THE LINK BETWEEN THE TIME AND SPACE CONCEPTS AND THE MEANING OF THE THEORY OF RELATIVITY

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INTRODUCTION

We would like to discuss here the meaning of time, and in particular the link that the concept of time has with the concept of space. As a matter of fact, standard space-time does contain a vector dimension for time that is additional to the three vector dimensions for space. One then supposes to think time independently of the three space dimensions. To the limit, one could think a time without space. This statement poses a problem for us. What meaning would have a time flowing independently of space (and of the matter it contains)? Leaving the domain of physics and turning toward the domain of cognitive sciences, one will notice that the concepts of time and space are extracted there from a single substratum of spatio-temporal relations between the material entities; they are not a priori separated from it. One thinks of the old experiments by J. Piaget, but there are a lot of others. From this unique substratum, the definition of space leans on the invariant relations between the material entities and that of time on their changing relations. Time has its first meaning in the changes of the positions of the material entities with respect to one another; to conceive a time independent of them, this is to push the abstraction to the absurd. Conversely, space has no structure (no real existence) without the material entities and without the time-consuming phenomena allowing to connect them and to compare their positions. The separation of space and time from a same substratum sends us back to the philosophical questions of immobility and mobility, or of permanence and change. In our present approach, it is crucial to understand that, from the epistemological point of view, these two groups of concepts are indissociably bound together. This is within this indissociable link that we would like to set the question of time.

By doing so, we will be brought to examine the theory of relativity. It is very often said that relativity establishes a link between time and space; but if one looks carefully at the things, one will note that this link is not a primary link: one already starts up with four dimensions, time and space being separated. This is only when comparing two frames in relative translation with respect to each other that one makes appear a link of a special type between the time and space variables (within the Lorentz relations). From the beginning, one has separated space from time and one has abandoned the whole discussion on the meaning of time. We will try to undertake this discussion here. We will be led to a new look on the theory of relativity, and will propose, not to abolish it, but to modify it in some respect. One knows that there are people opposing to this theory that have put in evidence a series of problems or of questions. Without claiming to review all these questions on an exhaustive manner, we suggest that our approach must allow to match the points of view between the opponents and the defenders of the theory of relativity. None of these two categories of physicists discuss the meaning of time the way we want to do it here. In this summary, we present the qualitative frame of our approach while giving some indications on its quantitative consequences.

1. A SPACE OF SPATIO-TEMPORAL RELATIONS BETWEEN THE MATERIAL ENTITIES

As we have said, rather than to start with space and time as separate, it appears to us necessary to begin with a unique space of spatio-temporal relations. It is then necessary to extract and separate from it, according to hypotheses or approximations that need be specified, time and space, two essential concepts to think. In this approach, one can proceed from two manners: in the first, one can start with a space of spatio-temporal relations constructed uniquely from the positions of the material entities compared with one another, then abstract time; in other words, time and space are first mixed, then one succeeds to separate them. In the second, one starts with a thinking frame where space and time already exist separately from each other, and one shows how they are bound. From our thought point of view, it may be easier to start with time and space as separated from each other, then to examine with which degree of uncertainty they are bound. These two approaches rejoin.

One will notice that to-day physics contains some aspects of redundancy when counting space and time parameters separately. The usual system allows different sets of values for the spatial and temporal coordinates, *all corresponding to the same appearance of the world*. One could for example imagine that the solid bodies that are around us inflate to a constant speed with time, while the speeds of all that moves would increase in the same proportion. That would change nothing in the appearance of the world because the relative positions and movements of the material entities would not change. This shows that the coordinates system is not narrowly adapted to reality; it apparently offers useless degrees of freedom. Other similar thought experiments would show that there is something too much, and we are encouraged to discuss the possible link between the space and time variables.

Let us settle in the current frame with three dimensions for space and one for time. In order to define time and space in an operational manner, one needs to measure them, and one is right away led to use such instruments as rules and clocks, as Einstein himself proposed. The examining of the functioning of these rules and clocks allows to present our approach. Actually, when one begins to study the world, one does have neither rules neither clocks independent of the phenomena, and that could allow to measure these and to attribute to them spatial and temporal properties. In the practice, it is necessary to choose among what is proposed to our investigation, tools that will be called rules (i.e. specified material points bound together), and tools that will be called clocks (i.e. material points defining a reference movement). Hypotheses must then be set. How can one be sure that a rule length does not vary when it is moved from a place to another? How can one be sure that a clock measures the same second now and all on time? The comparison with another rule or another clock brings us to a regression to the infinite. This question of the indecidability of the constancy of the standards has been seen by Poincaré. At a given moment, one has to make the assumption of the constancy of the space and time standards; this amounts to the two assumptions of a constancy of immobility and of a constancy of mobility respectively. These are assumptions of a philosophical or epistemological nature that are necessary to make in order to be able to progress; these are not like an hypothesis of a physical nature such as to say that the speed of light is constant.

In what respect is this discussion on rules and clocks connected with our interest on the link between time and space? Effectively, if one requires from the rule the property of immobility, of solidity, or of fixity of each extremity in comparison with the other (that allows to think and measure space), and if one requires from the clock the property of constant mobility (that allows to think and measure time), one is led to the earlier mentioned properties of these mobility and immobility hypotheses. From the philosophical point of view, they go together: *one cannot have one without the other*. One cannot proclaim the immobility without compare it in a way or another to a constant mobility, and one cannot either define the constant mobility without defining at the same time an immobility that serves as a landmark. To the epistemological level, this is an essential point. It is much discussed to-day. One speaks of pairs of contradictory notions: these are associated notions, that apparently exclude each other, but that one cannot think without each other. They lead us to so-called indecidability situations. This is to say that one must do choices to think that are not not strictly imposed by reality. Thus, so far as rules and clocks are concerned, we make, *within the same thought*, the hypothesis of the immobility of the extremities of the standard rule and the hypothesis of the uniform mobility of the movement of the standard clock. In other words, the uncertainty on the immobility is the same as the uncertainty on the constant mobility.

We can illustrate the associated construction of space and time variables with the positioning of material points on a one-dimensional spatial axis. The minimum of points to put is three, let be points 1, 2 and 3. With three points we will try to compute mobility and immobility, together with the positions and times associated to other material points that can be disposed on the axis. In other words, these three material points will have to serve us as clocks and rules, that do not exist independently of the material points and of the phenomena affecting them. The axis is not purely mathematical, it represents a one-dimensional simplification of the physical world. Along this axis, movement and immobility thus look as offered together to us by nature. Among the natural phenomena displaying movement, we thus choose a phenomenon to define time and to which we attribute a unit velocity (e.g. the propagation of sound or light, the movement of the sand in the egg timer etc.). Suppose here that one grants the mobility of point 3 when compared to points 1 and 2: point 3 will serve us to spot the time, and the distance between points 1 and 2 will serve us to measure space. In this disposal of the three points, we must therefore at the same time assume the constancy of the distance between points 1 and 2 and the constant mobility of point 3 compared to points 1 and 2. We cannot verify rigorously neither the constancy of the relations between 1 and 2 nor of the movement of 3 in comparison with 1 and 2. In the end, thanks to the preceding associated assumptions we are led to define both time and space variables from a single vector axis. One could make alternate hypotheses, consider for example that the distance between 1 and 2 varies slowly; but this is not separable of the "quicker" change than we affect to 3 in comparison with 1 and 2. In a way we arbitrarily decided to bind points 1 and 2 together and give to point 3 the mobility property.

In order this point of view may be better understood, suppose space and time variables be defined separately beforehand and affect to the three points the spatial coordinates $x_1(t)$, $x_2(t)$ and $x_3(t)$ that may depend on time t. In the preceding world reduced to one single dimension, the standard rule is actually defined by the difference x_2 -

x_1 and the time by the distance x_3 - x_1 (or x_3 - x_2). For any additional point 4, the position may be defined by the value x_4 - x_1 (or x_4 - x_2) and the time still by x_3 - x_1 (the unique time of the world is constructed on the common agreement on the position of a special indicator, here point 3). However, and to link with our approach, we must recognize that we know only the ratio $(x_4 - x_1)/ (x_2 - x_1)$ defining the position, and the ratio $(x_3 - x_1)/ (x_2- x_1)$ defining the time. These ratios furnish the only minimal set of parameters that maintain the same appearance to the world. A same value of these ratios authorizes different values for the space and time coordinates (when understood as independent) and puts into light the redundancy we discussed higher. In the preceding, space intervals and time intervals identify (other way to say that only the relative positions matter); they may be measured by the same phenomenon, as this is the case today where the meter and the second are defined within the same statement and both associated to the propagation of light to a constant-decided velocity.

This is with hypotheses of the preceding type that one does operate in the practice. If one looks at the way space is defined today for the positioning of satellites, it is based on a set of landmarks fixed to the earth. As opposed to the fixity of these indicators, time is measured by movements; for a long time, that of the sun with respect to the earth has been used. But if one lived a lot more slowly (at geological scale), one would notice than the earth and the mountains move and distort themselves. One would be then in a position where the former earth-bound indicators, that served to measure space, could serve to measure time. And in order to measure space, it would be necessary to lean on indicators that move less fast, as distant galaxies for example. Inversely, if one would live a lot faster, what appears to move, e.g. the sun in the sky or the waves of the sea, would appear immobile: it would be then necessary to find a quicker movement to measure the time, the propagation of the light for example. These thought experiments help us discuss the separation between the time and space concepts. One sees indeed that while changing the relative speeds of the material entities serving as landmarks, one passes continuously from parameters that one associates with space to parameters that one associates with time. *This reveals the difference in degree, not in nature, between these two parameters.* In connection with the spatio-temporal axis considered so forth, we can put an axis of relative speeds. Depending on the degree of precision of our instruments and on our choice, one can change one's position alongside this axis and change the boundary between what is affected to space and what is affected to time; but, as we will stress below, one cannot abolish the underlying hypotheses necessary to draw this boundary (e.g. the immobility hypothesis etc.) and the consequences it will have.

2. FIRST CONSEQUENCE: LORENTZ TYPE RELATIONS; CASE OF A SINGLE SPATIO-TEMPORAL DIMENSION

Let us examine a major consequence of associating time and space along a same axis. Let us start with two separated variables x and t. To reason, one must make hypotheses on the mobility and the immobility of material indicators disposed in different points of space. Actually, one must make the same hypotheses at different spots, or rather, one does not have a priori reason to make different hypotheses. Take again the previous earth example: we are doing experiments in Paris and in New York, and we say that the things happen in the same manner on both sides. But geologists know that, due to plate tectonics, Paris moves a little with respect to New York. The fact of not taking it into account leads to require from physical laws to have a special structure: we ask them to be the same despite a movement between Paris and New York to a small speed v. One thus obtains relations of Lorentz type (we designate here all the associated relations such as dilation and contraction of distances and times). These express the *inevitable distortions between, or uncertainties concerning, the space and time standards within the unit movement* (e.g. photon propagation) envisaged at distant points. The distortions on the standards induce similar distortions on all the space and time variables associated to all points. If one estimates to know the movement of Paris in comparison with New York, one can take it into account and apparently get rid of Lorentz relations. But this is to push back the problem, because it is necessary to a given moment to lean on spatial indicators that one supposes to be fixed, and write the laws of physics despite small unknown or indecidable movements of these indicators; there is a regression to the infinite and we cannot get rid of Lorentz type relations. Note that in the preceding, the small uncertainty velocity is normalized to the speed of the standard chosen phenomenon to measure time and space (e.g.light velocity). Another way to express the preceding is to say that we keep the same ratio of space to time standards in the different frames.

3. CASE OF THREE SPATIO-TEMPORAL DIMENSIONS

While considering so far a single space dimension, we have expressed a privileged link between time and space. Space has three dimensions. In order to do the passage from one to three dimensions, one must do for each dimension the same reasoning and assume the same associated hypotheses as previously, bearing on the mobility and the immobility of temporal and spatial indicators along the three axes. By consequence, this makes appear three "times", associated to three standard movements along the three axes. So far as Lorentz relations in the large are concerned, these will make appear distortions or uncertainties in relation with each of the three axes, with separate and independent play of each of the three coordinates v_x, v_y, v_z of the velocity along the three axes. The standard Lorentz relations do not show this symmetry. In our new formalism, three parameters thus appear to which corresponded a unique scalar time in the one dimensional case. One can say that these three parameters correspond to three spatial indicators along the three axes, and not that three times appear! One does not say that there are several times because there are several clocks. To define a unique scalar time for the three-dimensional space within a synchronization operation, various choices are possible using the set of the three spatial parameters. In any case, let us recall that the synchronization property is never known in the instant but is an hypothesis that is always more or less well verified afterwards.

4. OTHER CONSEQUENCES: A LOOK AT SOME PROBLEMS RAISED IN THE THEORY OF RELATIVITY

We have given a spatio-temporal meaning to each space vector dimension. When one goes to three dimensions, we have seen that it is necessary to envisage the dimensions in a more separate manner from the time point of view than what is done in the standard theory. This induces consequences on the manner to approach certain problems in the theory of relativity. Some problems arise at the mathematical level when one envisages frame movements with velocities that are not parallel to one of the coordinates axes. Usually, one envisages movements parallel to so-called Ox axis: the Lorentz relations then pertain to the two couples of variables (x, t) and (x', t'), and one keeps y = y' and z = z'. When one envisages movements of arbitrary direction, one loses the simplicity of the standard relations. This has been abundantly discussed by researchers. A solution to these problems may be in the use of so-called Thomas rotations or precessions. Actually, these additional rotations lead to paradoxes difficult to elucidate, and there still exists, from the strict mathematical point of view, inconsistencies in the application of this formalism to Maxwell equations. Without discussing all these questions in detail, it appears to us that *one loses the good functioning of the relativity formalism as soon as one loses the privileged coupling between the time variable and one spatial dimension*, that is to say as soon as one considers more than one single spatial dimension. This is the case for 3D-movements of frames along directions that are arbitrary with respect to the axes, for rotations etc. Our approach allows to avoid these problems. To our sense, it also allows to discuss other (if not all?) questions raised in the theory of relativity, such as those bearing on the use of the interval ds^2 (that must be restricted to the standard movement), etc.

Let us stress that our approach rests on philosophical considerations that are very general: it is not strictly constructed on special hypotheses bearing on the nature and propagation of light. It is interesting at this point to recall that Lorentz relations are met in other domains than light. To our understanding, one finds Lorentz type relations whenever one must make associated hypotheses of immobility and constant mobility for spatial and temporal indicators, related to the chosen standard movement that allows to measure space and time. This is indeed what one finds when one measures displacements and times by sound propagation. The associated hypotheses made in this case on the clocks that measure time by sound waves and on the spatial indicators lead to Lorentz type relations, as several authors have shown through different reasoning. Two interesting consequences in this comparison need be emphasized: first, the Lorentz-type relations are not incompatible with the existence of a privileged frame (here, the motionless frame where sound propagates) and express the uncertainties inherent to any separation between space and time variables from a unique substratum of spatio-temporal relations. Second, one does not need a Thomas rotation for the working of these sound clocks, provided one considers separately and in a symmetrical manner the different coordinates of the movement velocity along each axis.

5. TENSORIAL FORMALISM

To finish, let us notify that, to discuss the links between space and time variables, one can use a tensorial formalism. It opens to general relativity. In the presence of matter, light cannot go the same speed as in emptiness. In order to write the metric tensor, one must assume to be able to compare the speed of light with and without matter at the same place; this appears as a new hypothesis. At a given spot, metric tensor allows to define two types of variables, of covariant and contravariant type respectively, that are identified so long as the tensor is reduced to the unit tensor. On a general standpoint, one can give to these two types of variables, when applied to the (space/time standards determining) luminous movement, the container of a distance and of a time interval. The example of black holes gives a brief justification of this allocation: in a black hole, light spends an infinite time to go out, whereas the distance is finite. The correspondence between this infinite time and the distance that remains finite, is expressed by the correspondence between the two associated tensorial variables of the matter-bound metric tensor. This approach allows to write and generalize Lorentz type relations. The double tensorial aspect of the variables may help discuss Maxwell equations while giving a double tensorial value to the electric and magnetic fields.

6. CONCLUSION

In conclusion, it appears to us that the physical interpretation of the theory of relativity must take into account the effect of the essentially connected assumptions one must make to think both mobility and immobility and allowing to establish the time and space standards.

This text is based on published and unpublished works by B. Guy as well as those of a long list of researchers who have developed and discussed the theory of relativity and more general connected philosophical questions. The approach presented must be considered as preliminary and open to discussion.