part played by the second quantum of action in the electromagnetic interactions

In the four fundamental interactions between two elementary particles we are also dealing with a transport of momentum and energy along the distance between them and during the time needed to transmit the interaction. Thus the physical quantity called action plays here also its part. Let's consider first the electromagnetic interactions.

In 1934 A. Eddington, [6] indicated not only that:

(1) *The fine-structure constant is really the ratio of two natural units or atoms of action. The one (e^2/c) arising in electron electron theory and the other ($h/2\pi$) in radiation theory*[6] but also that

(2) The product of energy E_e of the interaction between two charged particles and the time t needed to transmit the interaction is always constant and equal to $h_e = e^2/c$.

It (the product $E_e\, t$) *is always the same, whether the two particles are close together or wide apart. If they are wide apart the energy is small but the light* *time is correspondingly increased. In symbols, if r is the distance apart, the energy is* e^2/r *and the time is* r/c*, so that the product is* e^2/c [6]

$$E_e\, t = (e^2/r)(\, r/c) = e^2/c$$

Note that also the product of the transmited momentum p_e and the distance r between the two charged particles is equal to e^2/c

$$p_e\, r = (E_e/c)r = [(e^2/r)/c]r = e^2/c$$

Developing Eddington's idea one can show [7,8,9] that the interaction between systems composed of charged particles are characterized always by integer multiples of the constant e^2/c. Since the charge of the first system $Q = Z_1 e$ and the charge of the second $q = Z_2 e$ are integer multiples of the elementary charge e (Z_1 = 1,2,3.. and Z_2 = 1, 2, 3...) therefore we can write

$$E_e\, t = (Qq/r)(r/c) = (Z_1 Z_2 e^2/r)(r/c) = Z_1 Z_2 (e^2/c)$$

As we can see the product of energy E_e of the interaction between two charged systems and the time t needed to transmit the interaction is always constant and equal to the integer multiple of the second quantum of action.

The fantastic development and results of Quantum Mechanics in which the unique fundamental basis of all quantisation is Planck's constant h caused the second quantum of action (e^2/c) to sink into oblivion. It continued to play an important part, but in the latent form, as a componant of the fine structure constant which is, at the same time, the coupling constant of electromagnetic interactions.

2. QUANTA OF ACTION CONNECTED WITH THE FOUR FUNDAMENTAL INTERACTIONS

In some papers [7,8,9] I have already shown that not only with the electromagnetic interactions is connected a new quantum of action (the second one), but there are also quanta of action connected with other interactions. As regards the presented above quantum of action connected with the electromagnetic interactions, we have to add here only that using the SI system of units we have to write this constant as follows

$$h_e = K\, e^2 /c = 7{,}699 \cdot 10^{-37} \text{ J} \cdot \text{s}$$

Where $K = 1/4\pi\varepsilon_o$ is the known coefficient used in the SI system of units. Einstein, Planck, Schrödinger and Eddington worked in the cgs system of units.

Following the method used by Eddington and assuming that all interactions propagate at velocity c we can show that also in the gravitational, weak and strong interactions the product of the respective energy of interactions and the time needed to transmit it is constant and constitute a quantum of action.

a. gravitational interactions

(1) when the two interacting particles have the same rest mass $m_1 = m_2$

$$E_G\, t = (G\, m^2/r)(r/c) = G\, m^2/c = h_G$$

In the case when m is respectively the rest mass of an neutron $m = m_n$ of a proton $m = m_n$ or of an electron $m = m_n$ we obtain the following values for h_G

$$h_{Gn} = G\, m_n^2/c = 6{,}24179 \cdot 10^{-73}\ \mathrm{J \cdot s}$$
$$h_{Gp} = G\, m_p^2/c = 6{,}2254 \cdot 10^{-73}\ \mathrm{J \cdot s}$$
$$h_{Ge} = G\, m_e^2/c = 1{,}8468 \cdot 10^{-79}\ \mathrm{J \cdot s}$$

(2) When the rest masses of the interacting partcles are different $m_a = m_b$

$$E_{Gab}\, t = (G\, m_a m_b\, /r)(r/c) = G\, m_a m_b/c = h_{Gab} = (h_{Ga}\, h_{Gb})^{1/2}$$

In the case when the interacting particles are a proton and an electron

$$E_{Gpe}\, t = (G\, m_p m_e\, /r)(r/c) = G\, m_p m_e/c = h_{Gpe} = (h_{Gp}\, h_{Ge})^{1/2} = 3{,}39073 \cdot 10^{-76}\ \mathrm{J \cdot s}$$

b. weak interactions

The weak interactions are shorte range interactions. The distance of interaction, as indicated by Białkowski [10], is pratically equal to the Compton wavelenght λ_C of the bosons Z^0, W^-, W^+ transmitting the interaction.

$$E_{w\,ABa}\, t = (1/4\, \pi)(g^2_{w\,ABa}\, /\, \lambda_{Ca})(\lambda_{Ca}\, /c) = (1/4\, \pi)(g^2_{w\,ABa}\ /c) = h_{w\,ABa}$$

Where A and B indicate the kind of interacting particles and a the kind of the boson transmitting the interaction and g_w is the charge of weak interactions

c. nuclear strong interactions

The nuclear strong interactions like the weak ones are shorte range interactions. The distance of interaction is also pratically equal to the Compton wavelenght λ_C of the meson transmitting the interaction.

$$E_{sABa}\, t = (1/4\, \pi)(g^2_{s\,ABa}\, /\, \lambda_{Ca})(\lambda_{Ca}\, /c) = (1/4\, \pi)(g^2_{s\,ABa}\ /c) = h_{s\,ABa}$$

Where A and B indicate the kind of interacting particles and a the kind of the meson transmitting the interaction and g_s is the charge of strong interactions

d. strong interactions between the quarks

Also in the interactions between quarks trasmited by gluons we are dealing with a transport of momentum and energy along the distance between them a during the time needed to transmit the interaction. Therefore also in this case we are able to introduce quanta of action that characterize such interactions. The problem, however is more complicated and must be presented in a separate paper.

3. COUPLING CONSTANTS AND QUANTA OF ACTION

Like the coupling constant of electromagnetic interactions is a ratio of two quanta of action in the same way the coupling constants of the other interactions are ratios of two quanta of action.

$$\alpha_e = 2\pi e^2/hc = 2\pi (e^2/c)/h = h_e/\hbar$$
$$\alpha_G = 2\pi\ Gm^2/hc = 2\pi (Gm^2/c)/h = h_G/\hbar$$
$$\alpha_w = 2\pi[(1/4\pi) g^2_w]/ hc = 2\pi [(1/4\pi) g^2_w /c)]/h = h_w/\hbar$$
$$\alpha_s = 2\pi [(1/4\pi) g^2_s]/ /hc = 2\pi [(1/4\pi) g^2_s/c)]/h = h_s/\hbar$$

As we can see the coupling constants depend only upon the quanta of action. The difference between them derivates only from the quanta of action of the four fundamental interactions

4. THE QUANTA OF ACTION AND THE FUNDAMENTAL LENGHTS OF QUANTUM MECHANICS (QM), ELECTROMAGNETISM (EM) AND THEORY OF GRAVITATION (TG)

As well known the fundamental lenght in QM is the Compton wavelenght $\lambda_C = h/mc$ of a particle. In EM the classical radius of a charged particle constitutes the fundamental lenght $r_e = Ke^2/mc^2$. It is interesting to note that using the quantum of action $h_e = Ke^2/c$ this fundamental lenght can be written as follows

$$r_e = h_e/mc$$

In the TG the gravitational lenght $r_G = Gm/c^2$ (called also „gravitational radius" or „Schwarzschild radius") constitutes the fundamental lenght. It is also interesting to note that using the quantum of actio $h_G = Gm^2/c$ also this lenght can be written in a similar way

$$r_G = h_G/mc$$

4. NEW QUANTA OF ACTION AND VIRTUAL BOSONS

As it is well known, according to the present-day physics of elementary particles the virtual bosons are tramsmitters of the four fundamental interactions. It is assumed that the interacting elementary particles are surrounded with clouds of virtual bosons. These bosons (gravitons, photons, bosons Z^0, W^-, W^+, mesons, gluons) are quanta of the respective fields (gravitational, electromagnetic, weak and strong). It is also assumed that particles interact with themselves. Such a selfinteraction consists in an emission and absorption of virtual bosons in the limits of Heisenberg uncertainty relations. The particles cannot emit and absorb real bosons. Such an emission and absorption would violate the conservation laws of energy and momentum.

The gravitational and electromagnetic interactions are long range interactions because their bosons, gravitons and photons, are massles. These interactions are transmited by bosons of all possible wavelenghts λ an all possible wave-periods T. In the acts of selfinteraction these bosons reach to great distances when their wavelenghts and wave-periods are great. Thus they can reach distant particles. When they meet at such a distance a particle then we are dealing with an exchange of virtual bosons.

The weak and nuclear strong interactions are short range interactions because their bosons, Z^0, W^-, W^+ and mesons, are rest mass particles. In these kinds of interactions, as it was already mentioned, the distance of interaction is practically equal to the Compton wavelenght of the bosons that transmit the interaction because in the acts of selfinteraction the virtual bosons are able to reach only the distance of the mentioned Compton wavelenght. The gluons are massless but they tranmit strong interactions only inside the particles made up of quarks therefore also these interactions are short range interactions.

The virtual bosons have the same properties like the real bosons except the energy and momentum that they transport. The real bosons transport energy and momentum equal to $E = h\nu$ and $p = h/\lambda$. The energy and momentum transmitted by virtual bosons are different then $E = h\nu$ and $p = h/\lambda$.

The energy transmitted by virtual bosons is equal to the energy of interaction E_a (where a indicates the kind of interaction). Since a virtual boson can reach another particle only at the distance equal to its wavelenght λ and during the time equal to its period T and since the transmitted energy is equal to the energy of interaction E_a therefore the wave properties λ and T of a virtual boson must be connected with E_a and p_a by means of other kinds of quanta of action. It is interesting to note that the other kinds of quanta of action are these introduced above using the Eddington method (energy of interaction x time needed to transmit the interaction).

a. Quantum of action connected with virtual photons

The quantum of actions connected with virtual photons is given by:

$$h_e = K e^2 /c = 7{,}69549 \cdot 10^{-37} \text{ J}\cdot\text{s}$$

Therefore:

(1) the energy and momenteum transmited by a virtual photon are given by

$$E_e = h_e \nu \qquad\qquad p_e = h_e /\lambda$$

(2) The Schrödinger equation for an electron present in the electromagnetic field of a nucleus is given by

$$- h/2m[\partial^2 \psi (x,t)/ \partial x^2] - (Zh_e c/ r)\psi (x,t) = ih[\partial\psi (x,t)/\partial t]$$

because the Coulomb potential is given by $V = - KZe^2/r = - Zh_e c/ r$

b. Quanta of action connected with virtual gravitons

The quanta of actions connected with virtual gravitons in the case of two neutrons, two protons and two electrons are given by:

$$h_{Gn} = G\, m_n^2/c = 6{,}24179 \cdot 10^{-73} \text{ J}\cdot\text{s}$$
$$h_{Gp} = G\, m_p^2/c = 6{,}2254 \cdot 10^{-73} \text{ J}\cdot\text{s}$$
$$h_{Ge} = G\, m_e^2/c = 1{,}8468 \cdot 10^{-79} \text{ J}\cdot\text{s}$$

In the case of interacting gravitationally proton and electron we have

$$h_{Gpe}= (h_{Gp}\, h_{Ge})^{1/2} = 3{,}39073 \cdot 10^{-76} \text{ J}\cdot\text{s}$$

Therefore:

(1) the energy and momenteum transmited by a virtual graviton between a proton and an electron are given by

$$E_{Gpe} = h_{Gpe}\, \nu= (h_{Gp}\, h_{Ge})^{1/2} \nu \qquad\qquad p_{Gpe}= h_{Gpe} /\lambda = (h_{Gp}\, h_{Ge})^{1/2}/\lambda$$

(2) The Schrödinger equation for an electron present in the gravitational field of a nucleus is given by

$$- h/2m[\partial^2 \psi (x,t)/ \partial x^2]- [(Z_1 h_{Gp}+ Z_2 h_{Gp})c/ r]\, \psi (x,t) = ih[\partial\psi (x,t)/\partial t]$$

because the Newton gravitational potential is given by

$$V = - G(Z_1 m_p+Z_2 m_n)/r = - (Z_1 h_{Gp}+ Z_2 h_{Gp})\, c/ r$$

Of course, the graviatational interactions are here so weak that they can be neglected. Note also that in the two last equation the defect of mass was not taken into consideration. It should be done because we are not dealing with a simple adition of the masses of protons and neutrons in the nucleus. Note also that gravitons are still only hypothetical entities.

c. Quanta of action connected with virtual bosons Z^0, W^-, W^+ transmitting the weak interactions.

The weak interactions are responsible for numerous decays and also for several scattering processes in which two particles interact**.** Let's consider a particular case of weak interactions i.e. the experimentaly ascertained scattering process

$$v_\mu + e \rightarrow v_\mu + e$$

in which the virtual neutral boson Z^o is the carrier of the interaction.

Also in this case the product of the energy of interaction $E_{w\,ve\,Z}{}^o$ between the two interacting particles and the time $T_{C\,Z}{}^o$ needed to transmit the interaction is always constant and constitutes the sought quantum of action.

$$E_{w\,v\,e\,Z}{}^o\, T_{C\,Z}{}^o = (1/4\,\pi)(g^2{}_{w\,v\,e\,Z}{}^o / \lambda_{C\,Z}{}^o)(\lambda_{C\,Z}{}^o /c) = (1/4\,\pi)(g^2{}_{w\,v\,e\,Z}{}^o /c) = h_{w\,v\,e\,Z}{}^o$$

$$h_{w\,v\,e\,Z}{}^o = 7{,}9218 \cdot 10^{-37} \text{ J}\cdot\text{s}$$

The energy and momentum transmited by the virtual boson Z^o in the considered scattering process are given by

$$E_{w\,ve\,Z}{}^o = h_{w\,v\,e\,Z}{}^o\ v_{C\,Z}{}^o \qquad p_{w\,v\,e\,Z}{}^o = h_{w\,v\,e\,Z}{}^o / \lambda_{C\,Z}{}^o$$

d. Quanta of action connected with the virtual mesons transmitting the nuclear strong interactions

In the case of two interacting nucleons the distance of interaction is pratically equal to the Compton wavelenght $\lambda_{C\,\pi}$ of the meson π which transmits the interaction. Also in this case the product of the energy of interaction $E_{sNN\,\pi}$ between two nucleons and the time $T_{C\,\pi}$ needed to transmit the interaction between two nucleons is always constant and constitutes the sought quantum of action.

$$E_{sNN\,\pi}\, T_{C\,\pi} = (1/4\,\pi)(g^2{}_{s\,NN\,\pi} / \lambda_{C\,\pi})(\lambda_{C\,\pi}/c) = (1/4\,\pi)(g^2{}_{s\,NN\,\pi}/c) = h_{sNN\,\pi}$$

$$h_{s\,NN\,\pi} = 1{,}55024 \cdot 10^{-33} \text{ J}\cdot\text{s}$$

The energy and momentum transmited by a virtual meson π between two nucleons are given by

$$E_{sNN\,\pi} = h_{s\,NN\,\pi}\, \nu_{C\,\pi} \qquad\qquad p_{s\,NN\,\pi} = h_{s\,NN\,\pi} / \lambda_{C\,\pi}$$

5. QUANTA OF ACTION AND RELATIVISTIC TRANSFORMATIONS

Since, according to the Relativity Theory the charges e, m_o , g_w , g_s and the constants G, K are invariant with respect to the relatistic transformations, therefore also the quanta of action h_e , h_G , h_w , h are invariant with respect to these transformations

Conclusions

We can say that Planck elementary quantum of action is connected with the motion of real particles. The quanta of the four fundamental interactions are connected with the motion of the virtual particles.

At the and of the XIX century M. Planck discovered his elementary quantum of action that became the basis for quantisation in Quantum Mechanics and has changed radically our image of the world of elementary particles. The indicated new quanta of action worked already in a hidden way as components of the coupling constants of the four fundamenteal interactions. I hope that they will play their part in the Physcs of XXI century. Recently E. Nelson [11] suggested that the the quantum of action connected with the electromagnetic interactions

$$h_e = K e^2 /c = 7{,}699 \cdot 10^{-37} \text{ J} \cdot \text{s}$$

plays a fundamental part in stochastic quantisation. Lets quote his words:

...quantum fluctuations are not of gravitational origin: one cannot construct a constant with the dimension of action from the gravitational constant G and the speed of light c. As is very well known, this can be done from c and the fundemental charge e. I conclude that quantum fluctuations may be of electromagnetic origin [11]

We can conclude that if the quanta of action connected with the four fundamental interactions (transmited by virtual bosons) are used in new kinds of quantisation our image of the the microscopic world will change once again radically.

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