The spin motion is the origin of special relativity and leads to Bohr's quantization rules and Planck's black body spectrum

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Abstract

A classical treatment (in stochastic electrodynamics with spin) is proposed to obtain Bohr's quantization rules and Planck's spectrum. The relevant relaxation times turn out to be of the order of nanoseconds. It is also explained why narrow spectral lines can be predicted in classical physics, together with the proportionality between frequency and radiated energy in a decay, without the concept of photon.

I. INTRODUCTION

Any attempt to derive some results of quantum mechanics (QM) from classical mechanics is interesting and worthwhile. This is the case of the recent paper by Carati and Galgani [1] where an analogue of Planck's formula is derived from classical statistical mechanics far from equilibrium. The equilibrium distribution function of the power spectral density $\rho(\omega)$ of a black body should therefore be that of Rayleigh and Jeans. The idea of Ref. [1] is that the low frequencies are rapidly excited because of collisions and they therefore rather quickly reach equilibrium. Actually, for $\hbar\omega \ll kT$ the Rayleigh-Jeans formula is very close to that of Planck. In high frequencies the relaxation times increase exponentially with ω, with the consequence that an analogue of the Planck's formula is obtained. Another consequence of Ref. [1] is that their spectrum $\rho_{C-G}(\omega)$ should gradually vary in time as it approaches the Rayleigh-Jeans spectrum ρ_{R-J}. This variation should be measurable in correspondence with the drastic departure of ρ_{C-G} from the Planck ρ_P, roughly starting from $\omega \simeq kT/\hbar$. An experimental apparatus of the kind used in the COBE satellite, able to measure the whole $\rho(\omega)$ in a few seconds, should detect the variation by repeating the measurements in the ω region $0.3 < \hbar\omega/kT < 1$. To address the measurement in the most convenient ω range it would be very useful if Carati and Galgani gave some order of magnitude for the relaxation times as functions of ω for a given T value.

There are, however, two criticisms against the work of Ref. [1]. The COBE satellite has found a perfect ρ_P for the cosmic background microwave radiation (CBMR) at 2.73 K in spite of the fact the CBMR has had the whole age of the universe to approach equilibrium, at least for $1 < \hbar\omega/kT < 2$. Moreover, astrophysical observations are in agreement with ρ_P starting from an age of $\sim 2 \times 10^9$ yr. The electromagnetic spectrum could significantly differ from ρ_P in the first minutes of the existence of the universe, corresponding to the nucleosynthesis era. But, contrary to the predictions of Ref. [1], the differences should regard low frequencies, less than the plasma frequency [2].

The second criticism regards rapid variations of temperatures, since even if Carati and Galgani do not at present succeed in predicting values for the relaxation times, the latter must be symmetric for excitation and deexcitation, i.e. for heating and cooling down. According to Carati and Galgani, the relaxation times increase exponentially with the angular frequency ω and, for $\omega \geq \omega_M$,where $\omega_M \simeq 2.82kT_1/\hbar$ is the ω value at which $\rho_P(\omega)$ is maximum at T_1, they should be very long. Consequently, the predicted $\rho_{C-G}(\omega, T)$ should be almost insensitive to rapid variations of temperature T for $\omega \geq \omega_M$. Let us consider a cavity with a small hole (that approximates rather well with a black body) at a temperature T_1. The relevant observed spectrum is $\rho_P(\omega, T_1)$. It is easy to cool down the cavity in a minute to a new $T_2 < T_1$ (for instance $T_2 \simeq T_1/2$), for which the new $\rho_{C-G}(\omega, T_2)$ should be practically equal to ρ_{R-J} for $\omega \ll kT_2/\hbar$, but (see Fig. 1) $\rho_{C-G}[\omega_M(T_1), T_2] \simeq \rho_{C-G}[\omega_M(T_1), T_1] \gg \rho_P[\omega_M(T_1), T_2]$.

This suggested experiment should be performed to show that there is no appreciable delay in reaching the Planck distribution. Although not aimed at accurately measuring hypothetical relaxation times, the ovens in which temperatures from 800°C to 3000°C are measured by means of the color [corresponding to the maximum $\rho(\omega)$ value in the Planck distribution] reach their maximum, steady brightness with no appreciable delay with respect to the temperature measured by a thermocouple (which is sensitive to the low frequency part of the Planck distribution, i.e., the one common to the Rayleigh-Jeans distribution). It is also a common observation that when we turn on an incandescent lamp, its brightness, related to the spectrum of the emitted light, reaches its steady value in a fraction of a second. The Carati-Galgani theory is therefore valid and correct only if a single temperature is considered (without transitories).

This drastic criticism is not at all against any attempt to derive ρ_P from classical physics. First of all, Carati and Galgani can improve their treatment and overcome the above difficulty. Moreover, all the criticisms raised against classical physics in textbooks of quantum mechanics are actually against an incomplete classical physics. The latter, in fact, requires that all the particles of the universe, having an accelerated motion, radiate so that there is a ubiquitous, stochastic electromagnetic field [3]. Assuming that the zitterbewegung, or spin motion, is the realistic version of the solution for a free electron derivable from the Dirac equation according to Barut and Zanghi [4], it turns out that the spin radiation is larger than 10^{12} times the atomic radiation[1]. Moreover, in our expanding universe, the spin motion, which is almost monochromatic, brings about [5] a power spectral density $\rho_{ZPF}(\omega)$ for the ubiquitous, stochastic e.m. field proportional to ω^3 up to and including the spin frequency ω_s

$$\rho_{ZPF}(\omega) = A\omega^3\theta(\omega_s - \omega), \tag{1}$$

where $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x < 0$. For $\omega < \omega_s$ the spectrum (1) is Lorentz invariant and special relativity can be derived from it since both sizes and atomic frequencies depend on it [5]. The proportionality constant A appearing in Eq.(1) turns out to be expressed in terms of the Hubble constant, the electron charge, the electron average density in the universe, and the speed c of light [5]. Comparing Eq.(1) with the expression of the zero-point field (ZPF) of QED

$$\rho_{QED}(\omega) = \hbar\omega^3(2\pi^2c^3)^{-1}, \tag{2}$$

it is therefore possible to relate $\hbar$ to the above-mentioned cosmological quantities [5]. Moreover, while $\rho_{QED} \to \infty$ for $\omega \to \infty$ (it is one of the divergences of QED), ρ_{ZPF} given by Eq.(1) is truncated at ω_s. The spin motion together with Eq.(1) allows one to derive the Schroedinger equation [5,8,9], and from that: $\rho_P(\omega)$. In this treatment there is no problem of time since equilibrium is reached in a very short time because the $\rho_{ZPF}(\omega)$ increases very strongly with ω.

To show in an elementary way how equilibrium is reached between absorbed and radiated power, in Sec. II we derive Bohr's quantization rules. In Sec. III we obtain the power spectral density of the black body with a superimposed ZPF.

We conclude in Sec. IV.

II. DERIVATION OF BOHR'S QUANTIZATION RULES

The Bohr quantization rules are derived from the balance of radiated power and the power absorbed from the ZPF. The latter must therefore be considered as real and not renormalized. On the contrary, in QED the ZPF is always renormalized for two resons: i) it has an infinite energy density given by the integral over all frequencies ω of Eq.(2); ii) as shown by Einstein, the kinetic energy of a charged particle immersed in a stochastic, electromagnetic (e.m.) field (such as the ZPF) should increase proportionally to time [10] and a free electron in vacuum should become a highly energetic cosmic ray in ten centimeters (as, for example, in the monitor of a PC) [11]. Both these drawbacks are overcome by means of a realistic zitterbewegung. Drawback i) is eliminated because of the finite energy density expressed by Eq.(1). Actually, we must consider all kinds of charged particles which, as a percentage in the universe, are practically electrons and the up and down quarks (in protons and neutrons) whose spin frequencies we denote by ω_u and ω_d, respectively. Consequently, there are three ω^3 ramps, each truncated at the corresponding spin frequency.

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[1] To obtain a completely classical treatment, the spin motion must be derived and not only hypothesized. This is what one of us is trying to do by means of a new model of particles and fields mentioned in Ref. [5] and using the recently found laws of refraction in moving media [6]. See also Ref. [7].

Finally, there are the quantum fluctuations at the Planck frequency ω_P so that the complete power spectral density of the real ZPF can be written as [12]

$$\rho_{\rm ZPF}(\omega) = \frac{\hbar\,\omega^3}{2\,\pi^2\,c^3}\left[\Theta(\omega_e-\omega)+\frac{N_u}{N_e}\left(\frac{\omega_e}{\omega_u}\right)^2\Theta(\omega_u-\omega)\right.$$
$$\left.+\frac{N_d}{N_e}\left(\frac{\omega_e}{\omega_d}\right)^2\Theta(\omega_d-\omega)\right]+\frac{\hbar\,\omega_e^4}{\pi^2\,c^3\,\omega_P}\,\exp\left[-\frac{(\omega-\omega_P)^2}{2\,\omega_P^2}\right]\,, \tag{3}$$

where N_u and N_d are the average number densities in the universe of the up and down quarks, respectively, and N_e the electron density. For $\omega<\omega_e$ the sum of the terms inside the square bracket is practically equal to unity since $\omega_e \ll \omega_u < \omega_d \ll \omega_p$ and Eq. (3) reduces therefore to Eq. (2). The $\rho(\omega)$ expressed by Eq. (3) is finite as well as its integration over ω from 0 to ∞. One of the drawbacks of a nonrenormalized ZPF is therefore overcome.

The second drawback ii), i.e., the huge acceleration an electron should undergo from a real ZPF, is solved because of two reasons. The first one is that $\rho(\omega)$ strongly decreases at the end of any ω^3 ramp and its integration is finite so that the acceleration (due to the Einstein-Boyer-Rueda mechanism [10]) of an electron is partially quenched. The second reason, much more important, is that the equation of motion, in non-relativistic approximation and neglecting the radiation reaction, is still strongly different from Newton's expression $m\,\mathbf{a}=\mathbf{F}$, and turns out to be given by [5]

$$m_*\mathbf{a}=\mathbf{F}\cdot\hat{\mathbf{n}}\hat{\mathbf{n}}\ , \tag{4}$$

where $\hat{\mathbf{n}}$ is the spin axis unit vector and m_* the inertial mass when $\hat{\mathbf{F}}=\hat{\mathbf{n}}$ (indeed, in this case Eq. (4) reduces to $m_*\mathbf{a}=\mathbf{F}$). When $\mathbf{F}\cdot\hat{\mathbf{n}}=0$ there is no net acceleration for the center of the spin revolution since the curvature of the electron trajectory increases during half circle and decreases in the subsequent half circle in such a way that no net effect remains. It appears as unusual that an electron accelerates parallelly to $\hat{\mathbf{n}}$ and not parallelly to the external field $\mathbf{E}$. However, $\hat{\mathbf{n}}$ precesses around $\mathbf{E}$ so that, on an average, $\langle\mathbf{a}\rangle$ is parallel to $\mathbf{E}$.

The precession of the spin axis $\hat{\mathbf{n}}$ appears very strange to people accustomed to QM where Pauli's famous definition is considered to hold:"spin is a classically unexplainable two valuedness". Our interpretation is that QM gives the <u>average</u> value for spin whose $\hat{\mathbf{n}}$ should be uniformly distributed in a half sphere with symmetry axis parallel to the magnetic field $\mathbf{B}$ for spin up and antiparallel to $\mathbf{B}$ for spin down. Actually, this interpretation explains [5] the reduction of the instantaneous spin angular momentum $\hbar$ to $\hbar/2$, the doubling of its gyromagnetic ratio, including the radiative correction if the electron diffusion along a spin revolution is evaluated by the free diffusion calculated in Ref. [3]. A uniform probability of distribution for $\hat{\mathbf{n}}$ implies, for any direction and in mathematical form, a zero measure. A zero measure (for spin lying on any directions) is just what Pitowski [13] assumed to obtain in a local way the same results as QM for the Einstein, Podolsky and Rosen "paradox". With a distribution of $\hat{\mathbf{n}}$, as also assumed by Barut [14], it is therefore possible to violate Bell inequalities by a local "classical" theory. Even the paramagnetic resonance can be explained classically as a precession of $\hat{\mathbf{n}}$.

After these clarifications, let us show why Eq. (4) drastically quenches the Einstein-Boyer-Rueda mechanism of acceleration that, otherwise, would also lead to the spontaneous self-ionization of atoms [15]. When an e.m. wave impinges on an electron, the electric field $\mathbf{E}$ produces a velocity variation $\delta\mathbf{v}$ parallel to $\hat{\mathbf{n}}$ [because of Eq. (4)]. Then the Lorentz force $\delta\mathbf{F}_L=e\delta\mathbf{v}\times\mathbf{B}$, due to the magnetic field associated to $\mathbf{E}$ in any train of e.m. waves, is perpendicular to $\delta\mathbf{v}$, hence to $\hat{\mathbf{n}}$ that is parallel to $\delta\mathbf{v}$ if no change of $\hat{\mathbf{n}}$ occurred during a half period of the e.m. wave. Now a $\delta\mathbf{F}_L$ perpendicular to $\hat{\mathbf{n}}$ produces no net acceleration of the center of the spin orbit according to Eq. (4). What said can be summarized as

$$\delta\mathbf{F}_L\cdot\hat{\mathbf{n}}=e\delta\mathbf{v}\times\mathbf{B}\cdot\hat{\mathbf{n}}\propto e\hat{\mathbf{n}}\times\mathbf{B}\cdot\hat{\mathbf{n}}=0\ . \tag{5}$$

Only when $\hat{\mathbf{n}}$ precesses, $\delta\mathbf{F}_L$ differs from zero because $\delta\mathbf{v}$ is no longer parallel to $\hat{\mathbf{n}}$. The Einstein-Boyer-Rueda mechanism of acceleration of an electron in the ZPF is strongly reduced since an electron with spin is only sensitive to the ZPF frequency when the latter is roughly equal to the electron's precession frequency. This mechanism can still justify the existence of the most energetic cosmic rays but the acceleration requires some thousand (or million) light years in the intergalactic space. The $\hat{\mathbf{n}}$ precession is larger the more intense is $\mathbf{E}$, hence the electron confinement. This consequence explains both the uncertainty principle and the diffraction of single electrons passing one or two slits [16].

At this point, with a real, nonrenormalized ZPF the Bohr conditions are derived by equating the radiated power P_r to the average power P_a absorbed from the ZPF. Now, P_r is given by the Larmor formula expressed by (we neglect the relativistic corrections that are very small for a hydrogen atom)

$$P_r = \frac{2}{3}\frac{e^2}{c^3}a^2 = \frac{2}{3}\frac{e^2}{c^3}\frac{v^4}{R^2}, \tag{6}$$

where e is the electron charge and the last step is valid for the uniform circular motion (as is in Bohr's theory). The average power P_a absorbed from a stochastic field by a harmonic oscillator of mass m and proper angular frequency ω_0 is given by

$$P_a = \frac{2}{3}\frac{e^2}{m}\pi^2\rho(\omega_0) \ , \tag{7}$$

where $\rho(\omega_0)$ is the power spectral density of the stochastic field calculated in correspondence of ω_0. A circular orbit is obtained with two harmonic oscillators so that, equating Eq. (7) to Eq. (6) we obtain $mv_1R_1 = \hbar$, which is Bohr's condition for the fundamental state. Actually, a pure circular motion cannot exist because of the random action of the ZPF. Although the ZPF little modifies an orbit during a single revolution, in the long run the orbit becomes elliptical with slow variations of eccentricity, major axis, and even of the orbit plane. The Bohr radius R_1 is only the most probable value to find the electron in a thin spherical shell around R_1.

Let us now consider the case that a ZPF fluctuation has produced a small variation of an initially orbit, transforming it into an elliptical orbit represented, in polar form, by

$$r = R(1 - \epsilon\cos\theta)^{-1}. \tag{8}$$

If the eccentricity $\epsilon \ll 1$, then Eq. (8) is equivalent, to within second order terms in ϵ, to

$$\begin{aligned} x &= R(\cos\theta + \epsilon\cos 2\theta), \\ y &= R(\sin\theta + \epsilon\sin 2\theta). \end{aligned} \tag{9}$$

In fact,

$$r = (x^2 + y^2)^{1/2} = R[1 + \epsilon^2 + 2\epsilon\cos\theta]^{1/2} \simeq R(1 + \epsilon\cos\theta) \simeq R(1 - \epsilon\cos\theta)^{-1}, \tag{10}$$

In a Keplerian motion there is conservation of angular momentum Γ so that, with the use of Eq. (8),

$$\omega = \frac{\Gamma}{mr^2} = \frac{\Gamma}{mR^2}(1 - \epsilon\cos\theta)^2 \simeq \omega_0(1 - 2\epsilon\cos\theta) \ . \tag{11}$$

Since $\theta = \int \omega dt$, we can solve Eq.(11) in an iterative way and we derive to first order

$$\theta \simeq \omega_0 t - 2\epsilon\sin\omega_0 t. \tag{12}$$

Substituting Eq.(12) into Eq.(8) we obtain the trajectory as a function of t. Since $\cos\theta$ is multiplied by ϵ and we are limiting our calculation to first order in $\epsilon \ll 1$, we can neglet the second term at the r.h.s. of Eq.(12). With $\theta \simeq \omega_0 t$, the first terms at the r.h.s. of Eq.(9) represent the main circular motion, considered as a deferent, on which there is a second circular motion (ϵ time the first one) considered as an epicycle. Since $\epsilon \ll 1$, the motion is practically circular so that Eq.(6) remains unaltered. What drastically changes is the absorbed power since now there are four harmonic oscillators. The epicycle rotates with angular velocity 2ω respect to the laboratory. However, since the epicycle rotates around a point that in turn rotates with ω, what is effective for the absorbed power is the relative frequency $2\omega - \omega = \omega$, i.e. the same frequency as the one of the deferent. Consequently, the absorbed power can be written as

$$P_a = 2n\frac{2}{3}\frac{e^2}{m}\pi^2\rho(\omega_0) \ , \tag{13}$$

with $n = 2$ (corresponding to 2 plane motions, hence 4 harmonic oscillators with the same ω). The Bohr orbit corresponds to $n = 1$, i.e. to 1 plane motion, hence to 2 harmonic oscillators.

More in general, a periodic elliptical motion can be expanded in Fourier series

$$\begin{aligned} x &= R[\cos\theta + \sum_{n=2}^{\infty}\epsilon_n\cos(n\theta + \varphi_n)], \\ y &= R[\sin\theta + \sum_{n=2}^{\infty}\epsilon_n\sin(n\theta + \varphi_n)], \end{aligned} \tag{14}$$

where φ_n are constant phases.

Each additional term corresponds to a circular motion which, being relevant to the same electron, is epicycloidal. If we limit to $n = 3$, we have an epicycle rotating with angular velocity 3ω (in the approximation $\theta = \omega t$) on another epicycle rotating with 2ω, in turn rotating on the deferent with ω. The relative, effective frequencies for absorption from the ZPF are $3\omega - 2\omega = 2\omega - \omega = \omega$, i.e. the same. Being $\epsilon_i \ll 1$, that is why P_r is practically given by Eq.(6) while P_a by Eq.(13).

If $\rho(\omega_0)$ is given by Eq.(3) [which reduces to Eq. (2) since atomic frequencies are much lower than ω_e], equating Eq.(13) to Eq.(6) with v_n and R_n (for v and R) and $v_n = \omega_n R_n$, we obtain

$$m v_n R_n = n\hbar, \tag{15}$$

i.e., Bohr's condition for quantization. We see that here we have the same ω_0 for radiation and absorption and not completely different frequencies for excitations and oscillators as in Carati-Galgani's theory [1]. That is why we obtain relaxation times of the order of some millions of atomic revolutions, corresponding to $\sim 10^{-10} - 10^{-9}$ s.

The simplicity of the calculations partially comes from the elimination of the random impulses due to the ZPF radiation pressure at high frequency, as shown by Eq.(5). In pure stochastic electrodynamics (pure SED without spin) the random impulses of the high frequency ZPF are the cause of the self-ionization of atoms [15], a result that destroyed pure SED. On the contrary, introduction of Eq. (4) limits the random impulses due to the radiation pressure to the precession frequency caused by the spin-orbit coupling. These much lower impulses have the effect of producing many oscillations between two states (two Bohr orbits in our case) when the atom is excited and begins to decay. Roughly, any oscillation between two states includes thousands of revolutions. Since the average decay corresponds to one million revolutions, there are thousands of oscillations. For one oscillation the spectral density has a mild maximum in correspondence to the weighted (with the net radiated power) average frequency $\langle\omega\rangle$ of radiation. For n oscillations the spectrum increases proportionally to n^2 for $\omega = \langle\omega\rangle$, and proportionally to n for all other frequencies that are not observable because compensated by the absorbed frequencies from the ZPF. That is why narrow lines are observed and the effective frequency for any transition is its $\langle\omega\rangle$.

Let us examine the transition between two states. If the radius R of the orbit changes very slowly we may consider it in quasi equilibrium so that

$$\frac{e^2}{R^2} = m\frac{v^2}{R}\ , \qquad \text{i.e.,} \qquad v = \frac{e}{(mR)^{1/2}}\ . \tag{16}$$

Substituting Eq.(16) into Eqs.(6) and (13) we obtain

$$P_r = 2e^6(3c^3m^2R^4)^{-1}, \tag{17}$$

and

$$P_a = 2ne^5\hbar(3c^3m^{5/2}R^{9/2})^{-1}. \tag{18}$$

With a given n, when $P_r = P_a$ we have stable equilibrium for a given $R_0(n)$ as an <u>average</u> effect. Actually, if $R_0 = R_0(n)(1+\xi)$ with $\xi > 0$ it is $P_r > P_a$ and R decreases. If $\xi < 0$, it is $P_r < P_a$ and R increases (see Fig.2). There are however the fluctuations of the ZPF (beside its average effect), which can easily destroy the small amplitude $\epsilon_n R$ of one of the epicyclic motion expressed by Eq.(14). The absorbed power loses two armonic oscillators and, as clearly seen in Fig.2, the radiated power is sensitively larger than the absorbed's and the electron motion becomes, on an average, a spiral motion towards the lower most probable orbit. The net radiated energy is twice the one of the ZPF corresponding to the net observable weighted average frequency $\langle\omega\rangle$. The factor 2 comes from being two the harmonic oscillators composing a circular (and also an elliptic) motion. Not more than two because the small epicyclic motions contribute to absorption but not to radiation. Now the ZPF spectrum can be written [see Eqs. (1) and (2)]

$$\rho_{ZPF}(\omega) = \frac{\omega^2}{\pi^2c^3}\frac{\hbar\omega}{2}\theta(\omega_s - \omega), \tag{19}$$

thus having an energy $\hbar\omega/2$ for each normal mode (for $\omega < \omega_s$), since ω^2/π^2c^3 represents the mode density. The doubling gives therefore the quantum relation for the energy E_r of the net, observable train of e.m. waves radiated because of an atomic transition

$$E_r = \hbar\langle\omega\rangle, \tag{20}$$

which is classically explainable as the doubling of the energy per normal mode of the stochastic electromagnetic field whose power spectral density is given by Eq.(19), without invoking the second quantization implying the concept of "photon". If, because of the random impulses due to the ZPF radiation pressure at the electron precession frequency, there are N oscillations between R_{n2} and R_{n1}, the total net radiated energy $E_{\text{net}} = \int dt(P_r - P_a)$ is independent of N and only depends on the initial and final states. The first principle of thermodynamics requires that E_{net} be equal to the energy difference between the two energy states (having the most probable corresponding R_n's), thus leading to the Bohr assumption

$$\hbar\langle\omega\rangle = E(n_1) - E(n_2). \tag{21}$$

One could ask why a strong fluctuation of the ZPF in the same direction of the radiation damping cannot destroy even the fundamental state. There are two reasons. The first is that, in an excited state, the ZPF can easily destroy the small modification, during each orbit, of the other oscillations with the same frequency (with respect to the main motion). As soon as one small additional epicycle is destroyed, the average absorbed power suddenly decreases and its value jumps from the curve corresponding to a given n to another with $n-1$, as shown in Fig.2. When $n = 1$ there is no possible other reduction. However one could think of a rare, persistent "negative" (i.e. in the same direction of the radiation damping) fluctuation of the ZPF such as to reduce to zero any motion of the electron. Here the second, stronger reason intervenes: there is always the spin motion, classically representable as a circular motion at the speed of light of an almost point-like electron (special relativity arises since one refers to the ideal center around which the electron gyrates [5]). Even if the considered strong negative fluctuation can reduce the electron revolution to the spin motion, the latter always remains and subsequent fluctuations ripristinate the average motion. That such reduction can occur is proved by the solution of the Schroedinger equation which gives a maximum for $\psi\psi^*$ in correspondence of the nucleus. Actually, the spin motion with radius R_s (the Compton radius) averages $\psi\psi^*$ in a sphere of radius R_s thus decreasing the absolute value of the electron energy and bringing about the Lamb shift.

To obtain other properties, as the selection rules, one should consider elliptic orbits even with large eccentricity, and also treat three-dimensional problems since, because of the random ZPF, the planes of the orbits precede and both major axes and eccentricities change. For instance, the fundamental state, although having an instantaneous angular momentum $\mathbf{\Gamma}$, in the long run the direction of $\mathbf{\Gamma}$ changes so that $\langle\mathbf{\Gamma}\rangle \simeq 0$. These results require expansions in spherical harmonics to solve the Schroedinger equation which is derived (and not postulated as in QM) in SED with spin [5,8]. It has been also possible to obtain a generalization of the Schroedinger equation containing additional terms [9], which gives very small (additional) variation (to the frequency of the spectral lines) of the order of one hundredth the Lamb shift [16]. Since the ZPF is brought about by the spin motion, potentially all QED is contained and even a new result has been obtained, as the radiative corrections for the free particle [3]. Other two results, not derivable from the Schroedinger equation but from QED and quantum statistics, are derived in SeCe. III, namely the black body spectrum and the decay rates of the excited states.

III. DERIVATION OF PLANCK'S BLACK BODY SPECTRUM

The Planck distribution is also easily derived in the classical stochastic electrodynamics (SED) with spin (that brings about the ZPF spectrum because of the universe expansion [5]). Consider N_1 oscillators, each with energy E_1, and N_2 oscillators, each with $E_2 > E_1$. The transition frequency ν from the lower energy state 1 to higher energy state 2 can be expressed as a proportionality to N_1 and the total absorbed power

$$\nu_{1\to 2} = AN_1 P_{\text{atot}}, \tag{22}$$

where A is a proportionality constant and P_{atot} the algebraic sum of the power P_{aZPF} absorbed from the ZPF, the P_{aT} absorbed from the thermal radiation, and $-P_r$ absorbed from the radiated power (which is a negative absorbed power)

$$P_{atot} = P_{aZPF} + P_{aT} - P_r. \tag{23}$$

On an average, $P_{aZPF} = P_r$ so that we derive from Eqs.(22) and (23)

$$\nu_{1\to 2} = AN_1 P_{aT}. \tag{24}$$

The inverse transition can be written as

$$\nu_{2\to 1} = AN_2 P_{trans}, \tag{25}$$

where the total transition power P_{trans} is now a positive radiated power (or a negative absorbed power) since the transition is from a higher to a lower energy state. The first two terms at the r.h.s. of Eq.(23) are still expressed in the same way (although they act as "negative" fluctuation, or as radiated power) while the third term is now positive

$$P_{trans} = P_{ZPF} + P_T + P_r. \tag{26}$$

Since, on an average, it is always $P_{ZPF} = P_r$, we derive from Eqs.(25) and (26)

$$\nu_{2\to 1} = AN_2(2P_{ZPF} + P_T). \tag{27}$$

Let us express the absorbed powers as in Eq.(7) [or as in Eq.(13), as well] and equate Eq.(24) to Eq.(27). We obtain, remembering that only the frequency ω of transition is observable (for simplicity, here we omit the symbol of average),

$$N_1\rho(\omega, T) = N_2[2\rho_{ZPF} + \rho(\omega, T)]. \tag{28}$$

The first term, inside the square bracket, substitutes the spontaneous decay coefficient introduced by Einstein.

Classically, the number of oscillators in each state is given by the Boltzmann distribution, so that

$$N_1/N_2 = e^{-E_1/kT}/e^{-E_2/kT} = e^{(E_1-E_2)/kT} \ . \tag{29}$$

Substituting Eqs.(19), (21), and (29) into Eq.(28) we derive the Planck distribution, i.e., $\rho(\omega, T) = \rho_P(\omega, T)$. The total power spectral density is the sum of the Planck and ZPF ones,

$$\rho_{\text{total}}(\omega, T) = \rho_{ZPF}(\omega) + \rho_P(\omega, T) = \frac{\hbar\omega^3}{\pi^2 c^3}\left[\frac{1}{2}\theta(\omega_e - \omega) + \frac{1}{e^{\hbar\omega/kT} - 1}\right] \ , \tag{30}$$

where we have introduced only the first ω^3 ramp of the zero-point field spectrum given by Eq.(3). As seen in Fig.3, the Planck contribution little modifies the ZPF even at a temperature $T = 2.3 \times 10^9$ K.

One of the main concern of this paper, in answer to Ref. [1], regards relaxation times. We therefore calculate the decay time between two atomic levels for small temperature, so that the thermal power P_T can be neglected in Eq.(27). Since both the radiation damping and the ZPF cooperate in the decay from a higher to a lower energy state, the power balance is

$$P_r + P_{ZPF} = 2P_r = -\frac{dU}{dt}, \tag{31}$$

where $U = -e^2/2R$ is the thermal energy. We take the expression (17) for the radiated power valid for circular orbits because the average variation of the radius r during a decay is $\sim 10^{-6}R$. The easy integration yields

$$\tau = \Delta t = \frac{3c^3m^2}{8e^4}(R^3_{n+1} - R^3_n), \tag{32}$$

which is half the classical value (neglecting the ZPF), and in agreement with the QED result (the Schroedinger equation, i.e. QM, implies no spontaneous decays). Obviously, since the ZPF is stochastic, the calculated Δt is not the same for all atoms but it represents the time constant of decay of large set of atoms.

The average, weighted period $T_{2\to 1}$ corresponding to a decay from $n = 2$ to $n = 1$, as derived from Eqs.(15) and (21) (i.e. from Bohr theory), is

$$T_{2\to 1} = \frac{8}{3}\frac{2\pi R_1}{\alpha c} \simeq 3.8 \times 10^{-16} \text{ s}. \tag{33}$$

With $n = 1$ we obtain from Eq.(32)

$$\tau = 1.4 \times 10^{-9} \text{ s} \simeq 3.6 \times 10^6 \ T_{2\to 1}, \tag{34}$$

thus justifying the use of Eq.(30) to obtain Eq.(32), as said after Eq.(31).

The relaxation times given by Eqs.(32) and (34) are strictly related to the ones relevant to the Planck spectrum when the temperature is changed abruptly and are of the order of some nanoseconds even for the high frequencies tail of the spectrum and not of $\sim 10^{10}$ years as required by Carati and Galgani [1].

IV. CONCLUSIONS

The present work has been stimulated by Carati and Galgani [1], although their attempt to classically derive the Planck spectrum, is shown in Sect. I to be valid only for a single temperature and with no transitories. The approach proposed here starts from the classical requirement that all the charged particles of the universe have an accelerated motion and must therefore radiate [3]. If the zitterbewegung (or spin motion as derived by Barut and Zanghi [4]) is considered in a realistic sense, the stochastic e.m. field is practically due to the spin motion and, because of the expansion of the universe, the resultant power spectral density turns out to be proportional to ω^3 for any kind of spin motion [5]. Since the average percentage of charged particles in the universe is roughly due in equal part to electrons and down quarks and in double part to up quarks, there are three ω^3 ramps as expressed by Eq.(3). The energy density of this e.m. stochastic field [called zero-point field (ZPF)] is therefore finite and there is no longer any need to perform a renormalization. The modified equation of motion (4) for a spinning particle strongly reduces the Einstein-Boyer-Rueda mechanism of acceleration [10], which otherwise would give an electron an energy of $\sim 10^{20}$ eV in 10 cm (in vacuum) [11]. If all is correct with a real, nonrenormalized ZPF, the Bohr radius R_1 is immediately obtained by equating the radiated power expressed by Eq.(6) to the power absorbed from the ZPF and given by Eq.(7) multiplied by 2 (the two harmonic oscillators composing a circular motion).

The fundamental state is therefore very easily derived. To obtain the excited states we consider an initially circular orbit modified by the action of the ZPF so as to become an ellipse (in agreement with the Keplerian motion which is little modified by the random, noncentral action of the ZPF). Since the variation of an orbit during the time of coherence of a ZPF wave train is very small, the eccentricity ϵ is much smaller than unity. Then Eq.(8) is equivalent to Eq.(9) or, more in general, to Eq.(14) which is the expansion in Fourier series of the elliptical motion. The n-th pair of the series (one term for the x component and the corresponding term for the y component) represents a circular motion with $n\theta \simeq n\omega t$ and is therefore interpretable as an epicycle on another epicycle of frequency $(n-1)\omega$. The relative frequency is always $n\omega - (n-1)\omega = \omega$, i.e. the same frequency of the first term which is a circular motion interpretable as a deferent. The absorbed power when n circular motions are excited from ZPF is therefore $2n$ times Eq.(7) (since any plane motion has two degrees of freedom) and is therefore given by Eq.(13). On the contrary, the radiated power remains the one of a circular motion because $\epsilon_n \ll 1$ in Eqs.(14). Equating Eq.(6) to Eq.(13) we obtain Bohr's conditions (15).

It is also interesting to examine the decay of an excited state. The ZPF can easily destroy one small epicyclic motion having an energy of two modes of the ZPF, i.e. $2\hbar\omega/2 = \hbar\omega$. The relation (20) between radiated energy and frequency is therefore justified in a classical way, without the concept of photon with its discretization of radiated energy. In the large band process of radiation, what is measured is only the weighted average angular frequency $\langle\omega\rangle$ of many oscillations (~ 1000) between the two states, produced by the random impulses of the ZPF radiation pressure that is active when the spin axis precedes (mainly because of the spin orbit coupling). Finally, $\hbar\langle\omega\rangle$, which is the net result of the difference between radiation and absorption between the initial and final states, must be equal to the energy difference between these two states [Eq. (21)], as required by the first principle of thermodynamics.

In Sect. III, a similar, detailed balance of transition frequencies (instead of powers as done in Sect. II) leads to the Planck spectrum using the classical statistics of Eq.(29). The procedure is similar to Einstein's but without introducing "ad hoc" spontaneous decay coefficients. The latter is substituted by the concomitant actions of the ZPF and radiation damping summarized by $2\rho_{ZPF}$ in Eq.(28). The relaxation times to equilibrium for a rapid change of temperature, are of the order of some millions of atomic transitions, i.e. of nanoseconds, contrary to the values required by Carati and Galgani [1], which should exceed the age of the universe. The rapid relaxation times are due to the detailed balance at any frequency, since the ZPF has a much wider spectrum than the atomic's, and its intensity is proportional to ω^3, corresponding to an energy $\hbar\omega/2$ per normal mode (at least for $\omega < \omega_0$). The relaxation time constant τ for an atomic transition is easily derived to be given by Eq.(32), which is experimentally confirmed. Our procedure is completely classical, as the one of Carati and Galgani who, as the majority of all authors, neglect the ZPF whose existence is required by classical physics. Actually, the stable elementary particles constituting matter are electrons and quarks up and down and, because of their confined motions, must have accelerations, hence radiation. The stochastic e.m. field thus brought about is the ZPF, whose power spectral density turns out to be expressed by Eq.(3) since it is greatly due to the spin radiation. The uniform expansion of the universe causes the ω^3 behavior for any main radiator, up to its spin frequency [5]. Only the spin motion as depicted by Barut and Zanghi [4] seems to be nonclassical. But this motion is a necessary consequence of the filament theory [7], derived in classical terms.

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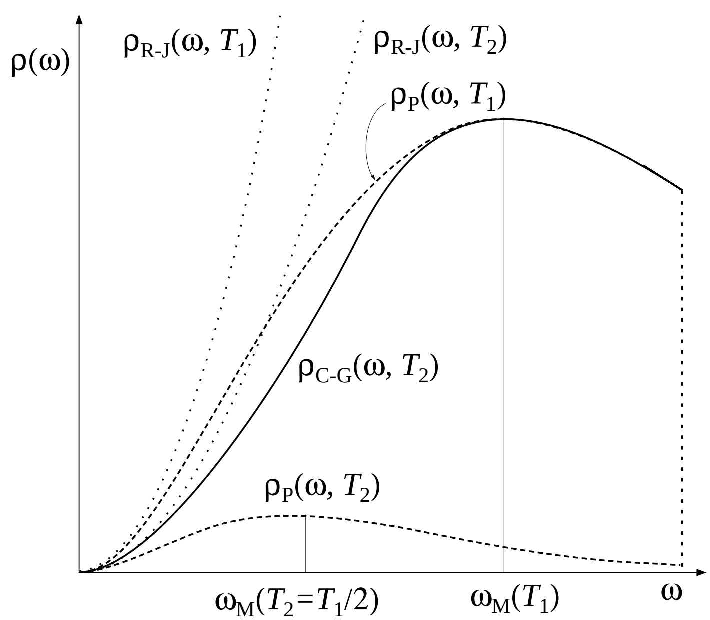

FIG. 1. Power spectral densities $\rho(\omega, T)$ [erg s cm^{-3}] vs. angular frequency ω [s^{-1}] for two temperatures, T_1 and $T_2 = T_1/2$ (in degrees Kelvin [K]). The subscript $R-J$ denotes Rayleigh-Jeans, $C-G$ stands for Carati-Galgani, and P for Planck. A rapid cooling down from T_1 to T_2 should leave $\rho_{C-G}[\omega > \omega_M(T_1), T_2]$ practically equal to $\rho_P(\omega, T_1)$ while for $\omega < \omega_M(T_2)$ it is $\rho_{C-G} \simeq \rho_{R-J}$. The connection $\rho_{C-G}(\omega, T_2)$ between these two regions is shown by a continuous line.

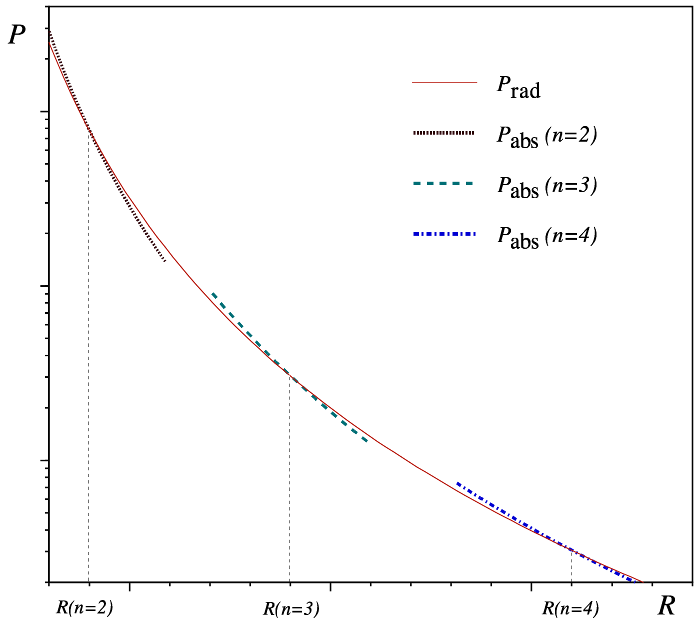

FIG. 2. Radiated power $P_{\rm rad}$ [erg s^{-1}] and absorbed power $P_{\rm abs}$ [erg s^{-1}] vs. radius R [cm] of the electron circular orbit. For any number n of plane orbits (in QM terms n is the principal quantum number) there is a stable point of intersection $P_{\rm abs}(n,R) = P_{\rm rad}(R)$ if the average effect of the zero-point field (ZPF) is considered. The ZPF fluctuations concordant with the radiation damping cause the transition from n to $n-1$.

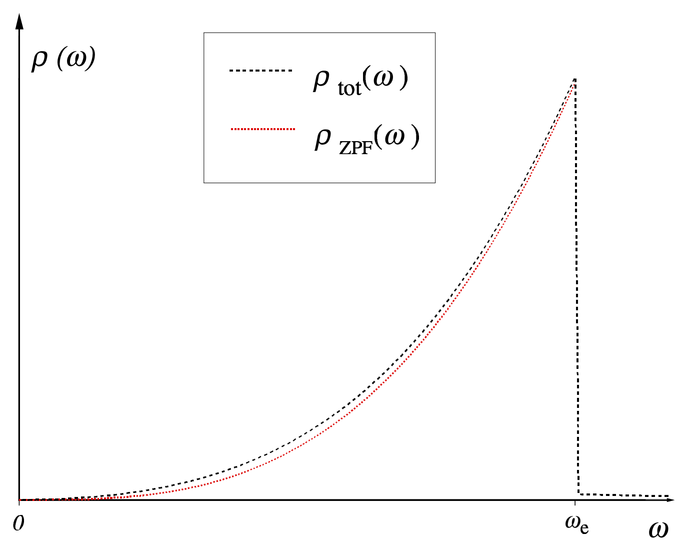

FIG. 3. Power spectral densities $\rho(\omega)$ [erg s cm^{-3}] vs. angular frequency ω [s^{-1}]. The Planck spectrum $\rho_P(\omega,T)$ is superimposed to the ZPF spectrum, thus giving the total power spectral density $\rho_{\rm tot}(\omega)$. Only the first ω^3 ramp of the ZPF is shown in Fig. 3. ω_e is the electron spin frequency and the considered temperature is $T \simeq 2.3\times10^9$ K. Even at this very high temperature $\rho_{\rm tot}$ little differs from $\rho_{\rm ZPF}$.