Correct interpretation of the Graneau, Phipps jr, and Roscoe experiment on electrodynamics

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Abstract

An experiment of impulsive electrodynamics [N. Graneau, T. Phipps, and D. Roscoe, Eur. Phys. J., D **15**, 87 (2001)] has been interpreted by the authors as a confirmation of Ampère's law, considering the force exerted on the mobile section as due only to another small section. The integration over all circuit gives zero longitudinal force by both Ampère's and Grassmann's laws. The correct interpretation of the experiment comes from the different air pressures in the two air gaps due to different solid angles for radiation and particle losses during the electrical discharge. Moreover, there is a larger number of ions hitting the bases of the smaller gap because of a larger useful solid angle. Finally, the ions are more trapped in the smaller gap because of a larger number of bounces. This interpretation leads to a better agreement with their experimental results.

I. INTRODUCTION

In a recent, interesting experiment (sketched in Fig. 1) Graneau *et al.* [1] have measured the force on a mobile section of length 55 mm between two air gaps. Their results are reported in Fig. 2 together with their theoretical interpretations. It immediately appears that the line interpolating their experimental results is strongly different from the continuous line representing their theoretical prediction. The latter has been obtained by Ampère's law applied to the mobile section and section 1 of Fig. 1. Strangely enough, Graneau *et al.* neglected the contribution due to the rest of the circuit, including the arcs in the two air gaps. The reason is that they are supporters of Ampère's law that predicts longitudinal forces between current elements and they want to confirm it at any cost. For readers' convenience we summarize the 150 years standing electrodynamics controversy regarding the elemental laws that express the force between two current elements $I_1 ds_1$ and $I_2 ds_2$, where I_1 and I_2 are the currents flowing in the elements (or segments) $d\mathbf{s}_1$ and $d\mathbf{s}_2$, respectively. The first historical expression has been given by Ampère and reads

$$\delta^2 \mathbf{F}_{A2} = -\frac{\mu_0 \hat{\mathbf{r}}}{4\pi r^2} I_1 I_2 \left(2\, d\mathbf{s}_1 \cdot d\mathbf{s}_2 - 3\, d\mathbf{s}_1 \cdot \hat{\mathbf{r}}\, d\mathbf{s}_2 \cdot \hat{\mathbf{r}}\right) \;, \tag{1}$$

where

$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1, \qquad \text{and} \qquad \hat{\mathbf{r}} = \mathbf{r}/r \;. \tag{2}$$

The other expression was obtained by Biot and Savart in a particular case, and by Grassmann in the general case, combining Laplace's second law

$$\delta^2 \mathbf{F}_2 = I_2 d\mathbf{s}_2 \times \delta \mathbf{B} \;, \tag{3}$$

with Laplace's first law

$$\delta \mathbf{B} = \frac{\mu_0}{4\pi} I_1 \delta \mathbf{s}_1 \times \frac{\hat{\mathbf{r}}}{r^2} \;, \tag{4}$$

thus deriving

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$$\delta^2 \mathbf{F}_{G2} = \frac{\mu_0}{4\pi r^2} I_1 I_2 \, d\mathbf{s}_2 \times (d\mathbf{s}_1 \times \hat{\mathbf{r}}) \; . \tag{5}$$

While Eq. (1) is symmetric in $d\mathbf{s}_1$ and $d\mathbf{s}_2$, thus being in agreement with the action and reaction principle, Eq. (5) is not symmetric and therefore does not comply with Newton's third principle. That is why Maxwell himself wrote, in his famous treatise, that "Ampère's expression will remain the queen of all the others". However, a current element cannot be a steady-state system. Consequently, the electromagnetic (e.m.) momentum changes so that the action and the reaction relevant to the forces exerted on the current elements must be violated. To disregard the impulse-momentum balance (including the e.m. one not known to Maxwell) is the main cause of all the criticisms raised against Grassmann's Eq. (5). On the other hand, some people consider as meaningless the discussion on the elementary laws just because a current element cannot be a steady-state system. But we show later that a current element is the superposition of equal and opposite point-like electric charges in relative motion so that the controversy can be solved.

When it is, for the unit vectors, $d\hat{\mathbf{s}}_1 = d\hat{\mathbf{s}}_2 = \hat{\mathbf{r}}$ we derive from Ampère expression (1)

$$\delta^2 \mathbf{F}_{A2} = \frac{\mu_0}{4\pi \, r^2} \hat{\mathbf{r}} \, I_1 I_2 \, ds_1 ds_2 \; , \tag{6}$$

while Grassmann's Eq. (5) gives $\delta^2 \mathbf{F}_{G2} = 0$. The presence of the longitudinal force (6) induced some people to think, one century ago, that the results derivable from Eqs. (1) and (5) were different even for closed circuits. But it was soon proved theoretically that in these cases the results were equal. Then some authors thought that different results could be obtained for the force exerted on a part of a circuit and due to the whole, same circuit. Two of them [2,3] performed somewhat similar experiments claimed to be in agreement with the predictions of Eq. (1) and in disagreement with those of Eq. (5). But two of us [4] have demonstrated that the two expressions give the same results even for the force exerted on a part of a circuit. Had the two mentioned experiments [2,3] been performed in a correct way they would have disproved the predictions of both Eqs. (1) and (5). A more careful experiment performed by some of us [5] gave results in agreement with standard electrodynamics. In the same paper [5] it was pointed out that the Pappas [2] experiment was unreliable for two reasons: (i) the existence of sharp angles, which imply a strong force not considered by Pappas [2], and (ii) the use of pulsed currents whose durations depend on the manual technique of the experimenter. The second experiment [3] is affected by the strong force, comparable to the useful one on the rest of the circuit, due to the electrical connection, and not considered by Phipps [3]. In fact, if the current is the same, the force is the same for circuits of different sizes but similar in shape. Since the two electrical connections used by Phipps [3] are just similar to the main circuit, the total force is three times that acting on the main circuit. For further discussion on the experiment of Ref. [5], see Refs. [6–8].

At this point, two of the main supporters of Ampère's expression (1), P. Graneau and N. Graneau [9], agree with the results of Refs. [4] and [5], i.e., that both theory [4] and experiment [5] show that the force exerted on a part of a circuit and due to the whole circuit confirms both Eq. (1) and Eq. (5), but they state that the measure of internal stresses would discriminate between them. However, internal reactions are defined as the stresses one must apply to the cross-section of an element out from the body (the circuit in our case) to keep it in equilibrium, by mechanically, but not electrically, isolating any small section of a circuit.

P. Graneau [10] states that experiments favour the Lorentz force

$$\delta \mathbf{F} = \delta q \mathbf{v} \times \mathbf{B} \tag{7}$$

in electron guns (i.e., on free electrons) and Ampère's law on current elements. In fact, Eq. (3) can be derived from Eq. (7) that is experimentally proved up to nine significant figures in mass spectrometers. The conduction current in a wire element is due to electrons moving at a speed $v = 400$ km/s and scattering against the ion lattice. The acceleration due to a small electric field $\mathbf{E}$ inside the wire produces an almost imperceptible bending of the electron trajectory along their free flights between two subsequent scatterings. The average speed of n electrons is

$$\langle \mathbf{v} \rangle = \frac{1}{n} \sum_{i=1}^{n} \mathbf{v}_i \tag{8}$$

and, in a wire in which a high current density is flowing, $\langle \mathbf{v} \rangle$ is of the order of 0.01 m/s. The resultant force on a wire element of length $\delta \mathbf{s}$ and cross-section S immersed in a magnetic field $\mathbf{B}$ is, according to Eqs. (7) and (8)

$$\delta \mathbf{F} = \sum_{i=1}^{n} e \mathbf{v}_i \times \mathbf{B} = en \langle \mathbf{v} \rangle \times \mathbf{B} \; , \tag{9}$$

where e is the electron charge. Now it is $n = N \, \mathbf{S} \cdot \delta \mathbf{s}$ where N is the numerical concentration and $\mathbf{S}$ the oriented, vector cross-section, so that, being $\hat{\delta \mathbf{s}} = \delta \mathbf{s} / \delta s = \langle \hat{\mathbf{v}} \rangle = \langle \mathbf{v} \rangle / |\mathbf{v}|$, we can write

$$e\,n\,\langle \mathbf{v}\rangle = e\,N\,\mathbf{S}\cdot\delta\mathbf{s}\langle \mathbf{v}\rangle = e\,N\,\langle \mathbf{v}\rangle\cdot\mathbf{S}\,\delta\mathbf{s} = \mathbf{j}\cdot\mathbf{S}\,\delta\mathbf{s} = I\,\delta\mathbf{s} \tag{10}$$

where $\mathbf{j} = e\,N\,\langle \mathbf{v}\rangle = \rho\,\langle \mathbf{v}\rangle$ is the current density and $\rho = e\,N$ the charge density of the electrons. Substituting Eq. (10) into Eq. (9) we obtain Eq. (3), i.e., Laplace's second law that is therefore equivalent, or derivable, from the Lorentz law, Eq. (7). Consequently, it is not possible to state that experiments favour Eq. (7) for free electrons and Ampère's law for current elements.

Moreover, if the e.m. forces on each element and on each electric charge composing the current element and due to the whole circuit are equally given by means of both Eqs.(1) and (5), even the internal stresses must be equal. Actually, denoting t^{ij} (with $i,j = 1,2,3$) the stress tensor of rational mechanics for continua, and considering as positive the tensile stresses, the statics equations of continua

$$\sum_{j=1}^{3} \frac{\partial t^{ij}}{\partial x^j} = -f^i \ , \tag{11}$$

predict the same stresses if the external forces f^i per unit volume are the same. In fact, it is

$$f^i = \frac{\delta F^i}{\delta V} = \frac{\delta F^i}{\delta\mathbf{S}\cdot\delta\mathbf{s}} \tag{12}$$

where δV is a volume element of length $\delta\mathbf{s}$ and cross-section $\delta\mathbf{S}$, and the force component δF^i on the considered element and due to the whole circuit is equally given by Eqs. (1) and (5). All the qualitative considerations of Ref. [11] are useless.

It is therefore clear that the application of either Ampère or Grassmann laws to the whole circuit of Fig.1 gives the same result. In particular there is no longitudinal force on the mobile element but only a small transversal force. As before said, N. Graneau recognized this conclusion in Ref. [9] so that his interpretation in Ref. [1] (perhaps influenced by Phipps and Roscoe) is a recrudescence of his initial error. Actually, the application of Grasmann's Eq. (5) requires a double, vector integration, i.e., a sixthfold scalar integration as done in Ref. [5], one around the whole circuit and other over the mobile section (which is a part of the circuit). The same double integration, one over the whole circuit, can be performed for the application of Ampère's Eq. (1). However, since the total action of the mobile part on itself is zero [because Eq. (1) satisfies action and reaction], one can omit the integration over the mobile part. It is therefore wrong to state (as done in Sect. 2 of Ref. [1]) that "the difference between the two laws, which is expressed by an exact differential quantity, no longer disappears since there is no closed loop integral, and the two laws therefore give rise to quite distinct predictions for this single-circuit circumstance". Indeed, the effect of the exact differential on the mobile rod is zero (because of action and reaction) and there is no difference in the predictions, as proved in Refs. [4] and [5]. The longitudinal forces experimented in Ref. [1] must therefore have another interpretation.

After correcting the values of the electrical parameters (given in Ref. [1]) in Sec. II, we give a rough interpretation of the experiment in Sec. III. We conclude in Sec. IV.

II. PARAMETERS OF THE ELECTRICAL CIRCUIT

The circuit used in Ref. [1] and shown in Fig. 1 can be schematized by an inductance L with in series a resistance R, and a capacitance C initially charged with a voltage V_0. The current is therefore

$$I = \frac{V_0\sqrt{C/L}}{\sqrt{1-R^2C/(4L)}}\exp\left(-\frac{Rt}{2L}\right)\sin\left(\frac{t}{\sqrt{LC}}\sqrt{1-\frac{R^2C}{4L}}\right) \ . \tag{13}$$

The minimum capacity used in Ref. [1] is $C_m = 3.34\ \mu$F, the measured angular frequency is the maximum one $\omega_M = 3.4\times 10^5\ \mathrm{s}^{-1}$ so that the inductance must be, if $R^2C/(4L) \ll 1$,

$$L = (C\omega^2)^{-1} = 2.59\ \mu\mathrm{H} \ , \tag{14}$$

instead of their declared value $L = 2.8\mu$H. Their measured time constant τ of the decay (5th column of table I of Ref. [1]) is $\tau = 54.3\ \mu$s. Consequently, we derive from Eq. (13), since $\tau = 2L/R$,

$$R = \frac{2L}{\tau} = 9.54\times 10^{-2}\ \Omega \ , \tag{15}$$

so that

$$R^2 C/(4L) = 2.93 \times 10^{-3} \ . \tag{16}$$

This value is much less than unity, so that the correction to Eq. (14) is in the fourth significant figure.

As derivable from Eq. (13), the amplitude of I for $t \to 0$ is $I_0 \simeq V_0 \sqrt{C/L}$, which has been measured and reported in the third column of table I of Ref. [1]. It is $I_0 = 42.9$ kA, whence

$$V_0 = I_0 \sqrt{L/C} = 37.7 \text{ kV} \ , \tag{17}$$

instead of the value $V_0 = 33$ kV reported in the first column of table I. We confirm the two corrected values (of L and V_0) by the agreement between the values of the dissipated power P calculated in two different ways. In the first way we express P by the Joule dissipated power averaged over half a period T

$$P = R\langle I^2 \rangle_{T/2} \simeq \frac{1}{2} R V_0^2 \frac{C}{L} \exp\left(-\frac{2t}{\tau}\right) = 8.74 \times 10^7 \exp\left(-\frac{2t}{\tau}\right) \text{ W} \ . \tag{18}$$

In the second way we start from the energy stored in C

$$\mathcal{E} = \frac{1}{2} C V_0^2 = 2.37 \times 10^3 \text{ J} \ . \tag{19}$$

Since

$$\mathcal{E} = \int_0^\infty P \, dt = \int_0^\infty P_0 \exp\left(-\frac{2t}{\tau}\right) dt = P_0 \tau/2 \ , \tag{20}$$

we obtain

$$P_0 = 2\mathcal{E}/\tau = \frac{2 \times 2.37 \times 10^3}{5.43 \times 10^{-5}} = 8.73 \times 10^7 \text{ W} \ , \tag{21}$$

in excellent agreement with Eq. (18). If we kept $V_0 = 33$ kV there would be a disagreement with a factor 1.3.

In the case of $C = 6.7$ μF, it is $\omega = 2.5 \times 10^5$ s^{-1}, hence $L = 2.39$ μH; $R_0 = 6.4 \times 10^{-2}$ Ω; $\mathcal{E} = 4.71 \times 10^3$ J, and $P_0 = 1.33 \times 10^8$ W.

III. ROUGH EVALUATION OF THE LONGITUDINAL MECHANICAL FORCES

In this section we present a simple, approximate model to determine the mechanical force acting on the mobile section of the electrical circuit, between the two air gaps.

The current I and the power P reach their maximum values in a short time $\Delta t = T_p/4$, where T_p is the period $2\pi/\omega$ of the damped oscillating current. In this time interval the energy $E = \int_0^{\Delta t} P \, dt$, dissipated mainly in the air within the gaps, increases the temperature and pressure causing a sudden rapid expansion of the gas.

The energy injected per unit volume of each gap is the same, and the molecules, ions, electrons and electromagnetic radiation bounce back and forth while hitting the upper and lower solid walls of the gap. However, according to this model, these particles have more chances to remain confined in the smaller than in the larger gap. Thus, the impulses transfered to the mobile rod by the expansion in the two gaps are different, being relatively larger the impulse from the small gap. It follows that the rod will be subjected to a net upward impulse that pushes it up.

In our model, part of the dissipated energy remains within the volume of the gap and transforms into kinetic energy E_k of the gas particles and the injected energy creates an expanding wave-front approximately in the radial direction from the center of the gap. The other part of the dissipated energy, that we denote by E_{loss}, flows through the lateral surface of the gap and corresponds to radiated power and other forms of work or energy loss through the lateral surface.

If $l = l_1 + l_2 = 20.5$ mm is the total gap of radius $r_g = 2.38$ mm (see Fig. 1), the energy dissipated in gap 1 reads

$$\left(\int_0^{\Delta t} P \, dt \right) \frac{l_1}{l} = E_{k1} + E_{\text{loss}\,1} \ . \tag{22}$$

With C a proportionality constant, we write

$$E_{\mathrm{loss}\,1} = C\left(\int_0^{\Delta t} P\,dt\right)\frac{S_1 l_1}{S_0 l}\ , \tag{23}$$

where $E_{\mathrm{loss}\,1}$ is the energy leaving the gap through the lateral surface $S_1 = 2\pi\, r_g l_1$, while $S_0 = 2\pi\, r_g^2 + 2\pi\, r_g l_1$ is the total boundary surface of gap 1. Since $\Delta t \ll \tau$ [where τ is the decay time constant appearing in Eq. (20)] we may take P_0 as a constant in Δt so that Eq. (23) becomes

$$E_{\mathrm{loss}\,1} = C\,P_0\,\Delta t\,\frac{l_1}{l}\,\frac{l_1}{r_g + l_1}\ . \tag{24}$$

To estimate C, we consider that, on account of the previous considerations, in the limit for $l_1 \to l \to \infty$ the electrical power is essentially completely lost into radiation and other forms of energy through the lateral surface, so that $C \simeq 1$.

The wave-front of the gas possessing the radial energy E_{k1} should reach the surface $\pi\, r_g^2$ of the mobile rod approximately in time Δt. What happens to the gas later is not important in this model because the relevant impulse is transmitted to the rod at the time of impact Δt. We suppose that the wavefront of the radial motion of the mass of gas takes place at a constant speed V, which we take first as an adjustable parameter determined by fitting the experimental data. Thus, we conveniently write $E_{k1} = \Pi_{1l}\, V/2$, where Π_{1l} is the linear radial effective momentum of the expanding gas.

With the above notations, Eq. (22) reads,

$$P_0\,\Delta t\,\frac{l_1}{l} = \frac{1}{2}\Pi_{1l}\,V + P_0\,\Delta t\,\frac{l_1}{l}\,\frac{l_1}{r_g + l_1}\ , \tag{25}$$

so that the effective radial momentum is

$$\Pi_{1l} = \frac{2\,P_0\,\Delta t}{V}\,\frac{l_1}{l}\left(1 - \frac{l_1}{r_g + l_1}\right) = \frac{2\,P_0\,\Delta t}{V}\,\frac{r_g}{l}\,\frac{l_1}{r_g + l_1}\ . \tag{26}$$

The fraction of impulse transmitted to the rod surface $\pi\, r_g^2$ is

$$\int_0^{\Delta t} F_1\,dt = 2\,\Pi_{1l}\,\frac{S_{\mathrm{rod}}}{S_0} = 2\,\Pi_{1l}\left(\frac{\pi\, r_g^2}{2\pi\, r_g^2 + 2\pi\, r_g l_1}\right) = \frac{2\,P_0\,\Delta t}{V}\,\frac{r_g}{l}\,\frac{r_g l_1}{(r_g + l_1)^2}\ . \tag{27}$$

The solid angle S_{rod}/S_0 appearing in Eq. (27) is meaningful only if all the expanding gas is contained in a sphere of radius $l_1/2$ with the centre placed in the centre of the gap. However, when $l_1/2 \ll r_g$, there are about $N = \langle l_c\rangle/l_1$ diameters of these spheres linearly distributed in the average chord $\langle l_c\rangle$. The average is easily calculated as

$$\langle l_c\rangle = \frac{2}{\pi}\int_0^{\pi/2} 2\,r_g\ \sin\alpha\ d\alpha = \frac{4}{\pi}\,r_g\ . \tag{28}$$

When, after a short initial time, the gas density has decreased but temperature and pressure are still high, the molecules of the gas within each sphere may bounce back and forth between the solid walls for about N times the case $l_1/2 \simeq r_g$ before leaving the gap. Thus, the momentum Π_{1l} becomes N times more effective.

Because of this, Π_{1l} in Eq. (27) should be corrected by a factor f_1 that we conveniently write as

$$f_1 = k\,(1 + N) = k\left(1 + \frac{4}{\pi}\,\frac{r_g}{l_1}\right)\ . \tag{29}$$

The constant k is determined by normalizing $f_1 = 1$ when $2\,r_g = l_1$. We find $k = 0.61$, so that

$$f_1 = 0.61\left(1 + \frac{4}{\pi}\,\frac{r_g}{l_1}\right)\ . \tag{30}$$

Similarly, it is

$$f_2 = 0.61\left(1 + \frac{4}{\pi}\,\frac{r_g}{l_2}\right)\ . \tag{31}$$

With $\Pi_{1l} \to f_1\Pi_{1l}$ in Eq. (27), the net upward impulse transmitted to the mobile rod of mass $m = 17.7$ g, is

$$mv = \int F_1\, dt - \int F_2\, dt = \frac{2\, P_0\, \Delta t}{V} \frac{r_g}{l} \Delta^* \ , \tag{32}$$

where

$$\Delta^* = r_g \left[\frac{l_1 f_1}{(r_g + l_1)^2} - \frac{l_2 f_2}{(r_g + l_2)^2} \right] \ . \tag{33}$$

The time interval Δt taken by the expanding wavefront of the exploding gas to reach the mobile rod surface is related to the velocity V. Since V will be determined empirically by data fitting, for convenience we write $\Delta t = r_g/V$.

The height h reached by the mobile rod is predicted to be

$$h = \frac{v^2}{2g} = \frac{1}{2g} \left(\frac{2P_0}{mV^2 l} r_g^2 \Delta^* \right)^2 \ . \tag{34}$$

By comparing the height h of Eq. (34) with empirical data, we determine the parameter V to be 1.8×10^3 m/s. In Fig. 2 we report a comparison between the values of h obtained with the phenomenological Eq. (34) and those obtained empirically in Ref. [1] and reported in Table I.

In conclusion, the geometric factors appearing in the rhs of Eq. (34) takes into account the fact that the energy dissipated in the volume of the gap, with respect the energy lost through the lateral surface, is relatively larger in the small lower gap than in the upper gap. For this reason, the dissipated volume energy produces an expansion with a momentum Π_l that is relatively more effective in the smaller gap, thus producing the upward push of the mobile rod.

During the expansion there is a pressure gradient directed from inside the gap outward. After the expansion the pressure inside reaches an equilibrium with the outer pressure.

In Fig. 2 we report our values of the impulse for different gap values and the theoretical interpolating curve. We see that it interpolates the experimental values much better than the almost straight line of Ref. [1]. The only exception regards the case $l_1 = 0$ since in this case many other factors intervene. First of all, the current at the beginning of the discharge flows through the metallic contact. The contact area is usually small so that it melts and evaporates, thus producing a pressure that detaches a little the mobile section. At this point our mechanism acts but with the decaying tail of the damped current, thus giving a smaller impulse.

IV. CONCLUSIONS

The interpretation of the considered experiment made by the authors [1] is completely wrong, as discussed in the Introduction. Indeed, they consider the action of a single wire element below the smaller gap instead of integrating along the whole circuit, including the current elements in the plasma during the discharge. The integration of Ampère's expression (1) over the whole circuit (for instance, force on $d\mathbf{s}_2$ and integration over $d\mathbf{s}_1$) gives the same result as the integration of Grasmann's expression(5), i.e. a transversal force on $d\mathbf{s}_2$ (with zero longitudinal force). This result is a theorem [4] and has also experimentally been verified [5]. Moreover, the value of the longitudinal force calculated by Graneau et al. [1] depends on the length δs_1 of the considered element in the circuit, since the force is proportional to δs_1, but inversely proportional to the square of the distance r between the centre of δs_1 and the centre of the mobile element between the two gaps. Even if they have chosen δs_1 at best, they succeed to have agreement with their experiment for a single value of the height d_1 of the smallest gap. Their predicted linear behaviour (see Fig. 2) is very far from the curved line joning their experimental values.

The interpretation of the experimental results is given in Sec. III by a simple energetic balance, taking into account the different losses in the two gaps, due to the different solid angles. There is also the longer time of action in the smaller gap due to the more bounces of the hot ions (in the electrical discharge) in the smaller gap.

A more refined treatment implies the solution of a system of five equation in the following five unknowns, namely: 1) air density N, 2) air pressure p, 3) air temperature T, 4) velocity V of the expansion of the air, 5) velocity of the expansion of the discharge channel. The equations are: 1) the equation for a perfect gas $p = NKT$, 2) a polytropic relation $T/T_0 = (N/N_a)^{0.1}$, 3) The Euler equation, 4) the continuity equation, 5) the energetic balance. Equations 1) and 2) are algebraic while 3), 4), and 5) are nonlinear differential equations. Moreover, the collision frequencies and the mean free paths of the electrons and ions in the discharge channel must be evaluated. All this very complicated treatment will be given in a future paper [8] where the agreement with the experimental values is obtained without the phenomenological value $V = 1800 m/s^{-1}$ introduced after Eq. (34). At the same time it will be shown why the rough treatment of Sec. III gives such excellent agreement with the experimental results of Ref. [1].

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l_1	$h3.3$	$h3.3$ren	$h5$	$h5$ren	$h6.7$	$h8.3$	$h8.3$ren	$h10$	$h10$ren
0	2.5	10.31	4.1	7.36	8.9				
0	2.5	10.31	4	7.18	11				
1	3	12.37	10.3	18.49	16				
1			5.8	10.41					
2	1.9	7.83	4.1	7.36	8.4				
2			3.3	5.93	6.6				
3						11.5	7.49		
4					1.3	2.8	1.82	3.3	1.48
4									
5					1				
8								0.6	0.27
10.2								0	0

TABLE I. Experimental values of the heights h [mm] reached by the mobile element vs the length l_1 [mm] of the lower (and smaller) gap. The values are relevant to different values of the capacities indicated after h (for example $h3.3$ corresponds to $C = 3.3\ \mu$F). The renormalized values have been obtained to have a unique plot, having assumed $C_0 = 6.7\ \mu$F as the reference capacity. The renormalized value for a generic C has been obtained as $h_{\text{ren}} = h(C_0/C)^2$.

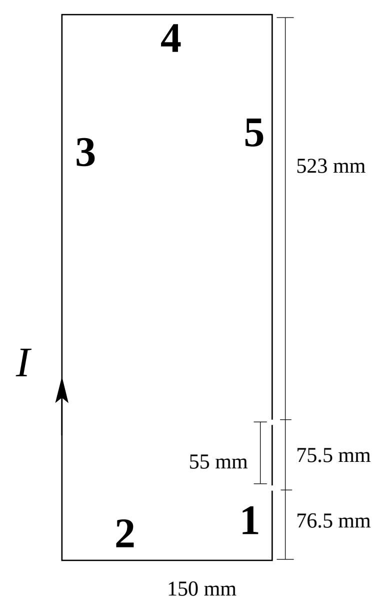

FIG. 1. Sketch of the electrical circuit used by Graneau, Phipps and Roscoe and taken from Fig. 6 of Ref. [1].

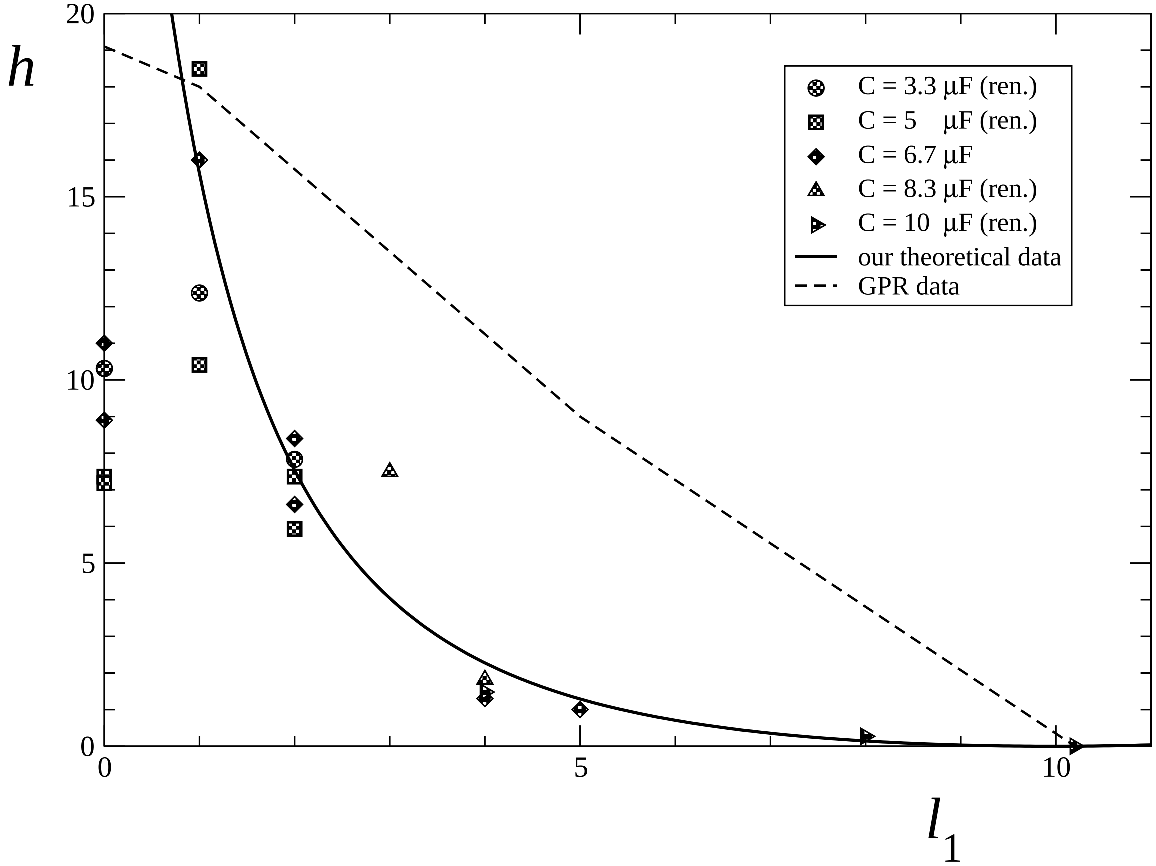

FIG. 2. Experimental points of the heights h [mm] reached by the mobile section vs the length l_1 [mm] of the smaller gap as given in Ref. [1] and renormalized in Table I. The dashed line is the theoretical prediction of Graneau, Phipps and Roscoe (GPR) data in Ref. [1]. The continuous line is our theoretical prediction that fits much better the experimental points.