COSMOMETRY

A new paradigm for relativity and cosmology

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The new paradigm consists of using the (radius of) curvature of space and time instead of velocity as the variable in transformations such as Lorentz. A detailed demonstration of the new paradigm is given for the Lorentz situation and an indication of how it could be applied is offered for the Hubble effect. The possibility that the Lorentz and Hubble effects could be connected emerged in the author's paper entitled " A hyperbolic generalisation of relativity" presented at PIRT VI in 1998. If this could nowbe confirmed it would, of course, have great significance for cosmology.

I.General

I.1 The model used in this paper is a simple schematic one of an absolute quadric representing infinity with a reference frame inside it. See Figure 1. Since any point inside the quadric is an infinite distance from the boundary no one point alone can be distinguished from another but distances between two points are significant. In other words there is no metric of single points; only of segments. Space can be distinguished from time by being susceptible to the rotations of its three axes. The group of rotations has a unique underlying algebra with multiplication table:

$$[L,M] = N \quad [M,N] = L \quad \{N,L] = M$$

the square brackets indicating Lie multiplication : LM-ML. The scenario is shown in Figure 1.

I.2 Two structures unique to hyperbolic geometry are essential to the explanation that follows: hyper-cycles and horo-cycles. Hyper-cycles which were originally called equidistant lines are lines with larger radii of curvature than geodesics and any geodesic has the potential for a range of hyper-cycles on either side of it, meeting it at its two infinite points. These lines can best be understood by considering their analogues in spherical geometry, lines of latitude. Elementary geometry shows that a line of latitude is shorter than the equator by a factor of cos(A) where A is the latitude and it is plain to see that this is due its radius of curvature being smaller than that of the sphere by the same factor. In hyperbolic geometry a segment of a hyper-cycle is longer than the corresponding one of its

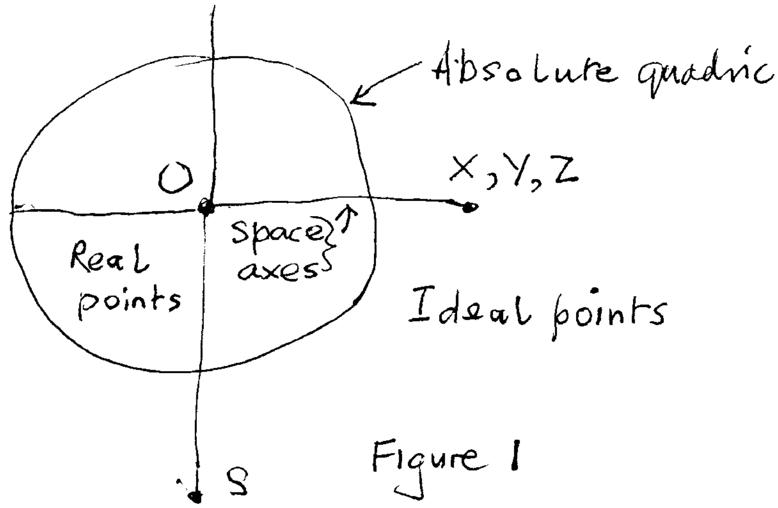


Figure 1

associated geodesic by a factor cosh(a) where a is the distance between them.

I.3 There is no analogue in either Euclidean or spherical geometry to horo-cycles. They are circles touching the absolute quadric. Alternatively they may be regarded as circles with centres at infinity. The metric along any horo-cycle is Euclidean.

I.4 A possible connection between Lorentz and Hubble follows readily from the above model. The origin of co-ordinates has a dual 3-space in the ideal region outside the absolute and the four axes intersect it at points X,Y,Z,S. S is enclosed by a surface that separates it from the three space points. We can now envisage two sets of transformation bearing in mind that a trans-formation normally involves just two operands. For example, in a spatial rotation x and y go to x' and y' leaving the z axis unchanged. In a Lorentz transformation one of the operands is the time variable.

I.5 We can also have a set of three transformations involving time with each of the space variables taken separately. This was the algebraic situation in the author's previous paper except that one transformation was applied to three separate pairs of variables. It seemed that this transformation was in some sense an inverse of Lorentz.

I.6 We begin by an analysis of Lorentz according to the new paradigm.

II Lorentz

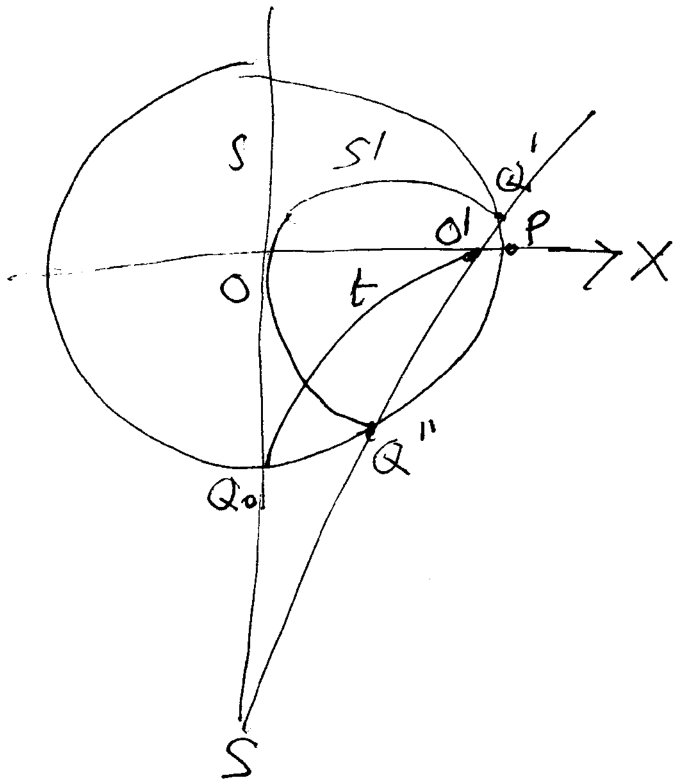
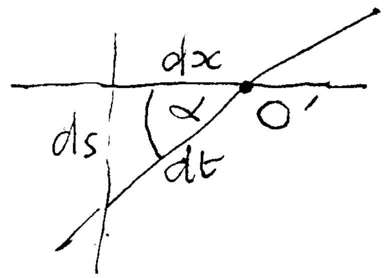


Figure2

II.1 The Lorentz situation is illustrated in Figure 2. OP represents the space axis and SOQ_0 the original time axis. S is the pole of OP in the sense that all geodesics from it cut OP normally. Now if we draw a second time line SO'Q''Q' the line Q''OO' is a hyper-cycle with Q''O'Q' as its axis or equator. If we take radius of curvature of the axis to be unity that of SO'Q''Q' is $\cosh\Delta x$ where Δx = OO'. We can calculate the value of this in the following way. Let us draw the (geodesic) line Q_0O' and provisionally designate the variable along it as t. Since Q_0O' and Q_0O meet at infinity and are therefore parallel then $\cos\alpha = \tanh\Delta x$ where α is the angle $OO'Q_0$. We can link up

Figure 3



with received theory by constructing a small right-angled triangle at O' with sides proportional to dx, dt and ds. See Figure 3. Then cos α = dx/cdt = v/c. Now ds/dt = sech α = $\sqrt{1 - v^2/c^2}$ which indicates that Q_0O' and Q_0O do indeed represent standard and proper time respectively. The space variable in the Lorentz transformation, dx/cds = (dx/cdt). (dt/ds) = sinh Δx as required. The time variable can be derived directly from the radius of curvature of the proper time line, cosh Δx.

II.2 It is important to appreciate the status of this picture. It is not, repeat not, geometry in the sense that the line OP represents actual space. It is a diagram representing relationships that incorporate the essense of hyperbolic geometry. It could perhaps be called metametry but this surely would be misunderstood. Hence the author's choice of cosmometry.

III The Hubble effect

III.1 The basic Hubble diagram is essentially the same as the Lorentz one, with the difference that in place of the single S axis we are dealing with three space ones X,Y,Z. In the Lorentz case the transformation has a single space axis, i.e. the one chosen by the direction cosines of rotation. The Lorentz transformation is directed. In the Hubble case orientation presumably results in a tilt of the whole space region. For simplicity we shall here assume that the observer is stationary relative to the matter in the universe and more particularly to the microwave radiation. The axis pictured is thus a representative of the three. Figure 4.

III.2 If we draw a second space line through our representative space centre X and let it cut the absolute at Q' and Q" the curve Q'OQ" is a hyper-cycle with radius of curvature cosh Δs where Δs = OO'. So much for the transformation of space but there seems to be no place in this picture for velocity.

III 3 As a first step towards making a place for it we have to make a speculative leap. Earlier we mentioned that the second set of transformations

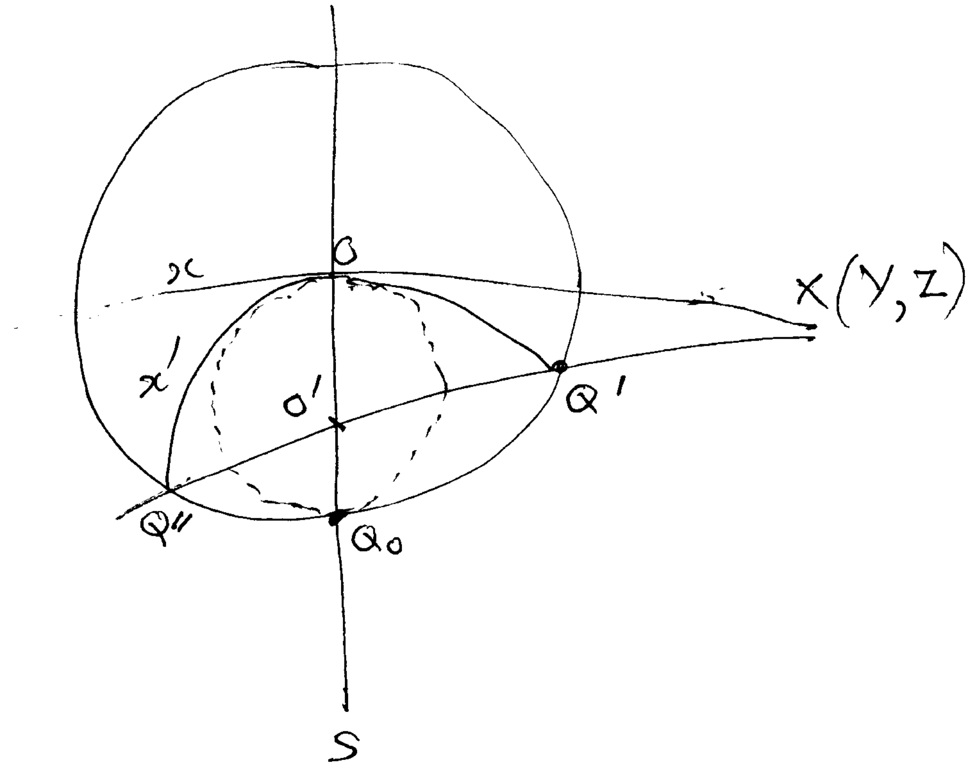


Figure 4

seemed to be inverses of Lorentz so we hazard a guess that we should be putting the origin (and the observer) at infinity, that is at Q_0. The geodesic from X would be a tangent to the absolute and the space of extension the horo-cycle touching the absolute at Q_0 and another geodesic through X. Unfortunately since the origin is at infinity distances from it are meaningless so we have to measure them from another point. The only one that makes sense is the previous origin at O on the other side of the horo-cycle. However we still have no velocity in our picture. We have to make another speculative leap but before we do we must introduce some analysis.

III.4. If we are using an origin at infinity we need functions with zeros at infinity in particular a counterpart of the hyperbolic Lorentz transformation. This seems to be $\begin{pmatrix} \coth s & \cos echs \\ \cos echs & \coth s \end{pmatrix}$. It is hyperbolic and with determinant unity. It is not a member of a continuous group like the Lorentz transformation but is has eigen-functions coth(s/2) and tanh(s/2). Despite their unpromising appearance these are analogues of exponentials but with values running in the opposite directions. For example, as s runs from zero to infinity coth(s/2) ruins from infinity to unity. The analogy is even more striking in that whereas exp(x) can be written as $\sqrt{\frac{1+\tanh(x)}{1-\tanh(x)}}$, coth(s/2) can be written $\sqrt{\frac{1+\sec h(s)}{1-\sec h(s)}}$, that is replacing tanh by sech. The importance of this analogy is that it suggests that velocity or some quantity associated with it behaves as a sech not a tanh as in the Lorentz case.

III.5. Now sech is a non-directional even function in contrast with tanh which is directional and odd. The author therefore proposes the following. Since sech(s) is non-directional we have to regard the whole absolute quadric as a single point. In geometrical terms this means that a radius from any point on the absolute is identical to another and carries the same variable, namely, standard time t. We can therefore draw a time line through the point where the axis OX cuts the absolute. See Figure 5. In principle we now have a diagram identical to the one in the Lorentz case. However the same analysis does not quite fit. The cosine of the angle α represents ds/dt which before equalled sech(Δs). From the geometry it is in fact tanh(Δs). One can only suppose that the trouble lies with the origin being at infinity whereas we have proceeded exactly as we did in the case of Lorentz where the origin would be at O. The mathematics of making this change of origin are not immediately obvious but it is arguable that tanh from the point of view of an origin at O should become sech if the origin is at Q_0.

III.6. The complement of α in Figure 5 represents dx/cdt and equals sech(Δs) in hyperbolic terms. So the ratio dx/cds = (dx/cdt)/(ds/dt) = sech(Δs)/tanh(Δs) = cosech(Δs). The companion component dt/ds = coth(Δs) so we have returned to our starting point with the putative Hubble matrix $\begin{pmatrix} \coth s & \cos echs \\ \cos echs & \coth s \end{pmatrix}$. This demonstrates that the Hubble diagram does in fact agree exactly with the analysis.

II.7 We have now shown that the Lorentz transformation has a companion transformation in which sech takes the place of tanh, the function representing velocity. Whereas Lorentz has an origin in finite space the new transformation has as its origin the whole extent of the quadric at infinity. There is thus a strong case for believing that the new transformation represents the Hubble effect.

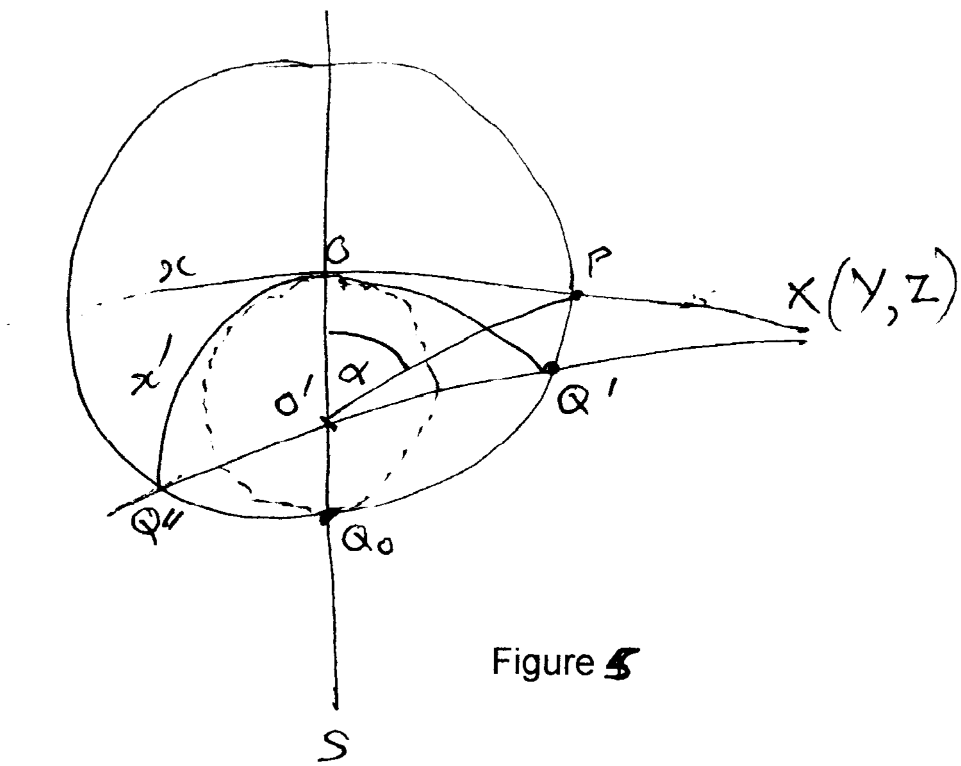


Figure 5

Concluding remarks

IV.1 If the results described above are accepted they are clearly of great significance. Hitherto the Hubble effect has been attributed to an actual expansion of the universe which in turn has been explained by an application of the General Theory of Relativity (GTR). If, on the contrary, it can be explained as a matter of geometry the status of GTR in this context becomes questionable.

IV.2 The philosophical implications are even greater. If the features of the universe as we see it can be explained by geometry, that universe is not evolving. There was no Big Bang; what we see was always there.