COMPATIBILITY AND UNIFORMITY IN THE RELATIVITY THEORY: PROBLEMS AND SOLUTIONS

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Abstract.

C*ompatibility* of time and spatial co-ordinate transformations, which is explained in the paper, is crucial for their validity and applications. It is shown that the Lorentz transformations and related speed transformations are pairwise compatible if and only if an arbitrary point moves with speed of light. This opens the problem of *compatibility* of the transformations. Another useful property of the transformations is their *uniformity*, which is also explained in the paper. The Lorentz transformations are not uniform.

The paper presents Newtonian generalisations of the Lorentz transformations, which are uniform and completely compatible. This means that they are compatible in spite the value of speed of an arbitrary point is also arbitrary. The generalised transformations are based on the properties of *time*, which confirm Newton's explanation of time. The paper establishes new results on velocity, mass and energy in the framework of the compatible uniform relativity for the general case.

Keywords: Basic physics**,** relativity theory, systems, time, Lorentz transformations.

1. INTRODUCTION

Newton's recognition in [28] of the importance of time scales and units appeared crucial for opening new directions in the relativity theory [10], [11], [14] – [17]. Independently of Newton, their importance was also recognised in the framework of dynamical systems with multiple time scales [12], [16], [27], [29] – [33].

Newton's explanation in [28] of the meaning and the sense of both *absolute time* and *relative time* was shown to be correct [10], [11], [14] – [17]. It incorporates Einstein's meaning of time relativity and represents the basis for further developments of the relativity theory in Newtonian sense that is not restrained when compared with a priori accepted constraints in the Lorentz – Einstein relativity theory, which caused the singularity of the Lorentz transformations.

A crucial point of Einstein's relativity theory is the postulate on the constancy of the value of velocity of light in vacuum relative to inertial frames. This is interpreted in some literature as invariance of the (numerical) value of light speed with respect to a choice of time unit and length unit. The (numerical) value of speed of anybody or of anything depends on the units of time and length. Although this holds also for speed of light, it has not been taken into account in the Lorentz – Einstein relativity theory [2] – [6], [19] – [23].

Another inherent point of Einstein's general relativity theory is invariance of the time-space length under the Gaussian transformation [2] – [6]. The Lorentz - Einstein relativity theory is based on and demands a priori equality of time scaling coefficients and of those of spatial co-ordinate scaling in the Lorentz transformations. Besides, their values are assumed constant a priori, as well, which was achieved by determining them for a priori accepted speed of an arbitrary point to be light speed [2] – [6], [19] – [23].

By relying on the nature of *time* it was shown in [10], [11], [14] – [17] that the time scaling coefficients should be different, as well as those of spatial co-ordinates scaling, which led to new results on time and spatial co-ordinate transformations. The Lorentz transformations represent their singular case. Consequently, new formulas were proved for the basic physical variables: speed, acceleration, force and energy, as well as for mass.

The paper provides analysis of compatibility of the co-ordinate transformations. Time (or, spatial) co-ordinate transformations are *compatible* if and only if they lead to an identity when they are combined (i. e. when the inverse transformation is applied to the transformation). *Pairwise compatibility* of the transformations expresses both compatibility between the time co-ordinate transformations and compatibility between the spatial co-ordinate transformations. *Entire compatibility* of the transformations means that they altogether lead to an identity when time co-ordinate transformations and spatial co-ordinate transformations are mutually combined. *Complete pairwise (entire) compatibility* of the transformations means, respectively, their pairwise (entire) compatibility in arbitrary case independently of both a position and speed of an arbitrary point. Their *partial* (*restrictive*) *pairwise (entire) compatibility* means, respectively, their pairwise (entire) compatibility only under the condition that an arbitrary point moves with speed of light. It will be shown in the sequel that the Lorentz transformations are restrictively pairwise compatible. They are not completely pairwise compatible. This opens a new problem in the relativity theory - the problem of *complete (pairwise, entire) compatibility of the transformations,* and poses the next questions:

Q1 Under which conditions the time and the spatial co-ordinate transformations are completely (pairwise, entire) compatible?

Q2 Under which conditions the velocity transformations are completely (pairwise, entire) compatible?

The new results provoke also the next questions:

Q3 What are implications of different time scales linked with the Gaussian generalisation of the Lorentz transformations on the relativity theory?

Q4 What are implications on dynamical systems with multiple time scales?

Q5 What are implications on speed, mass and energy?

The paper contributes with full replies to these questions in the framework of the uniform transformations. Uniformity of the transformations expresses their property that the same initial moment, the same time scale and time unit, i. e. the same time set and time axis, hold over (cover) the whole space. The new transformations represent Newtonian generalisations of the Lorentz transformations.

The main results are proved in all details due to their delicacy.

2. NOTATION

A time scale of a time axis T can be variously accepted. Different time scales are associated with T: "an original time scale" T that is not indexed and "i"-time scale T_i. A time value (instant, moment) measured in T-scale and T_i-scale is designated by t and t_i, respectively, $t \in T$ and $t_i \in T_i$, i = 1, 2, …, s.

The Cartesian product set $T\text{x}R^n$ is denoted by I, $I = T\text{x}R^n$. It is the *(n+1)-dimensional real integral vector space,* for short *the integral space.* A pair (t, x) is called *an event in I* [18]. It can happen only once due to the properties of *time*. The integral space corresponding to the time axis T_i and the frame R_i^n is I_i: $I_i = T_i \text{ x } R_i^n$, i = 1, 2, …, s.

Let origin O_i of co-ordinate system R_i^n, hence R_i^n itself, move with constant speed $\mathbf{v}_{Oi}$ relative to origin O of R^n and measured with respect to $t \in T$, i = 1, 2, ... , s. The Euclidean (or any other scalar) norm $\|.\|$ of speed vector $\mathbf{v}_{Oi} = v_{Oi}\mathbf{r}_0$ is denoted by v_{Oi}, $v_{Oi} = \|\mathbf{v}_{Oi}\|$ if measured relative to $t \in T$. The vector $\mathbf{r}_0$ is an arbitrarily chosen and fixed constant unity vector: $\|\mathbf{r}_0\| = 1$.

A light array passes an infinitesimal path (vector) $d\mathbf{r}_L$ during an infinitesimal time interval dt. They determine the light speed (vector) $\mathbf{c}(t) = d\mathbf{r}_L(t)/dt$ at moment t. The value of light speed $c(t) = \|\mathbf{c}(t)\|$ is constant in vacuum: $c(t) \equiv c$ [3: p. 15], [4: p. 26]. It is the light speed value with respect to the vacuum and measured relative to T, for short: the light speed value. Since the light propagates equally in all the directions then we can accept to consider a light signal and a translation of O_i together with R_i^n in a direction and sense of the vector $\mathbf{r}_0$ that is used to represent symbolically also the direction and the sense of movement of the spaces R_j^n, j = 1, 2, ... , s. Analogously, without losing in generality, we represent a position of arbitrary point P in the environment relative to origins O and O_i of R^n and R_i^n at the same moment by vectors $\mathbf{r}_P \in R^n$ and $\mathbf{r}_P^i \in R_i^n$ as $\mathbf{r}_P = \rho_P\mathbf{r}_0$, and $\mathbf{r}_P^i = \rho_P^i\mathbf{r}_0$, i = 1, 2, …, s. Their lengths are expressed by their norms $\rho_P = \|\mathbf{r}_P\|$ and $\rho_P^i = \|\mathbf{r}_P^i\|$, respectively, which may vary in time:

$\mathbf{r}_P(t_{(.)};t_{(.)0}) = \rho_P(t_{(.)};t_{(.)0})\mathbf{r}_0$ is the position vector of point P with respect to $O_{(.)}$ at $t_{(.)}$, (.) = i, j; i, j = 1, 2, …, s, which will be denoted also by $\mathbf{r}_i(t_i;t_{i0}) = \rho_i(t_i;t_{i0})\mathbf{r}_0$: $\mathbf{r}_P(t_i;t_{i0}) = \mathbf{r}_i(t_i;t_{i0})\mathbf{r}_0$.

Their velocities should be naturally permitted to depend on the reference integral space:

c_j^i is the scalar value of light speed measured with respect to the origin O_j of R_j^n and relative to t_i (rather than relative to t_j if $T_i \neq T_j$),

c_{ij} denotes both c_i^i and c_j^j if and only if $c_j^j = c_i^i$: $c_{ij} = \;\; = c_{ji} = c_i^i = c_j^j$; in the Lorentz transformations (L1), (L2) there is c_{ij} denoted simply by c,

$\mathbf{v}_P^{O_i}(t_i;t_{i0}) \equiv v_P^{O_i}(t_i;t_{i0})\mathbf{r}_0$ is the instantaneous speed vector of point P with respect to O_i at t_i and measured relative to t_i provided the initial moment was t_{i0}; $\mathbf{v}_P^{O_i}(t_i;t_{i0}) \equiv \mathbf{v}_P^{O_i} \equiv v_P^{O_i}\mathbf{r}_0$ if and only if speed vector $\mathbf{v}_P^{O_i}(t_i;t_{i0})$ is constant vector, and $\mathbf{v}_P^{O_i}(t_i;t_{i0}) \equiv \mathbf{v}_P^{O_i}(t_i) \equiv v_P^{O_i}(t_i)\mathbf{r}_0$ if and only if t_{i0} is known and fixed,

$\overline{\mathbf{v}}_P^{O_i}(t_i;t_{i0}) \equiv \overline{v}_P^{O_i}(t_i;t_{i0})\mathbf{r}_0$ is the average speed vector of P over $[t_{i0}, t_i]$ with respect to O_i at t_i and measured relative to t_i:

$$\overline{\mathbf{v}}_P^{O_i}(t_i;t_{i0}) = \frac{\mathbf{r}_i(t_i;t_{i0})}{t_i - t_{i0}} =$$

$$= \begin{cases} \mathbf{v}_P^{O_i}(t_{i0};t_{i0}) = \mathbf{v}_{P0}^{O_i}, t_i = t_{i0} \\ \dfrac{1}{t_i - t_{i0}} \displaystyle\int_{t_{i0}}^{t_i} \mathbf{v}_P^{O_i}(t_i;t_{i0})dt_i, t_i > t_{i0} \end{cases} =$$

$= \overline{\mathbf{v}}_P^{O_i}(t_i)$ for known and fixed t_{i0},

$\mathbf{v}_{O_k}^m \equiv v_{O_k}^m \mathbf{r}_0$ is the constant speed of O_k relative to O measured in terms of t_m, k, m = 1, 2, …, s,

$v_{O_i}^m \leq v_{O_j}^m$ is accepted, $m \in \{i, j\}$,

$\boldsymbol{v}_{ji}^k \equiv v_{ji}^k \mathbf{r}_0 \equiv (v_{O_j}^k - v_{O_i}^k)\mathbf{r}_0$ is the constant relative speed vector of O_j with respect to O_i measured all in terms of t_k, k ∈ {i, j}; notice that $\boldsymbol{v}_{ji}^i \equiv -\boldsymbol{v}_{ij}^i$, and that $\boldsymbol{v}_{ji}^i \neq \boldsymbol{v}_{ji}^j \equiv (v_{Oj}^j - v_{Oi}^j)\mathbf{r}_0 \equiv -\boldsymbol{v}_{ij}^j$ is possible if $T_i \neq T_j$,

$\boldsymbol{v}_{ji} \equiv v_{ji}\mathbf{r}_0$ denotes both $\boldsymbol{v}_{ji}^i$ and $\boldsymbol{v}_{ji}^j$ if and only if they are equal: $\boldsymbol{v}_{ji}^i = \boldsymbol{v}_{ji}^j = \boldsymbol{v}_{ji} = - \boldsymbol{v}_{ij}$. In the Lorentz transformations (L1) – (L4) there is v_{ji} denoted simply by *v*.

If and only if point P represents a light signal, then: $\mathbf{r}_P^O = \mathbf{r}_L = \mathbf{r} \in R^n$, $\mathbf{r}_L = \rho_L \mathbf{r}_0 = \rho \mathbf{r}_0$, $\mathbf{r}_P^{O_{(.)}} = \mathbf{r}_{L(.)} \in R_{(.)}^n$, and $\mathbf{v}_P^{O_{(.)}} = \mathbf{v}_L^{O_{(.)}} = \mathbf{c}_{(.)}^{(.)} = c_{(.)}^{(.)}\mathbf{r}_0$, (.) = i, j; i, j = 1, 2,…, s.

3. Time Features

It was shown that the following Newtonian explanation of the crucial features of *time* is correct [10], [11], [14] – [17], which we represent in the axiomatic form due to the physical reality and the experience:

Axiom 1. Time *is* ***an independent physical variable.*** *Its value is strictly monotonously continuously increasing independently of space and of all other (physical and mathematical) variables, processes and events.*

The ***value of time*** is called **moment** or **instant.** It is denoted by t and a subscript, e.g. t_2. An arbitrary instant will be denoted as time itself by t. The time value is determined accurately up to an unknown additive constant. *A* ***sequence of time values*** *determines uniquely the order of events happening.*

Moment (***instant***) reflects *an instantaneous internal physical situation* of a material object called its **age**. **Time value difference** (**time interval**) is used to ***measure duration*** *of a process, of a movement, of a rest or of the existence of somebody or of something*.

A co-ordinate axis used for a time axis is immovable relative to the environment. It is denoted by *T*. It is an ordered set of all instants:

$T = \{t: t \in R, t \in C^{(1)}(R), dt > 0\}$.

Once a time axis has been accepted with a fixed time scale including a fixed time interval unit, then the zero moment t = 0 should have been also accepted. Afterwards, we can select any instant $t_0 \in T$ for an initial instant. We accept that it has been chosen, known and fixed. It can be $t_0 = 0$, but need not.

Different materials can have *different speeds of evolution of physical processes*, which can hold also for the same material object at its different points or parts. For this reason, different time scales, different initial moments and/or different time units can be assigned to different material objects and/or to different parts of the same material object giving a relative meaning to *time* (in this sense that is Newtonian).

Newton himself [28: pp. 8 – 10] introduced and explained both the absolute and relative sense of *time*. Newton's explanation of the relativity of *time* incorporates that of Einstein [2, p. 20], [4, pp. 26 – 27], [5, pp. 23 – 40], which was explained and proved in [10], [11], [14] – [17].

4. Dynamical Systems with Multiple Time Scales

A large class of dynamical systems is adequately mathematically modelled by (S1) and (S2), [7] – [9], [12],

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}_1(t, \mathbf{x}, \mathbf{y}, M), \mathbf{x} \in R^p, \mathbf{y} \in R^s, \quad \text{(S1)}$$

$$\mathbf{f}_1(.): R \times R^p \times R^s \times R^{s \times s} \to R^p, M \in R^{s \times s},$$

$$M\frac{d\mathbf{y}}{dt} = \mathbf{f}_2(t, \mathbf{x}, \mathbf{y}, M), \quad \text{(S2)}$$

$$\mathbf{f}_2(.): R \times R^p \times R^s \times R^{s \times s} \to R^s.$$

The matrix M = diag$\{\mu_1\ \mu_2\ \dots\ \mu_s\}$ contains different (possibly small) parameters μ_i that enable the introduction of s different time scales and/or units, $\mu_i \in R^+$, i = 1, 2, ... , s, $R^+ =]0, \infty[$:

$$t_i - t_{i0} = \mu_i (t - t_0),$$
$$t_{i0} = \mu_i t_0, \mu_i \in R^+,\ i = 1, 2, \ldots, s. \quad (1)$$

By accepting constant time scaling coefficients μ_i we open a possibility for a uniform time scale over the whole space R_i^n.

5. COMPATIBILITY, LORENTZ TRANSFORMATIONS AND SPEED TRANSFORMATIONS

The Lorentz transformations have the next forms [1] – [6], [19] – [23]:

$$t_i - t_{i0} = \alpha[(t_j - t_{j0}) + \frac{v}{c^2}\rho_j(t_j; t_{j0})],$$
$$\alpha = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}, \quad (L1)$$

$$t_j - t_{j0} = \alpha[(t_i - t_{i0}) - \frac{v}{c^2}\rho_i(t_i; t_{i0})], \quad (L2)$$

$$\mathbf{r}_i(t_i; t_{i0}) = \lambda[\mathbf{r}_j(t_j; t_{j0}) + \boldsymbol{v}(t_j - t_{j0})],\ \lambda = \alpha, \quad (L3)$$

$$\mathbf{r}_j(t_j; t_{j0}) = \lambda[\mathbf{r}_i(t_i; t_{i0}) - \boldsymbol{v}(t_i - t_{i0})]. \quad (L4)$$

They are not uniform over space because the time co-ordinate transformations (L1) and (L2) depend on position $\rho_{(.)}(t_{(.)}; t_{(.)0})$ of arbitrary point P. This opens the problem of uniformity of the time co-ordinate transformations over space.

The time co-ordinate scaling factors and the spatial co-ordinate scaling factors are mutually equal in (L1) – (L4): $\lambda = \alpha$. Their value is constant:

$$\lambda = \alpha = \left[1 - (v/c)^2\right]^{-1/2} = \text{const}.$$

This is due to their determination for an arbitrary point moving with speed of light. If they were determined for arbitrary average speed $\mathbf{v}_P^{O_{(.)}}(t_{(.)})$ of arbitrary point P then they would be:

$$\alpha^* = \frac{1}{\sqrt{\left(1 + \frac{v\overline{v}_P^{O_j}(t_j)}{c^2}\right)\left(1 - \frac{v\overline{v}_P^{O_i}(t_i)}{c^2}\right)}}, \quad (L5)$$

$$\lambda^* = \frac{1}{\sqrt{\left(1 + \frac{v}{\overline{v}_P^{O_j}(t_j)}\right)\left(1 - \frac{v}{\overline{v}_P^{O_i}(t_i)}\right)}}. \quad (L6)$$

Evidently, $\mathbf{v}_P^{O_i}(t_i) \equiv \mathbf{v}_P^{O_j}(t_j) \equiv c \Rightarrow \alpha = \alpha^* = \lambda = \lambda^*$.

We can set the Lorentz transformations in a (quasi) linear form by using the definition of the instantaneous average speed of arbitrary point P, from which we get:

$$\mathbf{r}_i(t_i; t_{i0}) = \overline{\mathbf{v}}_P^{O_i}(t_i; t_{i0})(t_i - t_{i0}),$$

and

$$t_i - t_{i0} = \rho_i(t_i; t_{i0})\left[\overline{v}_P^{O_i}(t_i; t_{i0})\right]^{-1},$$

so that (L1) – (L4) take the next forms, respectively:

$$t_i - t_{i0} = \mu_{ij}(t_j - t_{j0}),$$

$$\mu_{ij} = \frac{1 + \frac{v\overline{v}_P^{O_j}(t_j)}{c^2}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \mu_{ij}[\overline{v}_P^{O_j}(t_j)], \quad (LL1)$$

$$\mu_{ij} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} = \text{const. if and only if } \overline{v}_P^{O_j}(t_j) \equiv c,$$

$$t_j - t_{j0} = \mu_{ji}(t_i - t_{i0}),$$

$$\mu_{ji} = \frac{1 - \frac{v\overline{v}_P^{O_i}(t_i)}{c^2}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \mu_{ji}[\overline{v}_P^{O_i}(t_i)], \quad (LL2)$$

$$\mu_{ji} = \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} = \mu_{ij}^{-1} \text{ if and only if } \overline{v}_P^{O_i}(t_i) \equiv c,$$

$$\mathbf{r}_i(t_i; t_{i0}) = \eta_{ij}\mathbf{r}_j(t_j; t_{j0}),$$

$$\eta_{ij} = \frac{1 + \frac{v}{\overline{v}_P^{O_j}(t_j)}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \eta_{ij}[\overline{v}_P^{O_j}(t_j)],\ \overline{v}_P^{O_j}(t_j) \neq 0, \quad (LL3)$$

$$\eta_{ij} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} = \mu_{ij} = \text{const. if and only if } \overline{v}_P^{O_j}(t_j) \equiv c,$$

and

$$\mathbf{r}_j(t_j; t_{j0}) = \eta_{ji}\mathbf{r}_i(t_i; t_{i0}),$$

$$\eta_{ji} = \frac{1 - \frac{v}{\overline{v}_P^{O_i}(t_i)}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \eta_{ji}[\overline{v}_P^{O_i}(t_i)], \quad (LL4)$$

$$\eta_{ji} = \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} = \eta_{ij}^{-1} \text{ if and only if } \overline{v}_P^{O_i}(t_i) \equiv c.$$

The equations (LL1) – (LL4) show that the Lorentz transformations (L1) – (L4) are homogeneous linear transformations with the gains $\mu_{ij}, \mu_{ji}, \eta_{ij}$ and η_{ji}, the values of which depend on the value $\overline{v}_P^{O_i}(t_i)$ or $\overline{v}_P^{O_j}(t_j)$ of speed of arbitrary point P in general. However, they are constant because the scaling factors α and λ in (L1) – (L4) were determined for:

$\overline{v}_P^{O_i}(t_i) \equiv \overline{v}_P^{O_j}(t_j) \equiv c$.

If we divide (LL1) by (LL2) and use (1), then we get:

$$(t_i - t_{i0}) = \mu_i(t - t_0) = \mu_{ij}(t_j - t_{j0}) = \mu_{ij}\mu_j(t - t_0),$$

$$(t_j - t_{j0}) = \mu_j(t - t_0) = \mu_{ji}(t_i - t_{i0}) = \mu_{ji}\mu_i(t - t_0).$$

These equations lead to:

$$\mu_{ij}[v_P^{O_j}(t_j)] = \frac{\mu_i}{\mu_j} = \text{const.} \Rightarrow v_P^{O_j}(t_j) \equiv v_P^{O_j} = \text{const.},$$

$$\mu_{ji}[v_P^{O_i}(t_i)] = \frac{\mu_j}{\mu_i} = \text{const.} \Rightarrow v_P^{O_i}(t_i) \equiv v_P^{O_i} = \text{const.},$$

so that, due to (LL3) and (LL4), the gains η_{ij} and η_{ji} obey the following:

$$\eta_{ij}[v_P^{O_j}(t_j)] = \eta_{ij}(v_P^{O_j}) = \text{const.},$$

and

$$\eta_{ji}[v_P^{O_i}(t_i)] = \eta_{ji}(v_P^{O_i}) = \text{const.}$$

These results show, under validity of (1), that the Lorentz transformations (L1) - (L4) are homogeneous linear transformations with constant gains $\mu_{ij}, \mu_{ji}, \eta_{ij}$ and η_{ji}. Besides, they demand for the value of speed of arbitrary point P to be constant, as well. In fact, let us repeat this once more,

$\overline{v}_P^{O_i}(t_i) \equiv \overline{v}_P^{O_j}(t_j) \equiv c$

was used in order to determine α and λ in (L1) – (L4).

Definition 1.

a) The transformations (L1) and (L2) [(L3) and (L4)] are, respectively, *compatible* if and only if they result in an identity after eliminating variables with the same subscript (either with "i" or with "j") from them by using the definition of the instantaneous average value of speed of an arbitrary point.
b) The transformations (L1) through (L4) are *pairwise compatible* if and only if both pairs [(L1), (L2)] and [(L3), (L4)] are compatible.
c) The transformations (L1) through (L4) are *entirely compatible* if and only if they altogether result in two (time co-ordinate and spatial co-ordinate) identities after eliminating variables with the same subscript (either with 'i" or with "j") from (L1) and/or (L2) by using (L3) and/or (L4), and from (L3) and/or (L4) by applying (L1) and/or (L2).
d) The transformations are *partially (restrictively) compatible* if and only if they are compatible only in the case when an arbitrary point moves with speed of light.
e) The transformations are *completely (fully) compatible* if and only if they are compatible in every case (nevertheless whether an arbitrary point moves with speed of light or not).

Definition 2. The time co-ordinate transformations are *uniform (over space)* if and only if they do not depend on a choice of arbitrary point P, i. e. if and only if they are independent of position $\rho_{(.)}(t_{(.)}; t_{(.)0})$ and speed $\mathbf{v}_P^{O_{(.)}}(t_{(.)})$ of the point P.

Theorem 1. The Lorentz transformations (L1) – (L4) are:

a) partially (not completely) pairwise compatible,
b) completely entirely compatible.

Proof. a) Let us eliminate, for example, the co-ordinates indexed by "j" from the pairs [(L1), (L2)] and [(L3), (L4)]. We start with the first pair by setting it into the next form that results from the definition of the average value of speed of arbitrary point P:

$$t_i - t_{i0} = \frac{\left(1 + \frac{v\overline{v}_P^{O_j}(t_j)}{c^2}\right)}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}(t_j - t_{j0}),$$

$$t_j - t_{j0} = \frac{\left(1 - \frac{v\overline{v}_P^{O_i}(t_i)}{c^2}\right)}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}(t_i - t_{i0}).$$

After eliminating $(t_j - t_{j0})$ we find that the next identity should hold if and only if (L1) and (L2) are compatible:

$$1 - \left(\frac{v}{c}\right)^2 \equiv \left(1 + \frac{v\overline{v}_P^{O_j}(t_j)}{c^2}\right)\left(1 - \frac{v\overline{v}_P^{O_i}(t_i)}{c^2}\right).$$

For this identity to hold it is necessary and sufficient that the next Lorentz Compatibility condition holds:

$$\overline{v}_P^{O_j}(t_j) \equiv \overline{v}_P^{O_i}(t_i) \equiv c. \qquad \text{(LC)}$$

This proves that (L1) and (L2) are only partially (not completely) compatible. By applying the same procedure to the second pair [(L3), (L4)] we discover that the next identity holds if and only if (L3) and (L4) are compatible:

$$1 - \left(\frac{v}{c}\right)^2 \equiv \left(1 + \frac{v}{\overline{v}_P^{O_j}(t_j)}\right)\left(1 - \frac{v}{\overline{v}_P^{O_i}(t_i)}\right).$$

For this identity to hold it is necessary and sufficient that (LC) is valid. This proves that (L3) and (L4) are only partially compatible. Altogether, (L1) - (L4) are only partially pairwise compatible, which proves the statement under a).

b) We eliminate, for example, the variables with the subscript “j” from (L1) and (L3) by using (L2), (L4) and the definition of the instantaneous average value of speed of arbitrary point P. As the result we get the next two identities that hold if and only if the transformations are entirely compatible:

$$t_i - t_{i0} \equiv \frac{[1-\frac{v v_P^{-O_i}(t_i)}{c^2}+\frac{v v_P^{-O_i}(t_i)}{c^2}-\frac{v^2}{c^2}]}{1-\left(\frac{v}{c}\right)^2}(t_i - t_{i0}) \equiv$$

$$\equiv (t_i - t_{i0}),$$

and

$$\mathbf{r}_i(t_i;t_{i0}) \equiv \frac{[1-\frac{v}{v_P^{-O_i}(t_i)}+\frac{v}{v_P^{-O_i}(t_i)}-\frac{v^2}{c^2}]}{1-\left(\frac{v}{c}\right)^2}\mathbf{r}_i(t_i;t_{i0}) \equiv$$

$$\equiv \mathbf{r}_i(t_i;t_{i0}).$$

These identities prove complete entire compatibility of the Lorentz transformations (L1) – (L4). Q. E. D.

The speed transformations, which result directly from (L1) – (L4) and from the definition of speed :

$$\mathbf{v}_P^{O_{(.)}}(t_{(.)}) = \frac{d\mathbf{r}(t_{(.)},t_{(.)0})}{dt_{(.)}}, (.) = i, j,$$

have the next forms :

$$\mathbf{v}_P^{O_i}(t_i) = \frac{\mathbf{v}_P^{O_j}(t_j)+\boldsymbol{v}}{1+\frac{v v_P^{-O_j}(t_j)}{c^2}}, \quad \mathbf{v}_P^{O_j}(t_j) = \frac{\mathbf{v}_P^{O_i}(t_i)-\boldsymbol{v}}{1-\frac{v v_P^{-O_i}(t_i)}{c^2}}.$$

Notice that entire compatibility of the speed transformations coincides with their pairwise compatibility. Therefore, we consider just their compatibility.

Theorem 2. The speed transformations resulting from the Lorentz transformations (L1) – (L4) are only partially (not completely) compatible.

Proof. After eliminating, for example, $\mathbf{v}_P^{O_j}(t_j)$ from the speed transformations we get the next identity that holds if and only if the speed transformations are compatible:

$$\left(1-\frac{v v_P^{-O_i}(t_i)}{c^2}\right)\left(1+\frac{1-\frac{v}{v_P^{O_i}(t_i)}}{1-\frac{v v_P^{-O_i}(t_i)}{c^2}}\frac{v v_P^{-O_i}(t_i)}{c^2}\right) \equiv 1-\frac{v^2}{c^2}.$$

For this identity to hold it is necessary and sufficient that $v_P^{-O_i}(t_i) \equiv c$. If we repeat the procedure by eliminating $\mathbf{v}_P^{O_i}(t_i)$ than we get $v_P^{-O_j}(t_j) \equiv c$. Altogether, for the speed transformations to be compatible it is necessary and sufficient that (LC) holds, i. e. that arbitrary point P moves with speed of light. Hence, the speed transformations are only partially compatible. Q. E. D.

These theorems show that there exists an open problem of compatibility of the Lorentz transformations and of the speed transformations.

6. Problem Statement

We wish to discover linear transformations that are completely both pairwise and entirely compatible, which are uniform over space, and which imply completely compatible speed transformations. In order to ensure uniformity, all the time scaling coefficients $\mu_{(.)}$, (1), will be assumed independent of a choice of arbitrary point P.

Problem. What are conditions on the scaling coefficients $\mu_i \in R^+$, $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, so that the equations (2) - (5),

$$t_i - t_{i0} = \alpha_j^i[(t_j - t_{j0}) + \frac{v_{ji}^j}{(c_j^j)^2}\rho_L(t_j;t_{j0})], \quad (2)$$

$$t_j - t_{j0} = \alpha_i^j[(t_i - t_{i0}) - \frac{v_{ji}^i}{(c_i^i)^2}\rho_L(t_i;t_{i0})], \quad (3)$$

$$\mathbf{r}_i(t_i;t_{i0}) = \lambda_j^i[\mathbf{r}_j(t_j;t_{j0}) + \boldsymbol{v}_{ji}^j(t_j - t_{j0})], \quad (4)$$

$$\mathbf{r}_j(t_j;t_{j0}) = \lambda_i^j[\mathbf{r}_i(t_i;t_{i0}) - \boldsymbol{v}_{ji}^i(t_i - t_{i0})], \quad (5)$$

hold, that they imply the identity (6),

$$\begin{bmatrix} \mathbf{r}_i^T & (t_i - t_{i0})[\mathbf{v}_P^{O_i}(t_i)]^T \end{bmatrix} G \bullet$$

$$\begin{bmatrix} \mathbf{r}_i^T & (t_i - t_{i0})[\mathbf{v}_P^{O_i}(t_i)]^T \end{bmatrix}^T \equiv$$

$$\begin{bmatrix} \mathbf{r}_j^T & (t_j - t_{j0})\left[\mathbf{v}_P^{O_j}(t_j)\right]^T \end{bmatrix} G \bullet \begin{bmatrix} \mathbf{r}_j^T & (t_j - t_{j0})\left[\mathbf{v}_P^{O_j}(t_j)\right]^T \end{bmatrix}^T, \quad (6)$$

that they are completely both pairwise and entirely compatible and that they imply completely compatible speed transformations?

The time co-ordinate transformations (2), (3) can be uniform over space because they do not depend on a choice of arbitrary point P that can move with speed different from speed of light. They depend on position $\rho_L(t_{(.)};t_{(.)0})$ of the light signal, which is uniform over space.

The matrix G = blockdiag{A -B} in (6), A and B are positive definite matrices with A = B possible but not required, A and $B \in R^{nxn}$. The block diagonal form of the matrix G reflects time independence of space (Axiom 1).

The time co-ordinate transformations and the spatial co-ordinate transformations (2) through (5) permit in general both different scaling coefficients and their independence of a position and of speed of arbitrary point P, which can be arbitrary. The transformations will be shown uniform and they are the basis for a new direction in the relativity theory – *the uniform relativity theory*.

The time-scale equations (1) are inherent for multiple time scale dynamical systems [10], [11], [14], [17]. The condition (6) expresses the Gaussian transformation of the time-space length. It is a crucial condition of Einstein's general relativity theory for validity of the co-ordinate transformations defined by (2) through (5), [2], [3].

If G = I is the identity matrix, then the transformation (6) becomes Euclidean, which is used in the special relativity theory.

7. Problem Solution

This section presents the new relativity theory fundaments. They enable us to establish new results on the basic physical variables and on mass.

Two cases should be distinguished and will be considered relative to the mutual relationship among scaling coefficients, which are constant in either case:

General case: $\alpha_i^j \neq \alpha_j^i$ and/or $\lambda_i^j \neq \lambda_j^i$.

Special case: $\alpha_i^j = \alpha_j^i$ and $\lambda_i^j = \lambda_j^i$.

Problem Solutions for the General Case

We will allow for the (numerical) value of any speed, hence of light speed, to depend on an integral space with respect to which it is determined.

Theorem 3. If speed of arbitrary point P is arbitrary, then, in order for the scaling coefficients $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j \neq \alpha_j^i$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j \neq \lambda_j^i$, to be constant and to obey the equations (2) through (5), and for (1) through (5) to imply (6), it is necessary and sufficient that speed of point P is constant and that the following equations hold for any choice of $\mu_i \in R^+$**:**

$$\alpha_j^i = \frac{\mu_i}{\mu_j}\frac{1}{1+\frac{v_{ji}^j}{c_j^j}} = \frac{c_j^j}{c_i^i}\frac{1}{1+\frac{v_{ji}^j}{c_j^j}}, \quad (7)$$

$$\alpha_i^j = \frac{\mu_j}{\mu_i}\frac{1}{1-\frac{v_{ji}^i}{c_i^i}} = \frac{c_i^i}{c_j^j}\frac{1}{1-\frac{v_{ji}^i}{c_i^i}}, \quad (8)$$

$$\lambda_j^i = \frac{1}{1+\frac{v_{ji}^j}{v_P^{O_j}}}, \quad (9)$$

$$\lambda_i^j = \frac{1}{1-\frac{v_{ji}^i}{v_P^{O_i}}}, \quad (10)$$

$$v_{ji}^j > \max\left\{-c_j^j, -v_P^{O_j}\right\}, \quad v_{ji}^i < \min\left\{c_i^i, v_P^{O_i}\right\}, \quad (11)$$

and

$$\frac{\mu_j}{\mu_i} = \frac{v_P^{O_i}}{v_P^{O_j}} = \frac{c_i^i}{c_j^j}, i, j = 1,2,\ldots,n. \quad (12)$$

Proof. *Necessity*. Let time scale coefficients $\mu_i \in R^+$, i = 1, 2, ... , s, be defined by (1). Let coefficients $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j \neq \alpha_j^i$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j \neq \lambda_j^i$, be constant and obey (2) through (5), and let (1) through (5) imply (6). Since $\rho_L(t_j;t_{j0})$ represents the position of the light signal measured relative to $I_j = T_j \times R_j^n$, then $\rho_L(t_j;t_{j0}) = c_j^j(t_j - t_{j0})$, j=1, 2, …, s. This, (1) and (2) yield:

$$t_i - t_{i0} = \mu_i(t - t_0) = \alpha_j^i\left(1+\frac{v_{ji}^j}{c_j^j}\right)(t_j - t_{j0}) =$$

$$= \alpha_j^i\left(1+\frac{v_{ji}^j}{c_j^j}\right)\mu_j(t - t_0), \; i, j = 1,2,\ldots,s,$$

so that:

$$\alpha_j^i = \frac{\mu_i}{\mu_j}\left(1 + \frac{v_{ji}^j}{c_j^j}\right)^{-1}, \ i, j = 1, 2, \ldots, s. \qquad (13)$$

This proves the first equation in (7). Besides, $\alpha_j^i \in R^+$ and (13) prove $v_{ji}^j > -c_j^j$ in the first inequality in (11). The first equation in (8) is analogously proved by combining $\rho_L(t_i;t_{i0}) = c_i^i(t_i;t_{i0})$, i = 1, 2, …, s, with (1) and (3). From the definition of the instantaneous average value of speed, from (1) and from (4) we deduce the following:

$$\mathbf{r}_i(t_i;t_{i0}) = \overline{v}_P^{O_i}\mu_i(t - t_0)\mathbf{r}_0 = \lambda_j^i(\overline{v}_P^{O_j} + v_{ji}^j)\mu_j(t - t_0)\mathbf{r}_0,$$

i, j = 1, 2, …, s, (14)

or,

$$\lambda_j^i = \frac{\mu_i \overline{v}_P^{O_i}(t_i)}{\mu_j \overline{v}_P^{O_j}(t_j)}\left(1 + \frac{v_{ji}^j}{\overline{v}_P^{O_j}(t_j)}\right)^{-1}, i, j = 1, 2, \ldots, s. \qquad (15)$$

We transform the right hand side of (6) as follows by using (1) and $\rho_i(t_i;t_{i0}) = \overline{v}_P^{O_i,i}(t_i)(t_i - t_{i0})$, i = 1, 2, …, s:

$$\begin{bmatrix} \mathbf{r}_i(t_i;t_{i0}) \\ (t_i - t_{i0})\overline{\mathbf{v}}_P^{O_i}(t_i) \end{bmatrix}^T G \begin{bmatrix} \mathbf{r}_i(t_i;t_{i0}) \\ (t_i - t_{i0})\overline{\mathbf{v}}_P^{O_i}(t_i) \end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix} \overline{v}_P^{O_j}(t_j)(\overline{v}_P^{O_j}(t_j))^{-1}\mu_j^{-1}\mu_j\mu_i(t - t_0)\overline{v}_P^{O_i}(t_i)\mathbf{r}_0 \\ \overline{v}_P^{O_j}(t_j)(\overline{v}_P^{O_j}(t_j))^{-1}\mu_j^{-1}\mu_j\mu_i(t - t_0)\overline{v}_P^{O_i}(t_i)\mathbf{r}_0 \end{bmatrix}^T G \bullet$$

$$\bullet \begin{bmatrix} \overline{v}_P^{O_j}(t_j)(\overline{v}_P^{O_j}(t_j))^{-1}\mu_j^{-1}\mu_j\mu_i(t - t_0)\overline{v}_P^{O_i}(t_i)\mathbf{r}_0 \\ \overline{v}_P^{O_j}(t_j)(\overline{v}_P^{O_j}(t_j))^{-1}\mu_j^{-1}\mu_j\mu_i(t - t_0)\overline{v}_P^{O_i}(t_i)\mathbf{r}_0 \end{bmatrix} \equiv$$

$$\begin{bmatrix} \overline{v}_P^{O_i}(t_i)(\overline{v}_P^{O_j}(t_j))^{-1}\mu_i\mu_j^{-1}\mathbf{r}_j(t_j;t_{j0}) \\ \overline{v}_P^{O_i}(t_i)(\overline{v}_P^{O_j}(t_j))^{-1}\mu_i\mu_j^{-1}(t_j - t_{j0})\overline{\mathbf{v}}_P^{O_j}(t_j) \end{bmatrix}^T G \bullet$$

$$\bullet \begin{bmatrix} \overline{v}_P^{O_i}(t_i)(\overline{v}_P^{O_j}(t_j))^{-1}\mu_i\mu_j^{-1}\mathbf{r}_j(t_j;t_{j0}) \\ \overline{v}_P^{O_i}(t_i)(\overline{v}_P^{O_j}(t_j))^{-1}\mu_i\mu_j^{-1}(t_j - t_{j0})\overline{\mathbf{v}}_P^{O_j}(t_j) \end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix} \mathbf{r}_j(t_j;t_{j0}) \\ (t_j - t_{j0})\overline{\mathbf{v}}_P^{O_j}(t_j) \end{bmatrix}^T G \begin{bmatrix} \mathbf{r}_j(t_j;t_{j0}) \\ (t_j - t_{j0})\overline{\mathbf{v}}_P^{O_j}(t_j) \end{bmatrix}.$$

The last identity implies:

$$\left(\mu_i \overline{v}_P^{O_i}(t_i)\right)\left(\mu_j \overline{v}_P^{O_j}(t_j)\right)^{-1} = 1.$$

This and (15) yield:

$$\lambda_j^i = \left(1 + \frac{v_{ji}^j}{\overline{v}_P^{O_j}(t_j)}\right)^{-1}, i, j = 1, 2, \ldots, s.$$

This and constancy of λ_j^i show that $\overline{v}_P^{O_j}(t_j)$ is constant: $\overline{v}_P^{O_j}(t_j) \equiv v_P^{O_j}(t_j) \equiv v_P^{O_j}$, hence $\overline{v}_P^{O_i}(t_i)$ is also constant: $\overline{v}_P^{O_i}(t_i) \equiv v_P^{O_i}(t_i) \equiv v_P^{O_i}$. Therefore,

$$\lambda_j^i = \left(1 + \frac{v_{ji}^j}{\overline{v}_P^{O_j}}\right)^{-1}, i, j = 1, 2, \ldots, s,$$

and

$$\left(\mu_i \overline{v}_P^{O_i}\right)\left(\mu_j \overline{v}_P^{O_j}\right)^{-1} = 1,$$

which prove the equation (9) and complete the proof of the first inequality in (11) due to $\lambda_j^i \in R^+$. The equation (10) and the second inequality in (11) are analogously proved. Since point P is arbitrary, then it can represent the light signal so that then $\overline{v}_P^{O_{(.)}} = c_{(.)}^{(.)}$, (.) = i, j, which implies the second equation in (12). The first equations in (7) and (8) together with (12) result, respectively, in the second equations in (7) and (8).

Sufficiency. Let time scale coefficients $\mu_i \in R^+$, i = 1, 2, ... , s, be defined by (1). Let (7) – (12) and all the conditions of the theorem statement hold. We start with (1) and (7):

$$t_i - t_{i0} = \mu_i(t - t_0)\alpha_j^i(\alpha_j^i)^{-1} =$$

$$= \mu_i(t - t_0)\alpha_j^i \frac{\mu_j}{\mu_i}(1 + \frac{v_{ji}^j}{c_j^j}), i, j = 1, 2, \ldots, s,$$

or, by using (1) for i = j, and $\rho_L(t_j;t_{j0}) = c_j^j(t_j - t_{j0})$:

$$t_i - t_{i0} = \alpha_j^i(1 + \frac{v_{ji}^j}{c_j^j})(t_j - t_{j0}) =$$

$$= \alpha_j^i\left[(t_j - t_{j0}) + \frac{v_{ji}^j}{(c_j^j)^2}\rho_L(t_j;t_{j0})\right], i, j = 1, 2, \ldots, s.$$

This proves validity of (2). Equation (3) is proved analogously by starting with $t_j - t_{j0}$. We transform $\mathbf{r}_i(t_i;t_{i0}) = v_P^{O_i}(t_i - t_{i0})\mathbf{r}_0$ by using $\rho_i(t_i;t_{i0}) = v_P^{O_i}(t_i - t_{i0})$, (1) and (9):

$$\mathbf{r}_i(t_i;t_{i0}) = v_P^{O_i}(t_i - t_{i0})\mathbf{r}_0 = v_P^{O_i}\lambda_j^i(\lambda_j^i)^{-1}(t_i - t_{i0})\mathbf{r}_0 =$$

$$= v_P^{O_i}\lambda_j^i\left(1 + \frac{v_{ji}^j}{v_P^{O_j}}\right)(t_i - t_{i0})\mathbf{r}_0 =$$

$$= v_P^{O_i}\lambda_j^i\left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right)\frac{\mu_i}{\mu_j}(t_j - t_{j0})\mathbf{r}_0 =$$

$$= v_P^{O_j}\lambda_j^i\left((t_j - t_{j0})+\frac{v_{ji}^j}{v_P^{O_j}}(t_j - t_{j0})\right)\mathbf{r}_0 =$$

$$= \lambda_j^i\left(\rho_j(t_j;t_{j0})+v_{ji}^j(t_j - t_{j0})\right)\mathbf{r}_0 =$$

$$= \lambda_j^i\left(\mathbf{r}_j(t_j;t_{j0})+v_{ji}^j(t_j - t_{j0})\mathbf{r}_0\right).$$

This proves (4). The equation (5) is analogously proved by starting with $\mathbf{r}_j(t_j;t_{j0}) = v_P^{O_j}(t_j - t_{j0})\mathbf{r}_0$. Let us now transform the left-hand side of (6) as follows by using $\mathbf{r}_i(t_i;t_{i0}) = v_P^{O_i}(t_i - t_{i0})\mathbf{r}_0$, $\mathbf{v}_P^{O_i} = v_P^{O_i}\mathbf{r}_0$, in view of constancy of speed $\mathbf{v}_P^{O_i}$, and (1):

$$\begin{bmatrix}\mathbf{r}_i(t_i;t_{i0})\\(t_i - t_{i0})\mathbf{v}_P^{O_i}\end{bmatrix}^T G\begin{bmatrix}\mathbf{r}_i(t_i;t_{i0})\\(t_i - t_{i0})\mathbf{v}_P^{O_i}\end{bmatrix}\equiv$$

$$\begin{bmatrix}v_P^{O_j}(v_P^{O_j})^{-1}\mu_j(\mu_j)^{-1}\mu_i(t-t_0)v_P^{O_i}\mathbf{r}_0\\v_P^{O_j}(v_P^{O_j})^{-1}\mu_j(\mu_j)^{-1}\mu_i(t-t_0)v_P^{O_i}\mathbf{r}_0\end{bmatrix}^T G\bullet$$

$$\bullet\begin{bmatrix}v_P^{O_j}(v_P^{O_j})^{-1}\mu_j(\mu_j)^{-1}\mu_i(t-t_0)v_P^{O_i}\mathbf{r}_0\\v_P^{O_j}(v_P^{O_j})^{-1}\mu_j(\mu_j)^{-1}\mu_i(t-t_0)v_P^{O_i}\mathbf{r}_0\end{bmatrix}.$$

This, $\mathbf{r}_j(t_j;t_{j0}) = v_P^{O_j}(t_j - t_{j0})\mathbf{r}_0$, $\mathbf{v}_P^{O_j} = v_P^{O_j}\mathbf{r}_0$, (1) and (12) imply:

$$\begin{bmatrix}\mathbf{r}_i(t_i;t_{i0})\\(t_i - t_{i0})\mathbf{v}_P^{O_i}\end{bmatrix}^T G\begin{bmatrix}\mathbf{r}_i(t_i;t_{i0})\\(t_i - t_{i0})\mathbf{v}_P^{O_i}\end{bmatrix}\equiv$$

$$\left[(v_P^{O_j}\mu_j)^{-1}v_P^{O_i}\mu_i\right]^2\begin{bmatrix}\mathbf{r}_j(t_j - t_{j0})\mathbf{r}_0\\(t_j - t_{j0})\mathbf{v}_P^{O_j}\end{bmatrix}^T G\begin{bmatrix}\mathbf{r}_j(t_j - t_{j0})\mathbf{r}_0\\(t_j - t_{j0})\mathbf{v}_P^{O_j}\end{bmatrix}\equiv$$

$$\equiv\begin{bmatrix}\mathbf{r}_j(t_j - t_{j0})\mathbf{r}_0\\(t_j - t_{j0})\mathbf{v}_P^{O_j}\end{bmatrix}^T G\begin{bmatrix}\mathbf{r}_j(t_j - t_{j0})\mathbf{r}_0\\(t_j - t_{j0})\mathbf{v}_P^{O_j}\end{bmatrix}.$$

This proves (6) due to constancy of the value of speed of arbitrary point P: $\bar{\mathbf{v}}_P^{O_{(.)}}(t_{(.)}) \equiv \mathbf{v}_P^{O_{(.)}}(t_{(.)}) = \mathbf{v}_P^{O_{(.)}}$, (.) = i, j, and completes the proof of sufficiency.

Compatibility. We should verify compatibility of the results. The equations (7) – (10) give the next form to the equations (2) – (5):

$$t_i - t_{i0} = \frac{\mu_i}{\mu_j}\frac{(t_j - t_{j0})+\frac{v_{ji}^j}{(c_j^j)^2}\rho_L(t_j;t_{j0})}{1+\frac{v_{ji}^j}{c_j^j}} =$$

$$= \frac{c_j^j}{c_i^i}\frac{(t_j - t_{j0})+\frac{v_{ji}^j}{(c_j^j)^2}\rho_L(t_j;t_{j0})}{1+\frac{v_{ji}^j}{c_j^j}},\ i, j = 1, 2, \ldots, s, \tag{16}$$

$$t_j - t_{j0} = \frac{\mu_j}{\mu_i}\frac{(t_i - t_{i0})-\frac{v_{ji}^i}{(c_i^i)^2}\rho_L(t_i;t_{i0})}{1-\frac{v_{ji}^i}{c_i^i}} =$$

$$= \frac{c_i^i}{c_j^j}\frac{(t_i - t_{i0})-\frac{v_{ji}^i}{(c_i^i)^2}\rho_L(t_i;t_{i0})}{1-\frac{v_{ji}^i}{c_i^i}},\ i, j = 1, 2, \ldots, s, \tag{17}$$

$$\mathbf{r}_i(t_i;t_{i0}) = \frac{\mathbf{r}_j(t_j;t_{j0})+v_{ji}^j(t_j - t_{j0})\mathbf{r}_0}{1+\frac{v_{ji}^j}{v_P^{O_j}}},\ i, j = 1, 2, \ldots, s, \tag{18}$$

$$\mathbf{r}_j(t_j;t_{j0}) = \frac{\mathbf{r}_i(t_i;t_{i0})-v_{ji}^i(t_i - t_{i0})\mathbf{r}_0}{1-\frac{v_{ji}^i}{v_P^{O_i}}},\ i, j = 1, 2, \ldots, s. \tag{19}$$

We replace $(t_j - t_{j0})$ from (17) into (16):

$$t_i - t_{i0} =$$

$$= \frac{\mu_i}{\mu_j}\left[\frac{\mu_j}{\mu_i}\frac{(t_i - t_{i0})-\frac{v_{ji}^i}{(c_i^i)^2}\rho_L(t_i;t_{i0})}{1-\frac{v_{ji}^i}{c_i^i}}+\frac{v_{ji}^j}{(c_j^j)^2}\rho_L(t_j;t_{j0})\right]\bullet$$

$$\bullet\left(1+\frac{v_{ji}^j}{c_j^j}\right)^{-1},$$

and use $\rho_L(t_{(.)};t_{(.)0}) = c_{(.)}^{(.)}(t_{(.)} - t_{(.)0})$ for (.) = i, j,

$$t_i - t_{i0} =$$

$$= \frac{\mu_i}{\mu_j}\left[\frac{\mu_j}{\mu_i}\left(1-\frac{v_{ji}^i}{c_i^i}\right)(t_i - t_{i0})\left(1-\frac{v_{ji}^i}{c_i^i}\right)^{-1}+\frac{v_{ji}^j}{c_j^j}(t_j - t_{j0})\right]\bullet$$

$$\bullet\left(1+\frac{v_{ji}^j}{c_j^j}\right)^{-1} =$$

$$= \left[(t_i - t_{i0}) + \frac{\mu_i}{\mu_j} \frac{v_{ji}^j}{c_j^j} (t_j - t_{j0}) \right] \left(1 + \frac{v_{ji}^j}{c_j^j} \right)^{-1} .$$

(1) permits us to replace $(t_j - t_{j0})$ by $(\mu_j/\mu_i)(t_i - t_{i0})$:

$$(t_i - t_{i0}) = \left[(t_i - t_{i0}) + \frac{\mu_i}{\mu_j} \frac{v_{ji}^j}{c_j^j} \frac{\mu_j}{\mu_i} (t_i - t_{i0}) \right] \left(1 + \frac{v_{ji}^j}{c_j^j} \right)^{-1} =$$

$$= \left(1 + \frac{v_{ji}^j}{c_j^j} \right) (t_i - t_{i0}) \left(1 + \frac{v_{ji}^j}{c_j^j} \right)^{-1} = (t_i - t_{i0}).$$

This proves compatibility between (16) and (17) independently of position and speed of any point P in the space. Hence, their compatibility is complete. We verify their compatibility with their relationships with velocities expressed by:

$(t_{(.)} - t_{(.)0}) = (c_{(.)}^{(.)})^{-1} \rho_L (t_{(.)}; t_{(.)0})$, (.) = i, j,

as follows by utilising (1) applied to (16):

$$t_i - t_{i0} = \frac{\mu_i}{\mu j} \frac{(t_j - t_{j0}) + \dfrac{v_{ji}^j}{\left(c_j^j\right)^2} \rho_L (t_j; t_{j0})}{1 + \dfrac{v_{ji}^j}{c_j^j}} =$$

$$= \frac{\mu_i}{\mu_j} \frac{\dfrac{\mu_j}{\mu_i}(t_i - t_{i0}) + \dfrac{v_{ji}^j}{\left(c_j^j\right)^2} c_j^j \dfrac{\mu_j}{\mu_i} (t_i - t_{i0})}{1 + \dfrac{v_{ji}^j}{c_j^j}} = t_i - t_{i0} .$$

This proves their compatibility. In the same way we prove compatibility of (17) with

$(t_{(.)} - t_{(.)0}) = (c_{(.)}^{(.)})^{-1} \rho_L (t_{(.)}; t_{(.)0})$, (.) = i, j.

In order to verify compatibility of the spatial co-ordinate transformations we replace at first $\mathbf{r}_j (t_j; t_{j0})$ by the right-hand side of (19) into (18), we use $\mathbf{r}_i (t_i; t_{i0}) = \mathbf{v}_P^{O_i, i} (t_i - t_{i0})$ and afterwards we apply (1) and (12):

$$\mathbf{r}_i (t_i; t_{i0}) = \frac{\dfrac{\mathbf{r}_i (t_i; t_{i0}) - v_{ji}^i (t_i - t_{i0}) \mathbf{r}_0}{1 - \dfrac{v_{ji}^i}{v_P^{O_i}}} + v_{ji}^j (t_j - t_{j0}) \mathbf{r}_0}{1 + \dfrac{v_{ji}^j}{v_P^{O_j}}} =$$

$$= \frac{\dfrac{\left(1 - \dfrac{v_{ji}^i}{v_P^{O_i}} \right) \mathbf{r}_i (t_i; t_{i0})}{1 - \dfrac{v_{ji}^i}{v_P^{O_i}}} + v_{ji}^j \dfrac{\mu_j}{\mu_i} \mu_i (t - t_0) \mathbf{r}_0}{1 + \dfrac{v_{ji}^j}{v_P^{O_j}}} =$$

$$= \frac{\mathbf{r}_i (t_i; t_{i0}) + v_{ji}^j \dfrac{\mu_j}{\mu_i} (t_i - t_{i0}) \mathbf{r}_0}{1 + \dfrac{v_{ji}^j}{v_P^{O_j}}} =$$

$$= \frac{\mathbf{r}_i (t_i; t_{i0}) + v_{ji}^j \dfrac{v_P^{O_i}}{v_P^{O_j}} (t_i - t_{i0}) \mathbf{r}_0}{1 + \dfrac{v_{ji}^j}{v_P^{O_j}}} =$$

$$= \frac{\left(1 + \dfrac{v_{ji}^j}{v_P^{O_j}} \right) \mathbf{r}_i (t_i; t_{i0})}{1 + \dfrac{v_{ji}^j}{v_P^{O_j}}} = \mathbf{r}_i (t_i; t_{i0}).$$

This proves compatibility between (18) and (19) for arbitrary point P rather than only for the light signal. Let us check their compatibility with their relationships with the velocities: $\mathbf{r}_{(.)} (t_{(.)}; t_{(.)0}) = v_P^{-O_{(.)},(.)} (t_{(.)} - t_{(.)0}) \mathbf{r}_0$ by using (1), (12), (18) and (19),

$$\mathbf{r}_i (t_i; t_{i0}) = \frac{v_P^{O_j} (t_j - t_{j0}) \mathbf{r}_0 + v_{ji}^j (t_j - t_{j0}) \mathbf{r}_0}{1 + \dfrac{v_{ji}^j}{v_P^{O_j}}} =$$

$$= v_P^{O_j} \frac{\mu_j}{\mu_i} (t_i - t_{i0}) \mathbf{r}_0 = v_P^{O_i} (t_i - t_{i0}) \mathbf{r}_0 = \mathbf{r}_i (t_i; t_{i0}).$$

They are compatible also with their relationships with the velocities. The equations (18) and (19) are completely compatible. Altogether, the transformations (16) through (19) are completely pairwise compatible. We will verify their complete entire compatibility as follows. We replace, respectively, $(t_j - t_{j0})$ and $\mathbf{r}_j(t_j;t_{j0})$ from (17) and (19) into (18):

$$\mathbf{r}_i(t_i;t_{i0}) = \left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right)^{-1} \bullet$$

$$\bullet \left[\mathbf{r}_j(t_j;t_{j0}) + v_{ji}^j \frac{\mu_j}{\mu_i} \frac{(t_i - t_{i0}) - \frac{v_{ji}^i}{(c_i^i)^2}\rho_L(t_i;t_{i0})}{1-\frac{v_{ji}^i}{c_i^i}} \mathbf{r}_0 \right] =$$

$$= \left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right)^{-1} \left[\mathbf{r}_j(t_j;t_{j0}) + v_{ji}^j \frac{\mu_j}{\mu_i} \frac{1-\frac{v_{ji}^i}{c_i^i}}{1-\frac{v_{ji}^i}{c_i^i}} (t_i - t_{i0})\mathbf{r}_0 \right] =$$

$$= \left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right)^{-1} \bullet$$

$$\bullet \left[\frac{\mathbf{r}_i(t_i;t_{i0}) - v_{ji}^i(t_i - t_{i0})\mathbf{r}_0}{1-\frac{v_{ji}^i}{v_P^{O_i}}} + v_{ji}^j \frac{\mu_j}{\mu_i}(t_i - t_{i0})\mathbf{r}_0 \right] =$$

$$= \left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right)^{-1} \left(\frac{1-\frac{v_{ji}^i}{v_P^{O_i}}}{1-\frac{v_{ji}^i}{v_P^{O_i}}} + \frac{\mu_j}{\mu_i}\frac{v_{ji}^j}{v_P^{O_i}} \right) \mathbf{r}_i(t_i;t_{i0}) =$$

$$= \left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right)^{-1} \left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right) \mathbf{r}_i(t_i;t_{i0}) = \mathbf{r}_i(t_i;t_{i0}).$$

This proves complete entire compatibility among the transformations (17) through (19). We prove in the same way complete entire compatibility among (16), (18) and (19). Altogether, the transformations (16) through (19) are completely entirely compatible. Since (16) through (19) are (2) through (5) together with (7) through (10), then they are completely both pairwise and entirely compatible. Q. E. D.

The equations (16) – (19) are beyond the Lorentz-Poincaré-Einstein relativity theory. The former are rational functions of v_{ji}^i / c_i^i, v_{ji}^j / c_j^j, $v_{ji}^i / v_P^{O_i}$ or $v_{ji}^j / v_P^{O_j}$, while the latter are not. The former contain the relative values of light speed, but not the latter. The former are proved completely pairwise and entirely compatible, while the latter are not completely pairwise compatible.

The time co-ordinate transformations (2), (3), (7), (8), i. e. (16), (17), are uniform (over space) because they do not depend on a choice of arbitrary point P.

The preceding theorem is based on the explained meaning of *time* in Newton's sense (Axiom 1), which is reflected by different scale factors in equations (2) through (5). They essentially differ from those by Lorentz. The above result is general. The special case will be examined in the sequel.

The preceding theorem takes the next form in the case the reference frames R_i^n and R_j^n move in parallel with the same speed: $\mathbf{v}_{ji}^i = \mathbf{v}_{ji}^j = \mathbf{0}$. This is important for dynamical systems (S1), (S2).

Corollary 1. If the reference frames R_i^n and R_j^n move with the same speed in the same direction and sense, and if speed of arbitrary point P is arbitrary, then, in order for the scaling coefficients α_i^j, α_j^i, $\alpha_i^j \neq \alpha_j^i$, λ_i^j and λ_j^i, $\lambda_i^j \neq \lambda_j^i$, to be constant and to obey the equations (2) through (5), and for (1) through (5) to imply (6), it is necessary and sufficient that the following equations hold for any choice of $\mu_i \in R^+$**:**

$$\alpha_j^i = \frac{\mu_i}{\mu_j} = \frac{c_j^j}{c_i^i}, \quad \alpha_i^j = \frac{\mu_j}{\mu_i} = \frac{c_i^i}{c_j^j}, \quad \lambda_j^i = \lambda_i^j = 1,$$

and

$$\frac{\mu_j}{\mu_i} = \frac{v_P^{O_i}}{v_P^{O_j}} = \frac{c_i^i}{c_j^j}, \ i,j = 1,2,\ldots,n.$$

The (numerical) value of light speed need not be the same with respect to $I_i = T_i \times R_i^n$ and $I_j = T_j \times R_j^n$ as soon as the time scales of T_i and T_j are different: $\mu_i \neq \mu_j$.

The spatial scaling factors $\lambda_i^j = \lambda_j^i = 1$ because R_i^n and R_j^n move with the same speed.

Problem Solution for the Special Case

In the case the relative values c_i^i and c_j^j of light speed are mutually equal, $c_i^i = c_j^j$, which has been accepted a priori in the relativity theory by following Lorentz and Einstein, then they are denoted by c_{ij} or by c_{ji}, $c_i^i = c_j^j = c_{ij} = c_{ji}$. This designates the same relative value of light speed with respect to I_i and I_j. Then, v_{ji}

denotes that v^{i}_{ji} and v^{j}_{ji} are mutually equal: $v^{i}_{ji} = v^{j}_{ji} = v_{ji} = -v_{ij} = -v^{i}_{ij} = -v^{j}_{ij}$.

Theorem 4. Let time scale coefficients $\mu_i \in R^+$, i = 1, 2, ... , s, be defined by (1). In order for coefficients $\alpha^{j}_{i} \in R^+$, $\alpha^{i}_{j} \in R^+$, $\alpha^{j}_{i} = \alpha^{i}_{j} = \alpha_{ij} = \alpha_{ji}$, $\lambda^{j}_{i} \in R^+$ and $\lambda^{i}_{j} \in R^+$, $\lambda^{j}_{i} = \lambda^{i}_{j} = \lambda_{ij} = \lambda_{ji}$ to be constant and to obey (2) through (5), and for (1) through (5) to imply (6) it is necessary and sufficient that the following equations hold for any choice of time scale coefficient $\mu_i \in R^+$:

$$\bar{v}^{O_i}_{P}(t_i) \equiv \bar{v}^{O_j}_{P}(t_j) \equiv v^{O_i}_{P}(t_i) \equiv v^{O_j}_{P}(t_j) \equiv \equiv c^{i}_{i} = c^{j}_{j} = c_{ij} = c_{ji}, i, j = 1, 2, \ldots, s, \quad (20)$$

$$v^{i}_{ji} = v^{j}_{ji} = v_{ij} = -v_{ji} = -v^{i}_{ij} = -v^{j}_{ij}, i, j = 1, 2, \ldots, s, \quad (21)$$

$$\alpha_{ij} = \lambda_{ij} = \frac{1}{\sqrt{1 - \frac{v^{2}_{ji}}{c^{2}_{ji}}}} = \alpha_{ji} = \lambda_{ji}, i, j = 1, 2, \ldots, s, \quad (22)$$

$$\mu_j = \mu_i \sqrt{\frac{1 - \frac{v_{ji}}{c_{ji}}}{1 + \frac{v_{ji}}{c_{ji}}}}, i, j = 1, 2, \ldots, s, \quad (23)$$

$$A = B. \quad (24)$$

The transformations (2) through (5), (21) through (23) are partially (but not completely) both pairwise and entirely compatible.

The proof of this theorem is very long. It is therefore omitted due to the space limitation. It is along the same lines as the proof of Theorem 2 in [17].

The equations (22) generalise the Lorentz formulas for α and λ, (L1), (L3), which have been basic in the relativity theory.

8. Speed Transformations

Since the transformations (2) – (5), (7) – (10), i. e. (16) – (19), are completely compatible only in the general case, then the speed transformations will be presented for that case.

Theorem 5. Let time scale coefficients $\mu_i \in R^+$, i = 1, 2, ... , s, be defined by (1). Let coefficients μ_i, $\alpha^{j}_{i} \in R^+$, $\alpha^{i}_{j} \in R^+$, $\alpha^{j}_{i} \neq \alpha^{i}_{j}$, $\lambda^{j}_{i} \in R^+$ and $\lambda^{i}_{j} \in R^+$, $\lambda^{j}_{i} \neq \lambda^{i}_{j}$, be constant and obey (7) through (10), and let (1) through (5) imply (6). Then speed $\mathbf{v}^{O_i}_{P}$ of arbitrary point P with respect to origin O_i of R^n_i and relative to t_i and speed $\mathbf{v}^{O_j}_{P}$ of the same point P with respect to origin O_j of R^n_j and relative to t_j are interrelated as follows:

$$\mathbf{v}^{O_i}_{P} = \frac{\mu_j}{\mu_i} \frac{\mathbf{v}^{O_j}_{P} + \mathbf{v}^{j}_{ji}}{1 + \frac{v^{j}_{ji}}{v^{O_j}_{P}}} = \frac{\mu_j}{\mu_i} \mathbf{v}^{O_j}_{P} = \frac{c^{i}_{i}}{c^{j}_{j}} \mathbf{v}^{O_j}_{P},$$

i, j = 1, 2, …, s, (31)

$$\mathbf{v}^{O_j}_{P} = \frac{\mu_i}{\mu_j} \frac{\mathbf{v}^{O_i}_{P} - \mathbf{v}^{i}_{ji}}{1 - \frac{v^{i}_{ji}}{v^{O_i}_{P}}} = \frac{\mu_i}{\mu_j} \mathbf{v}^{O_i}_{P} = \frac{c^{j}_{j}}{c^{i}_{i}} \mathbf{v}^{O_i}_{P},$$

i, j = 1, 2, …, s. (32)

The transformations are completely compatible.

Proof. Let all the conditions hold. Hence, Theorem 3 holds. The velocity is defined as:

$$\mathbf{v}^{O_{(.)}}_{P}(t_{(.)}) = \frac{d\mathbf{r}_{(.)}(t_{(.)}; t_{(.)0})}{dt_{(.)}}, (.) = i, j; i, j = 1, 2, \ldots, s. \quad (33)$$

By applying (16) and (18) to the right hand side of the preceding equation and by using $d\rho_L(t_j; t_{j0}) / dt_j = c^{j}_{j}$, $v^{j}_{ji}\mathbf{r}_0 = \boldsymbol{v}^{j}_{ji}$, (1) and (12) we find:

$$\mathbf{v}^{O_i}_{P} = \frac{d\left[\frac{\mathbf{r}_j(t_j; t_{j0}) + v^{j}_{ji}(t_j - t_{j0})\mathbf{r}_0}{1 + \frac{v^{j}_{ji}}{v^{O_j}_{P}}}\right] : dt_j}{d\left[\frac{\mu_i}{\mu_j} \frac{(t_j - t_{j0}) + \frac{v^{j}_{ji}}{(c^{j}_{j})^2}\rho_L(t_j; t_{j0})}{1 + \frac{v^{j}_{ji}}{c^{j}_{j}}}\right] : dt_j} =$$

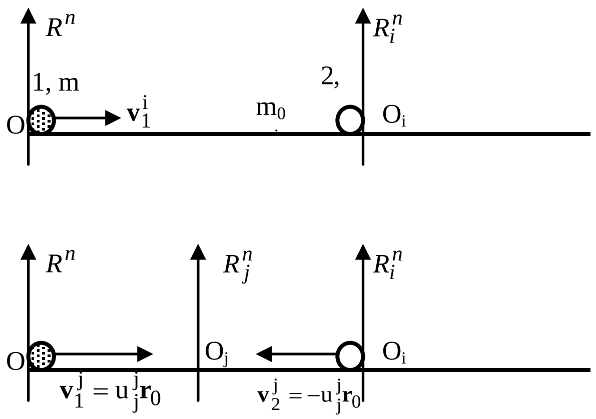
$$= \frac{\mu_j}{\mu_i} \frac{\mathbf{v}^{O_j}_{P} + \mathbf{v}^{j}_{ji}}{1 + \frac{v^{j}_{ji}}{v^{O_j}_{P}}} = \frac{c^{i}_{i}}{c^{j}_{j}} \frac{\mathbf{v}^{O_j}_{P} + \mathbf{v}^{j}_{ji}}{1 + \frac{v^{j}_{ji}}{v^{O_j}_{P}}} = \frac{\mu_j}{\mu_i} \mathbf{v}^{O_j}_{P}.$$

This proves (31). The equation (32) is analogously proved by using (1), (12), (17), (19), (33), $d\rho_L(t_i; t_{i0}) / dt_i = c^{i}_{i}$ and $v^{i}_{ji}\mathbf{r}_0 = \boldsymbol{v}^{i}_{ji}$. Complete compatibility of (31) and (32) is obvious. Q. E. D.

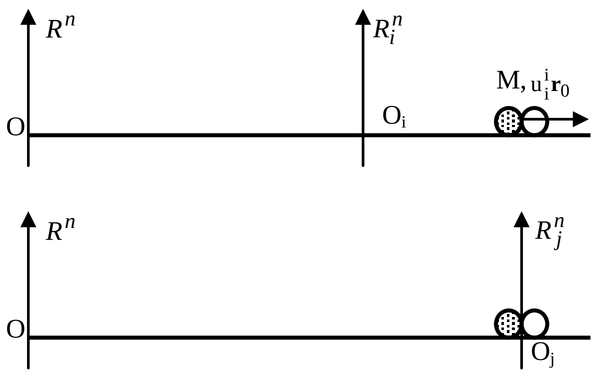
The equations (31) and (32) confirm the equations (12).

Velocity transformation equations (31) and (32) are beyond the Lorentz – Poincaré - Einstein relativity theory. The speed value ratios in the denominators of the right hand side quotients in (31) and (32) do not contain the light speed value, and the speed value $v_P^{O_{(.)}}$ appears in the denominators of the ratios $"v_{ji}^{(.)}/v_P^{(.)}"$, (.) = i, j, rather than in the numerators of the ratio $"vv_P^{O_{(.)}}/c"$ that represents the ratio in the speed transformations resulting from the Lorentz transformations, [21] - [24]. Formulas (31) and (32) contain the light speed values c_i^i and c_j^j relative to I_i and I_j, respectively. However, the light speed value is independent of time scale and/or unit used in the Lorentz – Poincaré - Einstein relativity theory [1] – [5], [21] - [24], [30].

9. MASS AND CO-ORDINATE TRANSFORMATIONS



Positions of two particles and of frames before the collision.



Positions of two particles and of frames after the collision.

Fig. 1.

Three inertial co-ordinate systems (reference frames) are accepted. Co-ordinate system R^n is the reference co-ordinate system for other two co-ordinate systems R_i^n and R_j^n. Particle No. 2 is tied with reference frame R_i^n before its collision with particle No. 1. We accept also that frame R_i^n does not move with respect to R^n. The second particle 2 is evidently at rest relative to R^n before the collision. Frame R_j^n is the "centre-of-mass" frame for particles 1 and 2. Both particles move relative to frame R_j^n. Their velocities relative to frame R_j^n have the same absolute value u_j^j measured in time scale T_j and before the collision, but they have the opposite sense. Their common speed after the collision equals zero relative to reference frame R_j^n. Both particles move evidently with common speed u_j^j with respect to frames R^n and R_i^n after the collision, when the speed is measured relative to T_j. Their common speed value is u_i^i with respect to frames R^n and R_i^n after the collision if the speed is measured relative to T_i. By following D'Inverno {[1]: pp. 45-47}, [14], [17], and by using (12), (31) and (32) we easily prove the next result valid for the general case:

Theorem 7. Let two identical particles 1 and 2 have the same mass m_0 at rest relative to reference integral space $I = T \text{x} R^n$. Let another two integral spaces $I_i = T_i \text{x} R_i^n$ and $I_j = T_j \text{x} R_j^n$ be accepted, $T_i \neq T_j$. Let space coordinate system R_i^n be at rest relative to R^n. Let particle 2 be at rest relative to R^n, and let it be linked with R_i^n before the collision. Let space coordinate system R_j^n be the "centre-of-mass" frame for particles 1 and 2. Then, mass m of particle 1 moving with constant speed $\mathbf{v}_1^i = v_1^i \mathbf{r}_0$ with respect to R^n and R_i^n, and measured relative to T_i is related to its mass m_0 at rest relative to R^n and R_i^n in the general case as follows:

$$m(\mathbf{v}_1^i) = \frac{1}{1 - \frac{v_1^i}{c_i^i}} m_0. \tag{35}$$

Proof is omitted due to the space limitation. It is similar to the proof of Theorem 4 in [17].

This result differs essentially from the formula for mass in the Einstein relativity theory. There is not a square root in the denominator and the quotient is not squared in (35), which exists in the formula of the Einstein relativity theory. Besides, mass depends on the relative value of light speed with respect to $I_i = T_i \mathrm{x} R_i^n$.

10. ENERGY AND COORDINATE TRANSFORMATIONS

The expression for energy E^{i} of a moving body relative to R_i^n now results from the general equation: $\mathrm{E}^{\mathrm{i}} = \mathrm{m}(\mathrm{c}_{\mathrm{i}}^{\mathrm{i}})^2$ and (35).

Theorem 8. Let two identical particles 1 and 2 have the same mass m_0 at rest relative to reference integral space $I=T\mathrm{x}R^n$. Let another two integral spaces $I_i = T_i \mathrm{x} R_i^n$ and $I_j = T_j \mathrm{x} R_j^n$ be accepted, $T_i \neq T_j$. Let space co-ordinate system R_i^n be at rest relative to R^n. Let particle 2 be at rest relative to R^n, and let it be linked with R_i^n before the collision. Let space co-ordinate system R_j^n be the "centre-of-mass" frame for particles 1 and 2. Then, energy E^{i} of particle 1 moving with constant speed $\mathbf{v}_1^{\mathrm{i}} = \mathrm{v}_1^{\mathrm{i}} \mathbf{r}_0$ with respect to R^n and R_i^n, and measured relative to T_i, is related in the general case to its energy $\mathrm{E}_0^{\mathrm{i}} = \mathrm{m}_0 (\mathrm{c}_{\mathrm{i}}^{\mathrm{i}})^2$ at rest relative to R^n and R_i^n by:

$$\mathrm{E}^{\mathrm{i}}(\mathbf{v}_1^{\mathrm{i}}) = \frac{1}{1 - \frac{\mathrm{v}_1^{\mathrm{i}}}{\mathrm{c}_{\mathrm{i}}^{\mathrm{i}}}} \mathrm{m}_0 (\mathrm{c}_{\mathrm{i}}^{\mathrm{i}})^2 = \frac{1}{1 - \frac{\mathrm{v}_1^{\mathrm{i}}}{\mathrm{c}_{\mathrm{i}}^{\mathrm{i}}}} \mathrm{E}_0^{\mathrm{i}}. \tag{36}$$

This result differs crucially from the formula for energy in the Einstein relativity theory. There is not a square root in the denominator and the quotient is not squared in (36), which exists in the formula of the Einstein relativity theory. Besides, energy depends on the relative value of light speed with respect to integral space $I_i = T_i \mathrm{x} R_i^n$.

A matter – energy explanation of the results obtained herein can be given by following Marmet [25].

11. CONCLUSION

Compatibility of the co-ordinate transformations means that application of the inverse transformation to the transformation results in an identity. If this holds for both pairs, the pair of time co-ordinate transformations and the pair of spatial co-ordinate transformations, then the transformations are *pairwise compatible*. If that holds for all four transformations altogether, then they are *entirely compatible*. If compatibility of the transformations is valid if and only if an arbitrary point moves with speed of light, then it is *partial (restrictive) compatibility*. However, compatibility is *complete* if and only if it holds for arbitrary value of speed of an arbitrary point.

Uniformity of the transformations means that the time co-ordinate transformations are independent of a choice of an arbitrary point (of its position and of its speed), i. e. that they hold uniformly over space.

It is proved that the Lorentz transformations are partially (but not completely) pairwise compatible and completely entirely compatible. The speed transformations resulting from the Lorentz transformations are only partially (pairwise and entirely) compatible. These results have opened the compatibility problem in the relativity theory.

It is also shown that the Lorentz transformations are ordinary homogeneous linear transformations with gains depending on speed of an arbitrary point. Consequently, they imply change of time and length units. Hence, numerical values of speeds change in general, which holds also for light speed. This was not taken into account in the Lorentz transformations.

Besides, it is explained why the Lorentz transformations are not uniform. This has raised the problem of uniformity of the transformations.

By referring to the theory of dynamical systems possessing multiple time scales, the time scaling factors are introduced in the relativity theory. They are defined by the equation (1).

The new co-ordinate transformations, which result from the properties of time expressed by Axiom 1 in the Newtonian sense, represent a generalisation of the Lorentz transformations. The time co-ordinate and the spatial co-ordinate scaling coefficients are all permitted to be mutually different a priori. This resulted in their new forms and in the new forms of the corresponding co-ordinate transformations. They are beyond those by Lorentz. They show that relative light speed value should be used in computations, i. e. that the value of light speed depends on the integral space relative to which it is measured. They have implied new formulas for speed, mass and energy. They are proved for the general case and their complete compatibility is verified, as well.

Acceleration is not considered because the demands for uniformity of the transformations and constancy of all the scaling factors resulted in the condition for constancy of the value of speed of an arbitrary point.

It is shown that the time and space co-ordinate transformations in the special case are subjected to stringent restrictions. One of them is that an arbitrary point should move with speed of light. They are uniform, but on the account of their compatibility only under such a restrictive condition, i. e. their compatibility is only partial (pairwise and entirely).

The results establish the basis for a new direction in the relativity theory - *the uniform relativity theory*. It emphasises that *time* is independent of space.

Bibliographical references

[1] R. D'Inverno, *Introducing Einstein's Relativity*, Clarendon Press, 0xford (1995).

[2] A. Einstein, *La théorie de la relativité*, Gauthier-Villars et Cie, Paris (1921).

[3] A. Einstein, *L'éther et la théorie de la relativité.* Gauthier-Villars et Cie, Paris (1921).

[4] A. Einstein, *The meaning of relativity.* Methuen & Co. Ltd., London (1950).

[5] A. Einstein, *Relativity.* University paperbacks, Methuen & Co. Ltd., London (1960).

[6] A. Einstein and L. Infeld, *L'évolution des idées en physique*, Flammarion, Paris (1938).

[7] Lj. T. Grujić, Singular perturbations and large-scale systems, *International Journal of Control*, **29**, 159-169, 1979.

[8] Lj. T. Grujić, Singular perturbations, large-scale systems and asymptotic stability of invariant sets, *International Journal of Systems Science*, **10**, 1323 - 1341, 1979.

[9] Lj. T. Grujić, Sets and singularly perturbed systems, *Systems Science*, **5**, 327 - 338, 1979.

[10] Lj. T. Grujić, *Time and Modeling*, Proc. 1996 IEEE International Conference on Systems, Man and Cybernetics, Beijing, **3** (1996), 2438-2443.

[11] Lj. T. Grujić, *Multiple Time Scale Systems, Time and Modeling*, Pre-prints of the IFAC-IMACS-IFIP Conference on Control of Industrial Systems, Belfort, France, **1**, (1997), 293-298; also in Proceedings of the IFAC Conference Control of Industrial Systems, Pergamon, Elsevier, **1** (1997), 189-194.

[12] Lj. T. Grujić, A. A. Martynyuk and M. Ribbens-Pavella, *Large Scale Systems Stability under Structural and Singular Perturbations*, Springer Verlag, Berlin (1987).

[13] Ly. T. Gruyitch, *Physical continuity and uniqueness principle. Exponential natural tracking control*, Journal: Neural, Parallel and Scientific Computing, Dynamic Publishers, Atlanta, USA, Vol. 6, No. 2, (1998), 143 - 170.

[14] Ly. T. Gruyitch, "Time, Relativity and Physical Principle: Generalisations and Applications", *Proc. VI International Conference Physical Interpretations of Relativity Theory*, London, (1998), 134-170.

[15] Ly. T. Gruyitch, "Time, Relativity and Physical Principle: Generalizations and Applications", Nelinijni Koluvanna, Kiev, Ukraine, **2**, No. 4, (1999), 465-489.

[16] Ly. T. Gruyitch, "Systems Approach to the Relativity Theory Fundaments", *Nonlinear Analysis*, **47**, (2001), 37-48.

[17] Ly. T. Gruyitch, "*Time* and Uniform Relativity Theory Fundaments", *Problems of nonlinear analysis in engineering systems*, No. 2(14), **7**, (2001), 1-28.

[18] R. E. Kalman, P. L. Falb and M. A. Arbib, *Topics in Mathematical System Theory.* Mc Graw Hill, New York (1969).

[19] H. A. Lorentz, *Versuch einer theorie der electricschen und optischen Erscheinungen in bewegten Körpern*, E. K. Brill, Leiden, (1895).

[20] H. A. Lorentz, *Das Relativitätsprinzip*, B. G. Teubner, Leipzig (1914).

[21] H. A. Lorentz, *The theory of Electrons*, B. G. Teubner, Leipzig (1916).

[22] H. A. Lorentz, *Deux mémoires de Henri Poincaré sur la physique mathématique*, Acta mathematica, Zeitscrift, **38** (1921), 294-308.

[23] H. A. Lorentz, A. Einstein and H. Minkowski, *Das Relativitätsprinzip*, B. G. Teubner, Leipzig (1923).

[24] H. A. Lorentz, A. Einstein, H. Minkowski and H. Weyl, *The Principle of Relativity*, Dover Publications, Inc., New York (1952).

[25] P. Marmet, *Einstein's Theory of Relativity versus Classical Mechanics*, Newton Physics Books, 2401 Ogilvie Road, Gloucester, ON, Canada, K1J 7N4, 1997.

[26] E. F. Mishchenko, Yu. S. Kolesov, A. Yu. Kolesov, N. Kh. Rozov, *Asymptotic Methods in Singularly Perturbed Systems*, Consultants Bureau, New York (1994).

[27] D. S. Naidu, *Singular Perturbation Methodology in Control Systems*, Peter Peregrinus Ltd., London (1988).

[28] I. Newton, *Mathematical Principles of Natural Philosophy - BOOK I. The Motion of Bodies,* (The first edition 1687), William Benton, Publisher, Encyclopaedia Britannica, Inc., Chicago (1952).

[29] R. E. O'Malley, Jr., *Introduction to Singular Perturbations*, Academic Press, New York (1974).

[30] I. Prigogine, *Etude Thermodynamique des Phénomènes Irréversibles*, Dunod, Paris (1947).

[31] I. Prigogine, *Non - Equilibrium Statistical Mechanics*, Interscience Publishers, John Wiley & Sons, New York (1962).

[32] D. R. Smith, *Singular-perturbation theory*, Cambridge University Press, Cambridge (1985).

[33] A. N. Tikhonov, *Systems of differential equations containing small parameters in the derivatives*, Math. Sbor. (in Russian), **31**, (1952), 576 – 586.