RELATIVITY WITH THREE DIMENSIONS OF TIME: SPACE-TIME VORTICES

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Abstract: The paper examines a theory of three dimensions of time, guided by multi-vector Geometric Algebra, that is consistent with Special Relativity, Maxwell's Equations, Klein-Gordon and Dirac equations. Rest mass and charge appear as operators in 'transverse time'. Advanced as well as retarded solutions to the wave-equation are retained to give circulations of signals in space and time: space-time vortices. The apparent lack of sources for the 'magnetic' field is explained. This new theory uses temporal boundary conditions to ensure causality and is the first theory to quantise photons without using already quantised harmonic oscillators. A new explanation is provided of the Einstein-Podolsky-Rosen paradox.

1. INTRODUCTION

Cramer's quantum theory [**1**] considers transactions between observers and observed objects with signals circulating back and forth in time. In fluid flow, Truesdell states that circulations, or vortices, are a 'peculiar and characteristic glory' of *three-dimensions* [**2**]. In inviscid fluids, whirl-pools persist with integer numbers of whirls until they interact with boundaries or other vortices. Hence taking Cramer and Tuesdell together, three dimensions of time might create space-time vortices that could form a natural basis for quantisation. Although Maxwell's curl equations give spatial vorticity there is no apparent temporal 'curl'.

This paper explores how three-dimensional (3-d) time allows photons to be identified as space-time vortices (STVs). This is a first theory to *derive* quantisation for photons from temporal boundary conditions without using already quantised harmonic oscillators [**3**]. In addition, 'transverse time' offers new interpretations of:
(a) rest mass;
(b) charge and current;
(c) entanglement [**4**, **5**, **6**, **7**].

Three dimensional time became attractive in studies of tachyons [**8**, **9**, **10**, **11**, **12** **13**, **14**] inspite of early criticisms [**15**] and difficulties [**16**, **17**, **18**, **19**, **20**]. The main criticisms arose from requiring of 3-d temporal space a similar homogeneity as that found in 3-d space where there is no preferred choice of reference axes. In reciprocal temporal space, such homogeneity would enable the three components of energy to interchange in unphysical ways [**15**]. Cole and Buchanan countered this by arguing that changes in direction of the temporal vector required energy [**21**]. In the present work, there is a preferred 'transverse' temporal plane Ot_1t_2 so that a 6-dimensional wave equation becomes a 'Klein-Gordon' equation:

$$-(\partial_{s1}{}^2+\partial_{s2}{}^2+\partial_{s3}{}^2)\Phi+M_o{}^2\Phi+\partial_{t3}{}^2\Phi = 0 \; ; \qquad (1)$$

where $M_o{}^2\Phi = (\partial_{t1}{}^2+\partial_{t2}{}^2)\Phi$; ($c = \hbar = 1$).

The orthogonal spatial axes $\{Os_n\}$ can have any orientation, but Ot_3, the axes of 'principal' time, remains normal to the temporal plane Ot_1t_2. The homogeneity of space means that a velocity $\boldsymbol{v}$ can always have its spatial changes taken along 'Os_3'. Temporal changes in $\boldsymbol{v}$ are postulated to be normal to Ot_1t_2 i.e. along Ot_3. Then Ot_1 , Ot_2, Os_1 and Os_2 are all transverse to $\boldsymbol{v}$ for a Lorentz transformation and therefore remain invariant. This ensures the Lorentz invariance of the temporal operator $M_o{}^2$. Hence M_o, in equ. (1) can be identified with *rest* mass.

Three dimensions of reciprocal time are now manifest by a transverse temporal operator $M_o{}^2$ associated with (rest mass energy)^2 and the principal temporal operator, $-\partial_{t3}{}^2$, associated with (free energy)^2, but the two forms can not be passively rotated into one another.

A further difficulty for 3d-time is the lack of consensus on changes required to Maxwell's equations. One of the most attractive proposals, that is also a starting point for future steps here, is that of Cole who retains the vector potential **A** and augments the usual scalar potential to a temporal vector **Φ** [**22**, **23**]. These lead to a 9-component **E** field in a 3 x 3 space-time array along with a new 3-component 'magnetic' field **W** augmenting the usual 3-component **B** field. Other workers have schemes some of which are more related to Cole [**24**, **25**, **26**, **27**] than others [**28**,**29**]. All methods recover Maxwell's equations in some limit.

'Geometric Algebra' (GA) is the algebra of multi-dimensional symmetry that for over three decades has been brilliantly promoted by David Hestenes along with others [**30**, **31**, **32**, **33**, **34**, **35**, **36**]. Space-Time Algebra (STA) is the branch of GA which specialises in 3d-space +1d-time. Maxwell's equations emerge as essential features of multi-vector symmetry in STA [**30**, **31**, **37**]. These successes suggest that the GA of '3+3' space-time should be studied if 3-d time is to be taken seriously. Using this method and insisting on a preferred 'transverse' temporal plane, classical Maxwell's equations are re-covered for **E** and **B** fields except that these fields are now necessarily complex with i ($i^2 = -1$) interpreted as a 90^o geometric rotation in the preferred transverse temporal plane Ot_1t_2 . Two conditions will be imposed:

(1) The orientation of any field 'in transverse time' is postulated to be unobservable, at least by classical measurements. This will mean that any complex term like $\psi = \psi_o \exp(i\theta)$ is unobservable. The phase factor will be shown to equivalent to a rotation of angle θ in the plane Ot_1t_2 . Hence terms like $\psi^*\psi$ are said to be invariant to rotations in transverse time (RTT invariant) and will be observable. Power, energy and polarisation will all be formulated in terms of RTT 'observable' quantities.

(2) The net power flowing backwards in time is postulated to be zero. This uni-directional power flow, will yield causality and quantisation of the electromagnetic fields.

2. OUTLINE OF PAPER

Section 3 examines a Geometric Algebra (GA) in 3-d space and 3-d time. To increase accessibility, GA wedge products are not exploited retaining vector products and curl that are so well known in 3-dimensions. For similar reasons the 2-d GA of transverse time is replaced with complex algebra.

To reduce the length, the work focuses on Maxwell's equations (sec.5). The derivation is related to that for Dirac's equation though the details for the latter are left for a future paper.

The resulting complex **E** and **B** fields lead to normal modes that are *analytic* complex functions with: (i) advanced and retarded solutions (ii) positive and negative frequencies (iii) positive and negative temporal chiralities. All these have essential features for space-time vortices. Promotion, demotion and annihilation operations will arise from complex variable theory and not quantum theory.

Section 6 considers concepts of space-time vortices (STVs) and vortex number conservation. Quantisation and causality emerge.

The work is rounded off by discussions on temporal chirality, polarisation and entanglement and an indication of the potential of the work.

3. '3+3' GEOMETRIC ALGEBRA

3.1 1-vectors and derivatives

Besides the book references given earlier, excellent web-tutorials for '3-d' GA and '3+1' STA are available [**38**, **39**, **40**] so this introductory section is restricted to outlining notation for the GA of '3+3' space-time used here.

Spatial and temporal unit 1-vectors are set as $\boldsymbol{\gamma}_s = \{\gamma_{s1}, \gamma_{s2}, \gamma_{s3}\}$ and $\boldsymbol{\gamma}_t = \{\gamma_{t1}, \gamma_{t2}, \gamma_{t3}\}$ respectively. The co-ordinate vector is taken as $\mathbf{X} = \{\mathbf{x}, \mathbf{t}\}$ where:

$$\mathbf{x} = (x_1, x_2, x_3)\ ;\ \mathbf{t} = (t_1, t_2, t_3)\ ;\ \text{with}$$
$$\underline{\mathbf{X}} = (\mathbf{x}.\boldsymbol{\gamma}_s + \mathbf{t}.\boldsymbol{\gamma}_t) = \Sigma_n (x_n \gamma_{sn} + t_n \gamma_{tn}) \qquad (2)$$

Bold quantities represent the components of 'vectors' but when underlined denote a GA multi-vector. A time-like metric is assumed : $t_1^2 + t_2^2 + t_3^2 - x_1^2 - x_2^2 - x_3^2$. Normalisation and orthogonality of the unit vectors requires:

$$\left.\begin{array}{l} \gamma_{tm}\gamma_{tn} = -\gamma_{tn}\gamma_{tm}\ (n \neq m);\quad \gamma_{tm}^2 = 1\ ; \\ \gamma_{sm}\gamma_{sn} = -\gamma_{sn}\gamma_{sm}\ (n \neq m);\quad \gamma_{sm}^2 = -1\ ; \\ \gamma_{sm}\gamma_{tn} = -\gamma_{tn}\gamma_{sm}\ (\text{all m and n}) \end{array}\right\} \quad (3)$$

Consider two arbitrary 1-vectors:
$\underline{\mathbf{V}} = \boldsymbol{\gamma}_s.\mathbf{P}_s + \boldsymbol{\gamma}_t.\boldsymbol{\Pi}_t$ and $\underline{\mathbf{W}} = \boldsymbol{\gamma}_s.\mathbf{Q}_s + \boldsymbol{\gamma}_t.\boldsymbol{\Theta}_t$,
then the multiplication rules of equs.(3) yield, using an 'obvious' vector product notation:

$$\underline{\mathbf{V}}\ \ \underline{\mathbf{W}} = -\mathbf{P}_s.\mathbf{Q}_s - \varphi_s\boldsymbol{\gamma}_s.(\mathbf{P}_s \times \mathbf{Q}_s) + \boldsymbol{\Pi}_t.\boldsymbol{\Theta}_t + \varphi_t\boldsymbol{\gamma}_t.(\boldsymbol{\Pi}_t \times \boldsymbol{\Theta}_t) + \boldsymbol{\gamma}_s.\boldsymbol{\gamma}_t.(\mathbf{P}_s\boldsymbol{\Theta}_t - \boldsymbol{\Pi}_t\mathbf{Q}_s) \qquad (4)$$

where $\varphi_s = \gamma_{s1}\gamma_{s2}\gamma_{s2}$ and $\varphi_t = \gamma_{t1}\gamma_{t2}\gamma_{t2}$ are respectively spatial and temporal pseudo-scalars or 3-vectors with $\varphi_s^2 = 1$ and $\varphi_t^2 = -1$. The full 6-d pseudo-scalar (or 6-vector) is S :

$$S = \varphi_t\varphi_s\ \ ;\ \ S^2 = 1; \qquad (5)$$

Pseudo 1-vectors or 5-vectors are given from $\gamma_{sn}S = -S\gamma_{sn}$; $\gamma_{tn}S = -S\gamma_{tn}$. Similarly $\gamma_{sn}\gamma_{tn}S = S\gamma_{sn}\gamma_{tn}$ gives a pseudo 2-vector or 4-vector while a pseudo 3-vector such as $\varphi_s S = -S\varphi_s$ remains a 3-vector (e.g. $\varphi_s S = \varphi_t$).

Temporal and spatial derivatives
$\partial_t = (\partial_{t1}, \partial_{t2}, \partial_{t3})$, $\partial_s = (\partial_{s1}, \partial_{s2}, \partial_{s3})$
give a 'D'Alembertian' space-time derivative:

$$\underline{\Box} = (-\partial_s.\boldsymbol{\gamma}_s + \partial_t.\boldsymbol{\gamma}_t) \qquad (6)$$

The negative sign in front of $\partial_s.\boldsymbol{\gamma}_s$ is a simplified method to avoid super/sub-scripting of co-/contra-variant forms. Observe that :

$$\underline{\Box}\ \underline{\mathbf{X}} = (-\partial_s.\boldsymbol{\gamma}_s + \partial_t.\boldsymbol{\gamma}_t)(\mathbf{x}.\boldsymbol{\gamma}_s + \mathbf{t}.\boldsymbol{\gamma}_t) = 6 \qquad (7)$$

where 6 is the invariant dimensionality of 3+3 space. Differentiating, using the multi-vector notation used here, gives

$$\begin{aligned}\underline{\Box}\ \underline{\mathbf{V}} &= (-\partial_s.\boldsymbol{\gamma}_s + \partial_t.\boldsymbol{\gamma}_t)(\boldsymbol{\gamma}_s.\mathbf{P}_s + \boldsymbol{\gamma}_t.\boldsymbol{\Pi}_t) \\ &= \partial_s.\mathbf{P}_s + \varphi_s\boldsymbol{\gamma}_s.(\partial_s \times \mathbf{P}_s) + \partial_t.\boldsymbol{\Pi}_t + \\ &\quad \varphi_t\boldsymbol{\gamma}_t.(\partial_t \times \boldsymbol{\Pi}_t) - \boldsymbol{\gamma}_s.\boldsymbol{\gamma}_t.(\partial_t\mathbf{P}_s + \partial_s\boldsymbol{\Pi}_t)\end{aligned} \qquad (8)$$

Retention of a curl operator in 3-dimensions highlights temporal vorticity ($\partial_t \times \boldsymbol{\Pi}_t$) in analogy with spatial vorticity ($\partial_s \times \mathbf{P}_s$) while space-time vorticity is formed from the 9 elements of ($\partial_t\mathbf{P}_s + \partial_s\boldsymbol{\Pi}_t$).

3.2 Passive Rotations

Rotations are formed in GA by 'rotors' such as $\underline{\mathbf{R}}_S = \exp(½\theta\ \gamma_{s1}\gamma_{s3})$ with the reversion given as $\underline{\mathbf{R}}_S^{\sim} = \exp(½\theta\ \gamma_{s3}\gamma_{s1})$. Then $\mathbf{V_{rot}} = \underline{\mathbf{R}}_S\,\underline{\mathbf{V}}\underline{\mathbf{R}}_S^{\sim}$ rotates $\underline{\mathbf{V}}$ in the $\gamma_{s1}\gamma_{s3}$ plane by an angle of θ but, for this rotor, γ_{s2} and all$\{\gamma_{tn}\}$are unaltered. Spatial homogeneity allows arbitrary passive rotations without changing the physics.

Rotors $\underline{\mathbf{R}}_{TT} = \exp(½\ \mu\ \gamma_{t1}\gamma_{t2})$ forming $\underline{\mathbf{V}}_{TT} = \underline{\mathbf{R}}_{TT}\,\underline{\mathbf{V}}\mathbf{R}_{TT}^{\sim}$ rotate $\underline{\mathbf{V}}$ within the plane Ot_1t_2. As for spatial rotations, any real angle μ will be permitted. There will be no preferred choice of direction for γ_{t1} or γ_{t2} within the plane Ot_1t_2.

Lorentz transformations, as in STA, are given by 'rotors' $\underline{\mathbf{R}}_L = \exp(½\alpha\ \gamma_{t3}\gamma_{s3})$. The principle direction of time, γ_{t3}, is always taken to be normal to the temporal plane Ot_1t_2. The use of rotors like $\underline{\mathbf{R}}_S$ can re-label any arbitrary spatial direction as γ_{s3} so the velocity $\boldsymbol{v}$ can always be taken to lie along the space-time direction γ_{s3} and γ_{t3}, while $|\boldsymbol{v}|$ determines the boost parameter α in the classical way. The transverse temporal and transverse spatial 1-vectors $\{\gamma_{t1},\ \gamma_{t2}\}$ and $\{\gamma_{s1},\ \gamma_{s2}\}$ are invariant under $\underline{\mathbf{V}}_L = \underline{\mathbf{R}}_L\underline{\mathbf{V}}\underline{\mathbf{R}}_L^{\sim}$ which partially interchanges principal time γ_{t3} and γ_{s3} ; γ_{t3} remaining normal to the transverse temporal plane Ot_1t_2.

Now as already proposed, the operator $(\partial_{t1}^2 + \partial_{t2}^2)$ is to be linked to (mass energy)^2 = M_o^2 . Setting M_o as the *rest* mass is consistent with $\underline{\mathbf{R}}_L\ (\gamma_{t1}\partial_{t1} + \gamma_{t2}\partial_{t2})\underline{\mathbf{R}}_L^{\sim}$ being Lorentz invariant. Finite energy and fields, combined with real M_o, require evanescent (i.e. decaying) waves in transverse temporal space. However $(-\partial_{t3}^2)$ is to be linked in the classical way ($\hbar$ = 1) to (free energy)^2 with oscillating waves . The two forms of wave cannot simply be passively rotated one into the other by rotors, like $\underline{\mathbf{R}}_T = \exp(½\phi\ \gamma_{t1}\gamma_{t3})$ which must in general be disallowed. Passive rotations through rotors $\underline{\mathbf{R}}_{ST} = \exp(\alpha\ \ \gamma_{tM}\gamma_{sN})$, (M =1, 2 ; N=1, 2, 3) should also normally be disallowed because some combinations of $\underline{\mathbf{R}}_{ST}$ and $\underline{\mathbf{R}}_L$ would mix evanescent fields and oscillating fields in transverse time. A direct interchange of mass energy and free energy is postulated to require interactive rotations, outside this paper's scope.

3.3 The multi-vectors

Following the discussion after equ. 5, there are: one scalar (0-vector) plus one pseudo-scalar (6-vector), six 1-vectors plus six pseudo 1-vectors (5-vectors), fifteen 2-vectors plus fifteen pseudo 2-vectors (4-vectors) , plus twenty 3-vectors giving 64 elements in total. To reduce the algebra, n-vectors and pseudo-n-vectors will be combined so that 'scalar' will refer to both scalar *plus* pseudo-scalar, '1-vector' refers to 1-vector *plus* pseudo 1-vector etc. Ordering of symbols will matter because the pseudo-scalar anti-commutes with all 1-vectors and the component parts may need to be identified at the end of the algebra. Spatial and temporal components can be specified so that a general multi-vector is formed from:

$$\begin{aligned}&\{\Psi\}_{\text{(1 of 0-vector)}}\\&+\{\boldsymbol{\gamma}_s.\mathbf{P}_s+\boldsymbol{\gamma}_t.\boldsymbol{\Pi}_t\}_{\text{(6 of 1-vectors)}}\\&+\{\varphi_t\boldsymbol{\gamma}_s.\mathbf{F}_s+\varphi_t\boldsymbol{\gamma}_t.\boldsymbol{\Phi}_t+\boldsymbol{\gamma}_t.\boldsymbol{\gamma}_s.\mathbf{G}_{st}\}_{\text{(15 of 2-vectors)}}\\&+\{\varphi_t\Xi+\varphi_t\boldsymbol{\gamma}_t.\boldsymbol{\gamma}_s.\mathbf{X}_{st}\}_{\text{(20 of 3-vectors)}}\end{aligned}\qquad(9)$$

where $\Psi = \psi + S\,\psi'$; $\mathbf{P_s} = \mathbf{p_s} + S\,\mathbf{p}'_s$;

$\boldsymbol{\Pi}_t = \boldsymbol{\pi}_t + S\,\boldsymbol{\pi}'_t$; $\mathbf{F_s} = \mathbf{f_s} + S\,\mathbf{f}'_s$;

$\boldsymbol{\Phi}_t = \boldsymbol{\phi}_t + S\,\boldsymbol{\phi}'_t$; $\mathbf{G}_{st} = \mathbf{g}_{st} + S\,\mathbf{g}'_{st}$;

$\Xi = \xi + S\,\xi'$; $\mathbf{X}_{st} = \mathbf{x}_{st} + S\,\mathbf{x}'_{st}$;

[$\mathbf{p_s} = \{p_{s1},\ p_{s2},\ p_{s3}\}$; $\boldsymbol{\phi}_t = \{\phi_{t1},\ \phi_{t3},\ \phi_{t3}\}$;

$\mathbf{g}_{st} = \{g_{sn\ tm}\}_{n = 1\text{-}3,\ m = 1\text{-}3}$ etc.]

Note that Equ. (9) is not unique. For example

$$\boldsymbol{\gamma}_s.\mathbf{P_s} = \boldsymbol{\gamma}_s.\mathbf{p_s} + \boldsymbol{\gamma}_s.(S\,\mathbf{p}'_s) = \mathbf{p_s}.\boldsymbol{\gamma}_s - (S\,\mathbf{p}'_s).\boldsymbol{\gamma}_s \neq \mathbf{P_s}.\boldsymbol{\gamma}_s$$

Here, as for the other components, both $\mathbf{p_s}$ and $\mathbf{p}'_s$ are real; complex amplitudes have yet to be considered. Rather than keep to generalities, the work focuses next on the preferred transverse temporal plane.

4. TRANSVERSE TIME

4.1 Preliminaries

In section 3 a preferred transverse plane is defined from the orthogonal 1-vectors γ_{t1} and γ_{t2} . The bi-vector $\gamma_{t1}\gamma_{t2} = i$ has $i^2 = -1$ with $\gamma_{t1}i = -i\ \gamma_{t1} = \gamma_{t2}$. The temporal gradient operator $\boldsymbol{\partial}_t.\boldsymbol{\gamma}_t$ can be written as

$$\begin{aligned}\boldsymbol{\partial}_t.\boldsymbol{\gamma}_t = \boldsymbol{\gamma}_t.\boldsymbol{\partial}_t &= (\partial_{t1} - i\partial_{t2})\,\gamma_{t1} + \partial_{t3}\,\gamma_{t3}\\ &= \gamma_{t1}\,(\partial_{t1} + i\partial_{t2}) + \gamma_{t3}\,\partial_{t3}\end{aligned}\qquad(10)$$

The differential operator for transverse time can be written as $\nabla_T = (\partial_{t1} - i\ \partial_{t2})$ so that $\nabla_T\ (t_1 + i\ t_2) = 2$ (the number of transverse temporal dimensions) while $\nabla_T^*\ (t_1 + i\ t_2) = 0$:

$$\boldsymbol{\partial}_t.\boldsymbol{\gamma}_t = \nabla_T\,\gamma_{t1} + \partial_{t3}\,\gamma_{t3} = \gamma_{t1}\nabla_T^* + \gamma_{t3}\,\partial_{t3}\qquad(11)$$

Now i commutes with itself and any combination of γ_{t3}, γ_{s1}, γ_{s2}, γ_{s3}. Transverse temporal components $(\gamma_{t1}\ a_{t1} + \gamma_{t2}\ a_{t2})$ can be represented as a complex component $\mathbf{A}_T$ in an Argand diagram:

$$\begin{aligned}\gamma_{t1}\,\mathbf{A}_T &= \gamma_{t1}\,a_{t1} + \gamma_{t2}\,a_{t2} = \gamma_{t1}\,(a_{t1} + \gamma_{t1}\,\gamma_{t2}\,a_{t2})\\ &= \gamma_{t1}\,(a_{t1} + i\,a_{t2})\end{aligned}\qquad(12)$$

It can be seen that $\exp(i\ \theta)$ represents an angle θ rotation in the plane Ot_1t_2. The commutation properties of i enable a single rotation operator to be used instead of the GA double-sided rotor notation.

The 'd'Alembertian' operator is of the form

$$\square = (-\partial_{\mathbf{s}}.\boldsymbol{\gamma}_{\mathbf{s}} + \nabla_T \gamma_{t1} + \partial_{t3}\gamma_{t3})$$
$$= (-\partial_{\mathbf{s}}.\boldsymbol{\gamma}_{\mathbf{s}} + \gamma_{t1}\nabla_T^* + \partial_{t3}\gamma_{t3}) \qquad (13)$$

If Φ is such that $\square\square\,\Phi = 0$, then

$$\square\square\Phi = -\partial_{\mathbf{s}}.\partial_{\mathbf{s}}\Phi + M_0^2\Phi + \partial_{t3}\partial_{t3}\Phi = 0 \qquad (14)$$

where $M_0^2 = \nabla_T^*\nabla_T = \nabla_T\nabla_T^*$, so that all such Φ satisfy a Klein-Gordon equation.

4.2 Complex multi-vectors

Using the notation of section 4.1, a general multivector $\underline{\mathbf{V}}$ may now be re-written as:

$$\underline{\mathbf{V}} = \Psi + \boldsymbol{\gamma}_{\mathbf{s}}.\mathbf{P}_{\mathbf{s}} + \gamma_{t3}\Pi_{t3} + \gamma_{t1}\boldsymbol{\Pi}_T + \{\gamma_{t3}\boldsymbol{\gamma}_{\mathbf{s}}.\, i\mathbf{F}_{\mathbf{s}} + i\Phi_{t3} - \gamma_{t3}\gamma_{t1}\, i\,\boldsymbol{\Phi}_T + \gamma_{t3}\boldsymbol{\gamma}_{\mathbf{s}}.\mathbf{G}_{\mathbf{s}t3} + \gamma_{t1}\boldsymbol{\gamma}_{\mathbf{s}}.\mathbf{G}_{\mathbf{s}T}\} + \gamma_{t3}\, i\,\Xi + \boldsymbol{\gamma}_{\mathbf{s}}.\, i\,\mathbf{X}_{\mathbf{s}t3} - \gamma_{t3}\gamma_{t1}\boldsymbol{\gamma}_{\mathbf{s}}.\, i\,\mathbf{X}_{\mathbf{s}T} \qquad (15)$$

A complex notation may be defined as in table 1 allowing equ. (15) to be grouped into even /odd grade multivectors with subscripts e/o.

TABLE 1	
$(\mathbf{P}_{\mathbf{s}} + i\,\mathbf{X}_{\mathbf{s}t3}) \rightarrow \mathbf{A}_{\mathbf{osC}}$;	$(\Pi_{t3} + i\,\Xi) \rightarrow \Phi_{oC}$;
$\boldsymbol{\Pi}_T \rightarrow \boldsymbol{\Phi}_{oT}$;	$i\,\mathbf{X}_{\mathbf{s}T} \rightarrow \mathbf{A}_{\mathbf{os}T}$;
$(\Psi + i\,\Phi_{t3}) \rightarrow \Phi_{eC}$;	$(\mathbf{G}_{\mathbf{s}t3} + i\mathbf{F}_{\mathbf{s}}) \rightarrow \mathbf{A}_{\mathbf{esC}}$;
$-i\,\boldsymbol{\Phi}_T \rightarrow \boldsymbol{\Phi}_{eT}$;	$\mathbf{G}_{\mathbf{s}T} \rightarrow -\mathbf{A}_{\mathbf{es}T}$;

Notice that $i\,S = S\,i$ so, for example, $i\mathbf{A}_{\mathbf{osC}}$ or $i\,S\,\mathbf{A}_{\mathbf{osC}}$ remains of the form $(\mathbf{p}^{//}_{\mathbf{s}} + i\,\mathbf{x}^{//}_{\mathbf{st}}) + S\,(\mathbf{p}^{///}_{\mathbf{s}} + i\,\mathbf{x}^{///}_{\mathbf{st}})$. Obviously care has to be taken in the order of $\boldsymbol{\gamma}_{\mathbf{s}}.\mathbf{A}_{\mathbf{osC}}$ as opposed to $\mathbf{A}_{\mathbf{osC}}.\boldsymbol{\gamma}_{\mathbf{s}}$, evaluating $(\mathbf{A}_{\mathbf{osC}})^2$ and finally separating out the pseudo-vector components.

The multivector $\underline{V}$ then splits into even and odd grade complex components:

$$\underline{\mathbf{V}} = \underline{\mathbf{V}}_{\mathbf{e}} + \underline{\mathbf{V}}_{\mathbf{o}} \qquad (16)$$

where

$$\underline{\mathbf{V}}_{\mathbf{e}} = [\Phi_{eC} + \gamma_{t3}\boldsymbol{\gamma}_{\mathbf{s}}.\mathbf{A}_{\mathbf{esC}} + \gamma_{t3}\gamma_{t1}\Phi_{eT} + \boldsymbol{\gamma}_{\mathbf{s}}.\gamma_{t1}\mathbf{A}_{\mathbf{es}T}] ;$$

$$\underline{\mathbf{V}}_{\mathbf{o}} = [\gamma_{t3}\Phi_{oC} + \boldsymbol{\gamma}_{\mathbf{s}}.\mathbf{A}_{\mathbf{osC}} + \gamma_{t1}\Phi_{oT} + \gamma_{t3}\boldsymbol{\gamma}_{\mathbf{s}}.\gamma_{t1}\mathbf{A}_{\mathbf{os}T}] ;$$

A general Lorentz rotor $\underline{\mathbf{R}}_L = \exp(\alpha\,\gamma_{t3}\,\gamma_{sN})$ will not change Φ_{eC}, $(\gamma_{t3}\,\gamma_{sN}\,\mathbf{A}_{\mathbf{es}NC})$, $(\gamma_{t1}\boldsymbol{\Phi}_{oT})$, $(\gamma_{t3}\gamma_{sN}\gamma_{t1}\mathbf{A}_{\mathbf{os}N})$, $(\gamma_{sM}\mathrm{A}_{osMC})$ or $(\gamma_{sM}\,\gamma_{t1}\mathrm{A}_{esMT})$ $(M \neq N)$. General Lorentz rotors (including spatial rotations) will transform terms $\{\boldsymbol{\gamma}_{\mathbf{s}}.\mathbf{A}_{\mathbf{osC}} + \gamma_{t3}\Phi_{oC}\}$ and $\{\boldsymbol{\gamma}_{\mathbf{s}}.\,\gamma_{t1}\mathbf{A}_{\mathbf{es}T} + \gamma_{t3}\gamma_{t1}\Phi_{eT}\}$ like conventional '3+1' space-time 4-vectors even though $\mathbf{A}_{\mathbf{osC}}$, $\mathbf{A}_{\mathbf{es}T}$ etc. are necessarily complex. Note that $\gamma_{t1}\underline{\mathbf{V}}_{\mathbf{o}} = [-\gamma_{t3}\gamma_{t1}\Phi_{oC} - \boldsymbol{\gamma}_{\mathbf{s}}.\gamma_{t1}\mathbf{A}_{\mathbf{osC}} + \Phi_{oT} + \gamma_{t3}\boldsymbol{\gamma}_{\mathbf{s}}.\mathbf{A}_{\mathbf{os}T}]$ has a 'similar' structure to that of $\underline{\mathbf{V}}_{\mathbf{e}}$. However the even-multivectors form a sub-algebra, where even-multivectors multiply into even multi-vectors. This sub-algebra is known as the spinor algebra and is considered to be closely related to the Dirac equation [**41**, **42**] with the Dirac Ψ being considered to be rotors determining the properties of the electron.

Assume that the first derivatives of all *even* multivectors are zero:

$$(-\partial_{\mathbf{s}}.\boldsymbol{\gamma}_{\mathbf{s}} + \partial_{t3}\gamma_{t3} + \nabla_T\gamma_{t1})\,[\Phi_{eC} + \gamma_{t3}\boldsymbol{\gamma}_{\mathbf{s}}.\mathbf{A}_{\mathbf{esC}} + \gamma_{t3}\gamma_{t1}\Phi_{eT} + \boldsymbol{\gamma}_{\mathbf{s}}.\gamma_{t1}\mathbf{A}_{\mathbf{es}T}] = 0 \qquad (17)$$

From Equ.(18), it can be shown (though not in this paper) that a Dirac equation emerges:

$$i\,\partial_{t3}\boldsymbol{\Psi} + i\,(\partial_{\mathbf{s}}.\boldsymbol{\alpha})\,\boldsymbol{\Psi} - \boldsymbol{\beta}\mathbf{M}_{\mathbf{o}}\,\boldsymbol{\Psi} = 0 \qquad (18)$$

The $\boldsymbol{\beta}$ and set of $\boldsymbol{\alpha}$ $(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,\boldsymbol{\alpha}_3)$ matrices are found to be 8 x 8 instead of 4 x 4 and can be normalised so that $\boldsymbol{\beta}^2 = 1$ with $\mathbf{M}_{\mathbf{o}}^2 = \nabla_T\nabla_T^* = \nabla_T^*\nabla_T$ consistent with equation (1). It can also be shown that $\boldsymbol{\beta}\,\boldsymbol{\alpha}_n = -\,\boldsymbol{\alpha}_n\,\boldsymbol{\beta}$ (n =1,2,3), a commutation property as in the classic Dirac equation [**43**, **44**]. The 8 complex elements, instead of 4, allow for the extra dimensions of transverse time. The matrices $\boldsymbol{\alpha}_{\mathbf{n}}$ are found to be Hermitian and ensure that the usual continuity equation holds:

$$\partial_{t3}(\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}) + \partial_{\mathbf{s}}.(\boldsymbol{\Psi}^\dagger\,\boldsymbol{\alpha}\,\boldsymbol{\Psi}) = 0 \qquad (19)$$

The interaction, using this formalism, between e.m. fields and electrons has not yet been developed and any discussion is omitted here.

In this paper, the detailed justification of these results is left for another publication in the interests of a more focussed discussion on Maxwell's equations.

5. MAXWELL'S EQUATIONS

5.1 The Potentials

In STA, the Maxwell potentials are formed from $\underline{\mathbf{V}} = \gamma_{t3}\,\Phi + \boldsymbol{\gamma}_{\mathbf{s}}.\mathbf{A}$ where Φ and **A** are real. A similar assumption can be made with 3+3 GA: $\underline{\mathbf{V}} = \gamma_{t3}\,\Phi_{oC} + \boldsymbol{\gamma}_{\mathbf{s}}.\mathbf{A}_{\mathbf{oC}}$ where $\Phi_{oC} = \Phi_{oR} + i\,\Phi_{oI}$; $\mathbf{A}_{\mathbf{oC}} = \mathbf{A}_{\mathbf{oR}} + i\,\mathbf{A}_{\mathbf{oI}}$. Again spatial rotations and Lorentz transformations transform Φ_{oC} and $\mathbf{A}_{\mathbf{oC}}$ in the same way as Φ and **A** in STA. If the transverse temporal axes are rotated then the complex fields will change by a phase factor exp $i\theta$.

An equivalent assumption is made to that in equ. (17) but now with the first derivatives of all *odd* multivectors set to zero:

$$(-\partial_{\mathbf{s}}.\boldsymbol{\gamma}_{\mathbf{s}} + \partial_{t3}\gamma_{t3} + \nabla_T\gamma_{t1})\,[\gamma_{t1}\Phi_{oT} + \gamma_{t3}\boldsymbol{\gamma}_{\mathbf{s}}.\gamma_{t1}\boldsymbol{A}_{\mathbf{os}T} + \gamma_{t3}\,\Phi_{oC} + \boldsymbol{\gamma}_{\mathbf{s}}.\boldsymbol{A}_{\mathbf{osC}}] = 0 \qquad (20)$$

This equation splits into various multivector components:

$$-\gamma_{t1}\,\gamma_{t3}[\partial_{t3}\,\Phi_{oT} - \partial_{\mathbf{s}}.\boldsymbol{A}_{\mathbf{os}T} - \nabla_T^*\boldsymbol{\Phi}_{oC}] = 0 \qquad (21)$$

$$\gamma_{t1}\gamma_s . [\partial_{t3}\boldsymbol{A}_{osT} - \partial_s \Phi_{oT} - \nabla_T{}^* \boldsymbol{A}_{osC} - iS(\partial_s \times \boldsymbol{A}_{osT})_s] = 0 \quad (22)$$

$$[\partial_{t3}\, \Phi_{oC} + \partial_s .\, \boldsymbol{A}_{osC} + \nabla_T\, \Phi_{oT}] = 0 \quad (23)$$

$$\gamma_{t3}\gamma_s . [\partial_{t3}\boldsymbol{A}_{osC} + \partial_s \Phi_{oC} + \nabla_T \boldsymbol{A}_{osT} + i\, S\, (\partial_s \times \boldsymbol{A}_{osC})_s] = 0 \quad (24)$$

Note that **E** and **B** fields in STA are bivectors, so it will be equ. (24) [and (28) below] that will be expected to be the cornerstone in defining these fields. With 3+3 space-time, additional fields as well as the original STA fields may be expected but only to the extent that these additional fields have appropriate changes under spatial rotations and Lorentz transformations, as will be seen shortly.

5.2 The Fields E and B

Now setting $t_3 = t$ and removing the explicit grades of vectors

$$\partial_t\, \Phi_{oT} - \partial_s . \mathbf{A}_{osT} = \nabla_T{}^*\, \Phi_{oC} = G_T \quad (25)$$

$$\partial_t \mathbf{A}_{osT} - \partial_s \Phi_{oT} - i\, S(\partial_s \times \mathbf{A}_{esC})_s = \nabla_T{}^* \mathbf{A}_{osC} = \mathbf{F}_T \quad (26)$$

$$\partial_t\, \Phi_{oC} + \partial_s . \mathbf{A}_{osC} = -\nabla_T\, \Phi_{oT} = G_C \quad (27)$$

$$\partial_t \mathbf{A}_{osC} + \partial_s \Phi_{oC} + i\, S\, (\partial_s \times \mathbf{A}_{osC})_s] = -\nabla_T \mathbf{A}_{osT} = \mathbf{F_C} \quad (28)$$

In order to retain non-zero sources and non-zero fields, $G_C = -\nabla_T\, \Phi_{oT}$ and $\mathbf{F_C} = -\nabla_T \mathbf{A}_{osT}$ are retained as non-zero. This is not at variance with requiring $\nabla_T{}^*\nabla_T\, \Phi_{oT} = \nabla_T{}^*\nabla_T\, \mathbf{A}_{osT} = 0$ and $\nabla_T\nabla_T{}^*\, \Phi_{oC} = \nabla_T\nabla_T{}^* \mathbf{A}_{osC} = 0$ for zero photon mass. Operating by $\nabla_T{}^*$ on equ. (27) and (28) automatically sets both sides to zero if $G_T = \nabla_T{}^*\, \Phi_{oC} = 0$ and $\mathbf{F}_T = \nabla_T{}^* \mathbf{A}_{osC} = 0$. Equations (25) and (26) imply that:

$$\partial_t\, G_C - \partial_s .\, \mathbf{F_C} = 0 \quad (29)$$

$$\partial_t \mathbf{F_C} - \partial_s G_C - i\, S(\partial_s \times \mathbf{F_C})_s = 0 \quad (30)$$

Then in 3+1 space time, treating transverse time as unobservable, it is $\partial_t\, G_C$ that appears as the 'source' for the fields $\partial_s .\, \mathbf{F_C}$ while $\partial_s\, G_C$ appears to be the 'source' for the fields $[\partial_t \mathbf{F_C} - i\, S(\partial_s \times \mathbf{F_C})_s]$.

Now, the theory has been arranged so that in general one could/should write ($G_C + S\, G_C{}^{/}$) instead of just the complex G_C and so on with all the components (see immediately after equ. 9). To every complex vector (or scalar), the complex pseudo-vector (pseudo-scalar) is an important complement in this 3+3 space-time analysis. With all these pseudo-vector components made explicit, equations 25-28 now have to be written as

$$\partial_t\, \Phi_{oC} + \partial_s . \mathbf{A}_{osC} = G_C\ ; \quad (31)$$

$$\partial_t\, \Phi_{oC}{}^{/} + \partial_s . \mathbf{A}_{osC}{}^{/} = G_C{}^{/}; \quad (32)$$

$$\partial_t \mathbf{A}_{osC} + \partial_s \Phi_{oC} + i(\partial_s \times \mathbf{A}_{osC}{}^{/})_s] = \mathbf{F_C} \quad (33)$$

$$\partial_t \mathbf{A}_{osC}{}^{/} + \partial_s \Phi_{oC}{}^{/} + i\ (\partial_s \times \mathbf{A}_{osC})_s] = \mathbf{F_C}{}^{/} \quad (34)$$

$$\partial_s . \mathbf{F_C} = \partial_t\, G_C\ ;\quad \partial_s .\, \mathbf{F_C}{}^{/} = \partial_t\, G_C{}^{/}\ ; \quad (35)$$

$$\partial_t\, \mathbf{F_C} - \partial_s\, G_C - i\, (\partial_s \times \mathbf{F}^{/}{}_C)_s = 0 \quad (36)$$

$$\partial_t\, \mathbf{F}^{/}{}_C - \partial_s\, G^{/}{}_C - i(\partial_s \times \mathbf{F_C})_s = 0 \quad (37)$$

The excitation in the observable 3+1 space is caused from the complex terms G_C and its pseudo-vector counterpart $G_C{}^{/}$. With appropriate complex multipliers χ and ξ , it is possible to combine these two excitations into one with a magnitude Q and another with zero excitation:

$$\chi G_C + \xi G_C{}^{/} = 0\ ;\quad \xi G_C + \chi G_C{}^{/} = Q\ ; \quad (38)$$

with a normalisation given as $\xi^2 - \chi^2 = 1$. The ξ and χ have the symmetries of a complex scalar and a complex pseudoscalar respectively so that Q has the same symmetries as G_c.

New fields can now be created from the combinations as below:

$$\Phi_{oC}{}^{//} = \chi\Phi_{oC} + \xi\Phi_{oC}{}^{/} = 0\ ; \quad (39)$$

$$\mathbf{A}_{osC}{}^{//} = \chi\mathbf{A}_{osC} + \xi\mathbf{A}_{osC}{}^{/} = 0 \quad (40)$$

$$\mathbf{E} = \partial_s\, (\xi\Phi_{oC} + \chi\Phi_{oC}{}^{/}) + \partial_t\, (\xi\mathbf{A}_{osC} + \chi\mathbf{A}_{osC}{}^{/}) = \xi\mathbf{F_C} + \chi\mathbf{F_C}{}^{/} \quad (41)$$

$$i\, \mathbf{B} = i\ [\partial_s \times (\xi\mathbf{A}_{osC} + \chi\mathbf{A}_{osC}{}^{/})_s] = \chi\mathbf{F_C} + \xi\mathbf{F_C}{}^{/} \quad (42)$$

This then gives the 'Maxwell equations'

$$\partial_t \mathbf{B} - (\partial_s \times \mathbf{E})_s = 0 \quad (43)$$

$$\partial_t\, \mathbf{E} - \partial_s\, Q + (\partial_s \times \mathbf{B})_s = 0 \quad (44)$$

$$\partial_s . \mathbf{B} = 0\ ;\quad \partial_s .\, \mathbf{E} = \partial_t\, Q\ ; \quad (45)$$

In this 3+1 representation of 3+3 space time, the **E** and **B** 'electromagnetic' fields become combinations of bivectors and 4-vectors (not to be confused with the 4-vectors of 3+1 space-time). The question of why there are no sources for the B-field in Maxwell's equations is now answered in this work by the statement that the B-field is that part of the electro-magnetic field which does not have sources for its divergence. *The lack of magnetic sources is part of the definition of* **B**.

5.3 Sources

Equs.(44) and (45) define sources $\partial_s\, Q$ and $\partial_t\, Q$ these are referred to as 'current density' $\boldsymbol{J}$ and 'charge density' ρ . Reversing time and space reverses ρ and $\mathbf{J}$ in keeping with PCT [**45**]. On examining $\rho = -\partial_t\, \nabla_T\, (\xi\Phi_{oT} + \chi\Phi_{oT}{}^{/})$, this does not look like a spatial density but more like a temporal frequency. The reconciliation comes later when it will be noted that the interpretation of these equations is in terms of exciting *units* of space-time vortices. The charge density is then a frequency of excitation of STVs , the current density is the spatial frequency of excitation of these STVs.

The current continuity is given from:

$$\partial_t\rho + \partial_s . \mathbf{J} = \partial_t^2 Q - \partial_s . \partial_s Q = 0. \qquad (46)$$

Even though $G_C = -\nabla_T \Phi_{oT}$ is non-zero, $\nabla_T^* \nabla_T \Phi_{oT} = 0$ for massless photons, so that $\nabla_T^* Q = 0$ and $\nabla_T\nabla_T^* Q = 0$ in the 6d wave equation. It may be thought that if Q represents electrons then Q should have mass and $\nabla_T\nabla_T^* Q = M_o^2 Q$ but Q represents only the *excitation* of the electromagnetic fields and an excitation need not have mass. Nevertheless both charge and mass have now been shown to be tied into the structure of the fields in transverse time, opening up future lines of research with potential for a more direct test of this theory.

The 3+3 space time formalism has allowed Maxwell's equations to be fully recovered along with charge, current and an explanation of why there are no magnetic sources. The key change from the classical version is that the equations are necessarily complex. The proposed new path, following STA as closely as possible, differs from Ref. **27** where the two components of the spatial **E** field were associated with direction γ_{t1} and direction γ_{t2} respectively. Further evidence/argument is required to give a definitive decision over the choice of theory because either choice appears to enable similar outcomes.

5.4 Conjugation

One conventional use of *i* occurs with temporal variations exp $(-i\omega t)$ [cf. engineering texts with exp $(j\omega t)$] that avoid separate equations in $\cos(\omega t)$ and $\sin(\omega t)$. This is quite different from the geometrical significance in this paper connected with transverse time where we have seen that the phase factor exp $i\theta$ is equivalent to a rotation in transverse time and is by postulate unobservable. Only RTT invariant terms like E^*E are observable. Conjugation reverses the sign of *i* . If one thinks of a set of axes $\gamma_{t1},\gamma_{t2},\gamma_{t3}$ then conjugation in effect changes the order of γ_{t1} and γ_{t2} and so changes the '*handedness*' or *chirality* of time Features of temporal chirality are discussed further after discussing the modal solutions to Maxwell's equations. The fields, as at present defined, remain with *positive* chirality until further notice.

5.5 RTT invariant power

The real world of power and energy can be related to the complex world through standard techniques for Poynting's theorem [**46**] giving:

$$\iint_S [(\mathbf{E}^* \times \mathbf{B}) + (\mathbf{E} \times \mathbf{B}^*)].\mathbf{n}\, dS$$
$$+ \iiint_V [(\mathbf{E}^*. \mathbf{J}) + (\mathbf{E} . \mathbf{J}^*)]\, dV =$$
$$- \iiint_V \partial_t [(\mathbf{E}^*. \mathbf{E}) + (\mathbf{B}^*. \mathbf{B})]\, dV \qquad (47)$$

This is interpreted in the usual way. The outward power flow from the surface combined with the energy lost within the enclosed volume equals the rate of loss of the stored energy. This form of Poynting's theorem is RTT invariant {e.g. $\mathbf{E}^*. \mathbf{E} = [\exp(i\theta)\, \mathbf{E}]^*[\exp(i\theta)\, \mathbf{E}]$ etc.}. Hence energy and power flow are RTT 'observables'.

5.6 Normal Modes

The field pair {**E** , **B**} can be treated in the same manner as Cohen-Tannoudji et al. [**47**] to find normal modes with real wave-vectors **k** where $\mathbf{k.k} = k^2$; $k > 0$. Even though *i* now implies a rotation of the transverse temporal axes Ot_1 and Ot_2 by 90° one may still write $\cos kx = \frac{1}{2}[\exp(-ikx) + \exp(ikx)]$ and $\sin kx = \frac{1}{2}i[\exp(-ikx) - \exp(ikx)]$. Complex Fourier analysis of **E** and **B** can still be used e.g.:–

$$\mathbf{E}(\mathbf{r}, t) = \Sigma_k \mathbf{E}\hat{}(\mathbf{k}, t) \exp(i\, \mathbf{k.r}) \qquad (48)$$

An integral may replace the summation as appropriate but here we are focussing on a particular value of **k**. With real fields $\mathbf{F}\hat{}(\mathbf{k}, t)^* = \mathbf{F}\hat{}(-\mathbf{k}, t)$ [**47**], but this is *not true* with complex fields. From div **E** = 0 = div **B** :-

$$\mathbf{k.E}\hat{}(\mathbf{k}, t) = 0 \;;\; \mathbf{k.B}\hat{}(\mathbf{k}, t) = 0 \;; \qquad (49)$$

From vector identities: curl curl $\mathbf{E} = k^2 \mathbf{E}$.
Hence, with $k > 0$,

$$-k\,(k\,\mathbf{E}) = \partial_t (\text{curl}\, \mathbf{B}) \;;$$
$$k\,(\text{curl}\, \mathbf{B}) = \partial_t (k\,\mathbf{E}) \;; \qquad (50)$$

$$i\,\partial_t \boldsymbol{\alpha} - k\,\boldsymbol{\alpha} = 0 \;;$$
$$k\,\boldsymbol{\alpha} = (k\,\mathbf{E} + i\, \text{curl}\, \mathbf{B}) \;; \qquad (51)$$

$$i\,\partial_t \boldsymbol{\beta} + k\,\boldsymbol{\beta} = 0 \;;$$
$$k\,\boldsymbol{\beta} = (k\,\mathbf{E} - i\, \text{curl}\, \mathbf{B}) \;; \qquad (52)$$

The $\boldsymbol{\alpha}$ modes have only 'positive' frequencies, while $\boldsymbol{\beta}$ modes have only 'negative' frequencies, where 'positive' implies the variation $\exp(-ik\,t)$; $k > 0$. Complex **E** and **B** mean $\boldsymbol{\alpha}^* \neq \boldsymbol{\beta}$ unlike ref. **47** with real **E** and **B**.

Any complex signal with only one sign of frequency, creates an analytic signal [**48**] in the principal time *t* . As discussed in Appendix 1, the groupings in equs. (51) and (52) create complex vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ which are analytic in a *complex time* $t = t + i\,\tau$. The analytic normal modes, appropriately normalised by some factor N, can now be expressed as:-

$$\boldsymbol{\alpha}(\mathbf{k}) = N^{-\frac{1}{2}}\,[\mathbf{E}\hat{}(\mathbf{k}, t) - (\mathbf{k}/k) \times \mathbf{B}\hat{}(\mathbf{k}, t)] \qquad (53)$$

$$\boldsymbol{\beta}(\mathbf{k}) = N^{-\frac{1}{2}}\,[\mathbf{E}\hat{}(\mathbf{k}, t) + (\mathbf{k}/k) \times \mathbf{B}\hat{}(\mathbf{k}, t)] \qquad (54)$$

The normal modes are the outcome of Fourier analysis of space and consequently are *non-local* in space though local in k-space. To localise the energy in space, there must be a superposition of modes with different k-vectors.

From equ.(49), both **E**^ and **B**^ are normal to **k** and hence $\mathbf{k}.\boldsymbol{\alpha} = \mathbf{k}.\boldsymbol{\beta} = 0$. Consequently $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ each have two independent spatial components normal to the vector **k**. We order these components and write the scalar product $\boldsymbol{\alpha}(\mathbf{k})^*.\boldsymbol{\alpha}(\mathbf{k})$ as $\boldsymbol{\alpha}(\mathbf{k})^\dagger\,\boldsymbol{\alpha}(\mathbf{k})$ [similarly for $\boldsymbol{\beta}(\mathbf{k})$].

The stored 'energy' U and 'energy transfer' **P** are given in a normalised form as:-

$$U(\mathbf{k}, t) = \langle \tfrac{1}{2}(\mathbf{E}\hat{}^*.\mathbf{E}\hat{} + \mathbf{B}\hat{}^*.\mathbf{B}\hat{})\rangle = (1/2)N\langle\, \boldsymbol{\alpha}(\mathbf{k})^\dagger\boldsymbol{\alpha}(\mathbf{k}) + \boldsymbol{\beta}(\mathbf{k})^\dagger\boldsymbol{\beta}(\mathbf{k})\rangle \qquad (55)$$

$$\mathbf{P}(\mathbf{k}, t).(\mathbf{k}/k) = \langle \tfrac{1}{2}\,(\mathbf{k}/k).[\mathbf{E}\hat{}\times\mathbf{B}\hat{}^* + \mathbf{E}\hat{}^*\times\mathbf{B}\hat{}]\rangle = (1/2)N\langle\, \boldsymbol{\alpha}(\mathbf{k})^\dagger\boldsymbol{\alpha}(\mathbf{k}) - \boldsymbol{\beta}(\mathbf{k})^\dagger\boldsymbol{\beta}(\mathbf{k})\rangle \qquad (56)$$

where < > denotes integration over some volume, say V. It is tempting to assert that the β modes with negative frequencies have negative energy [**26**] , but equ. 55 shows a positive contribution to U(**k**). While equ. 56 shows that the α and β contributions to **P**(**k**) are in the same direction as the group velocity for these modes.

5.7 Promotion/demotion/annihilation

Only one value of wave-vector **k** is considered for the moment. Define the 'rotor'

$$\Theta(\mathbf{k}, k) = \exp[i(\mathbf{k.r} - kt)] \qquad (57)$$

The normal modes can be designated as a 'retarded' mode $\boldsymbol{\alpha}(\mathbf{k}) = A_k\,\Theta(\mathbf{k}, k)$ and an 'advanced' mode $\boldsymbol{\beta}(\mathbf{k}) = B_k\,\Theta(\mathbf{k}, -k)$ with an opposite group velocity to $\boldsymbol{\alpha}(\mathbf{k})$. The amplitudes A_k and B_k are to be normalised later. Now consider a 'modulator' $\mathbf{a}_\kappa^\dagger = A_\kappa\Theta(\boldsymbol{\kappa}, \kappa)$, ($\kappa > 0$, $\boldsymbol{\kappa} \parallel \mathbf{k}$) as another analytic signal which *modulates* $\boldsymbol{\alpha}(\mathbf{k})$ in frequency and thereby generates (or promotes) a higher frequency analytic mode as follows:-

$$\boldsymbol{\alpha}(\mathbf{k} + \boldsymbol{\kappa}) = \mathbf{a}_\kappa^\dagger\,\boldsymbol{\alpha}(\mathbf{k}) \qquad (58)$$

where $\boldsymbol{\alpha}(\mathbf{k}+\boldsymbol{\kappa}) = A_{(k+\kappa)}\,\Theta(\mathbf{k}+\boldsymbol{\kappa}, k+\kappa)$ with $(k+\kappa) > 0$. The normalisation again will be determined later. Similarly a 'modulator' $\mathbf{a}_\kappa = A'_\kappa\Theta(\boldsymbol{\kappa}, \kappa)^{-1}$ ($\kappa > 0$, $\boldsymbol{\kappa} \parallel \mathbf{k}$), can modulate the fields and demote $\boldsymbol{\alpha}(\mathbf{k})$ to a new lower frequency mode:

$$\boldsymbol{\alpha}(\mathbf{k} - \boldsymbol{\kappa}) = \mathbf{a}_\kappa\,\boldsymbol{\alpha}(\mathbf{k}) \qquad (59)$$

where $\boldsymbol{\alpha}(\mathbf{k}-\boldsymbol{\kappa}) = A_{(k-\kappa)}\,\Theta(\mathbf{k}-\boldsymbol{\kappa}, k-\kappa)$: $(k-\kappa) > 0$ Again, normalisation will be determined later.

As discussed, Fourier analysis in space gives non-local amplitudes so that the normal modes $\boldsymbol{\alpha}(\mathbf{k})$ and $\boldsymbol{\beta}(\mathbf{k})$ are non-local. Changing $\boldsymbol{\alpha}(\mathbf{k})$ to $\boldsymbol{\alpha}(\mathbf{k}\pm\boldsymbol{\kappa})$ gives frequency modulation resulting in a new non-local modal amplitude. If the energy is to be localised then there has to be a group of such modes. Modulation is the start of producing a group of relevant non-local modes.

This process of modulation by a signal $\mathbf{a}_\kappa^\dagger$ may look trivial but $\boldsymbol{\alpha}$ is a complex *analytic* function of time with positive frequencies. Multiplying analytic functions together (modulation) has special rules [**48,49**]. If the modulation process attempts to make $(k-\kappa) \le 0$ then, from the Appendix 1, $\mathbf{a}_\kappa\,\boldsymbol{\alpha}(\mathbf{k}) = 0$. This result for analytic signals states that *attempts at reversing the sign of the frequency in an analytic signal, through modulation, results in making that signal zero*: *i.e. causes annihilation.* The real and imaginary components in the modulated signal are each cancelled. This is standard complex signal analysis; it does not require quantum theory. However, just as in quantum theory, promotion and demotion operations can result in 'ladders' of frequencies $k \pm N\kappa$ with corresponding wave-vectors. The frequency κ is not at present restricted to any particular value but care must be taken if 'annihilation' occurs.

Frequency reversal through modulation annihilates a mode, however frequency reversal *is* permissible via conjugation. From the Appendix 1, $\boldsymbol{\alpha}$ is analytic in the lower half complex plane of t while $\boldsymbol{\alpha}^*$ is analytic in the upper half plane. The demotion (promotion) operator for $\boldsymbol{\alpha}^*$ is equivalent to the promotion (demotion) operator for $\boldsymbol{\alpha}$ but still any reversal of frequency by modulation leads to annihilation. It follows that for the conjugated normalised fields one may write

$$\boldsymbol{\alpha}(\mathbf{k} + \boldsymbol{\kappa})^\dagger = \boldsymbol{\alpha}(\mathbf{k})^\dagger\,\mathbf{a}_\kappa;\;\; \boldsymbol{\alpha}(\mathbf{k} - \boldsymbol{\kappa})^\dagger = \boldsymbol{\alpha}(\mathbf{k})^\dagger\,\mathbf{a}_\kappa^\dagger \qquad (60)$$

Now in this case the modulation operators $\mathbf{a}_\kappa$ and $\mathbf{a}_\kappa^\dagger$ are to operate backwards on the modal vectors $\boldsymbol{\alpha}(\mathbf{k})^\dagger$. Again it is essential that $(k-\kappa) > 0$ if annihilation is to be avoided.

Similar promotion and demotion operations arise for $\boldsymbol{\beta}(\mathbf{k})$. These are $\mathbf{b}_\kappa^\dagger = B_\kappa\Theta(\boldsymbol{\kappa}, -\kappa)$ and $\mathbf{b}_\kappa = B'_\kappa\Theta(\boldsymbol{\kappa}, -\kappa)^{-1}$ with the same rule for analytic signals that annihilation occurs whenever one attempts to force $(k-\kappa) \le 0$.

Indeed $\boldsymbol{\beta}^*(-\mathbf{k})$ and $\boldsymbol{\alpha}(\mathbf{k})$ have identical analytic properties.

These are key steps. Complex Maxwell's equations, with analytic modes, automatically mean that modulation can promote, demote and annihilate these modes creating ladders of frequencies. However a specific ladder has not yet been established. Annihilation does not require any appeal to quantised harmonic oscillators or minimum energy states.

6. SPACE-TIME VORTICES

6.1 Concept

Equs. (55, 56) are disturbing because, for a given $\mathbf{k}$ ($k > 0$), they contain both retarded and advanced waves. Traditionally the advanced solutions are discarded, but not here. Now these equations 55/56 can be repeated with $\mathbf{k}$ replaced by $-\mathbf{k}$. In this case $\boldsymbol{\beta}(-\mathbf{k}) = B_{-k}\,\Theta(-\mathbf{k}, -k)$ is the retarded wave while $\boldsymbol{\alpha}(-\mathbf{k}) = A_{-k}\,\Theta(-\mathbf{k}, k)$ is the advanced wave. It will be proposed that, for one given temporal chirality, the energy is transported through the agency of two retarded waves $\boldsymbol{\alpha}(\mathbf{k})$, $\boldsymbol{\beta}(-\mathbf{k})$ *and* two advanced waves $\boldsymbol{\beta}(\mathbf{k})$, $\boldsymbol{\alpha}(-\mathbf{k})$ with all four contributing positively to the energy U within any volume V but β-modes contributing negatively to the power flow P (equ.56). The simultaneous appearance of positive and negative frequencies is required in any temporal Fourier analysis where any change of field $\delta\mathbf{F}$ occurs at a definite time so that $\delta\mathbf{F}(t) = 0$ for $t < t_o$ [**49**]. Feynman used this argument to justify anti-particles [**50**]. Related arguments also justify the appearance of both +/–$\mathbf{k}$ wave-vectors.

The concept of the space-time vortices (STV) is that with a single 'direction' of wave-vector, both advanced and retarded waves with both positive and negative frequencies are excited. That is four waves appear with wavevectors +/– $\mathbf{k}$ and frequencies +/–k for a given $\mathbf{k}$**.** Signals circulate in time and space. The STV may not look like a conventional whirlpool, nevertheless one unit of energy is assumed to be associated with each vortex loop: a loop being one complete wavelength along the combined forward *and* reverse paths in either α or β modes. Increasing the number of loops increases the energy, while decreasing the number of loops decreases the energy. Energy per loop *for a given type of excitation* will be assumed constant - but yet to be determined.

A brief comparison may be made between an STV and a *lossless* resonant microwave cavity [**51**]. The number of vortex loops within a fixed length, like the number of nodes in a resonant cavity is a relativistic invariant and is proportional to the frequency. So the energy per vortex loop must be proportional to the frequency for compatibility with special relativity.

6.2 Vortex excitation and detection

It is supposed that vortices already exist. For example, the quantum mechanical ground state could, in this work, be replaced with a ground state of vortices. The problem is how to excite and detect the vortices so that there is a uni-directional flow of energy without violation of causality or special relativity.

Equ. 36 show that modes are excited through $\mathbf{J}^*.\mathbf{E}$ or $\mathbf{J}.\mathbf{E}^*$. Although the detailed interaction has yet to be worked out, $\mathbf{J}$ must be the source of the modulators $\mathbf{a}^\dagger_\kappa = A_\kappa\,\Theta(\boldsymbol{\kappa}, \kappa)$ and similarly the source of $\mathbf{a}_\kappa$. Writing a complex field $\boldsymbol{\alpha} = \boldsymbol{\alpha}_{t1} + i\,\boldsymbol{\alpha}_{t2}$ is a short cut for $\gamma_{t1}\boldsymbol{\alpha} = \gamma_{t1}\boldsymbol{\alpha}_{t1} + \gamma_{t2}\,\boldsymbol{\alpha}_{t2}$. Then noting that $i\,\gamma_{t1} = -\gamma_{t1} i$, the application of GA shows that the order of modulation matters:

$$\mathbf{a}^\dagger_\kappa \mathbf{a}^\dagger_\kappa \gamma_{t1}\boldsymbol{\alpha} = \gamma_{t1}(\boldsymbol{\alpha}\,\mathbf{a}_{2\kappa})\,;$$
$$\mathbf{a}^\dagger_\kappa \gamma_{t1}\boldsymbol{\alpha}\,\mathbf{a}^\dagger_\kappa = \gamma_{t1}\,\boldsymbol{\alpha} \qquad (61)$$

It can be seen, dependent on order of events, how the 'same' double modulation can give either a double change in the frequency of a mode, or leave the frequency unaltered.

As noted, $\boldsymbol{\alpha}(\mathbf{k})$ and $\boldsymbol{\beta}(\mathbf{k})$ are spatially non-local mode amplitudes. Frequency modulation of $\boldsymbol{\alpha}(\mathbf{k})$ and $\boldsymbol{\beta}(\mathbf{k})$ remains a non-local effect. Consequently if one modulates $\boldsymbol{\alpha}(\mathbf{k})$ and also maintains that each vortex loop of the α modes represents a unit of energy, then such modulation has given a non-local change of energy! If this was the only effect, then it would be at variance with all concepts of causality and special relativity. However by modulating all four waves (wavevectors +/– $\mathbf{k}$ and frequencies +/–k) a possible method is found of conserving the non-local energy but yet obtaining energy transfer.

Modulate the retarded mode $\boldsymbol{\alpha}(\mathbf{k}) \to \boldsymbol{\alpha}(\mathbf{k} + \boldsymbol{\kappa})$ *and* the advanced mode $\boldsymbol{\alpha}(-\mathbf{k}) \to \boldsymbol{\alpha}(-\mathbf{k} - \boldsymbol{\kappa})$. These two waves together will form vortex loops (c.f. standing waves as in a resonant cavity) with the modulation increasing the numbers of α vortex loops from the original $\boldsymbol{\alpha}(\mathbf{k})$ and $\boldsymbol{\alpha}(-\mathbf{k})$. The *increase* in numbers of vortex loops is compensated by a simultaneous modulation of the advanced mode $\boldsymbol{\beta}(\mathbf{k}) \to \boldsymbol{\beta}(\mathbf{k} - \boldsymbol{\kappa})$ and the retarded mode $\boldsymbol{\beta}(-\mathbf{k}) \to \boldsymbol{\beta}(-\mathbf{k} + \boldsymbol{\kappa})$ giving a *reduction* in the number of β vortex loops. There would then be no net non-local change in the number of loops and thus no additional non-local change of energy. The quantity $[\mathbf{P}(\mathbf{k}).\mathbf{k} + \mathbf{P}(-\mathbf{k}).(-\mathbf{k})]$ gives an inward flow of energy in the +**k** fields compensated by an outward flow of energy in the –**k** fields at the excitation. There is then a reaction back on the exciting electron from the advanced fields, as proposed by Wheeler and Feynman [**52**].

Detection of these excited vortices is assumed to be an entirely separate process from excitation, occuring at a different time. It requires a further modulation of the modes, now by the detecting electron. This modulation of the modes discerns the difference between advanced and retarded waves but observes the same rule that the total number of vortex loops over any extensive non-local region shall not change in order not to create global energy changes from a local modulation. This could be achieved through modulations which change $\alpha(\mathbf{k}+\boldsymbol{\kappa}) \rightarrow \alpha(\mathbf{k}+2\boldsymbol{\kappa})$, $\alpha(-\mathbf{k}-\boldsymbol{\kappa}) \rightarrow \alpha(-\mathbf{k}-\boldsymbol{\kappa}+\boldsymbol{\kappa})$, $\beta(\mathbf{k}-\boldsymbol{\kappa}) \rightarrow \beta(\mathbf{k}-\boldsymbol{\kappa}+\boldsymbol{\kappa})$, $\beta(-\mathbf{k}+\boldsymbol{\kappa}) \rightarrow \beta(-\mathbf{k}+2\boldsymbol{\kappa})$. Considering the inteference of the advanced and retarded waves in any length L, it is found that the number of α-vortex loops remain as $(k+\kappa)L$ while the number of β-vortex loops remains as $(k-\kappa)L$. Non-local loop numbers are conserved and global energy is not changed. However there is a power flow P: $=(1/2)N <[\alpha^\dagger\ \alpha_{(\mathbf{k}+2\kappa)} - \beta^\dagger\ \beta_{(-\mathbf{k}+2\kappa)}]_{forward} + [\alpha^\dagger\ \alpha_{(-\mathbf{k})} - \beta^\dagger\ \beta_{(\mathbf{k})}]_{reverse}>$ (62)

The final energy U must be similarly be given from:

$$U = (1/2)N<[\alpha^\dagger\ \alpha_{(\mathbf{k}+2\kappa)} + \beta^\dagger\ \beta_{(-\mathbf{k}+2\kappa)}]_{forward} + [\alpha^\dagger\ \alpha_{(-\mathbf{k})} + \beta^\dagger\ \beta_{(\mathbf{k})}]_{reverse}> \quad (63)$$

Here the modes have been re-grouped into forward waves and reverse waves. The energy transferred in the reverse direction $[\alpha^\dagger\ \alpha_{(-\mathbf{k})} - \beta^\dagger\ \beta_{(\mathbf{k})}]_{reverse}$ balances out to zero with the normalisation for α and β modes being taken to be the same. It has already been noted that $\beta(-\mathbf{k}+2\boldsymbol{\kappa})^*$ has the same analytic properties as $\alpha(\mathbf{k}-2\boldsymbol{\kappa})$ so with normalisation, the scalar $\beta^\dagger\ \beta_{(-\mathbf{k}+2\kappa)}$ may be numerically calculated from $\alpha^\dagger\ \alpha_{(\mathbf{k}-2\kappa)}$ giving the forward energy and power flow as if in positive frequency modes alone:

$$U_f = ½N<[\alpha^\dagger\ \alpha_{(\mathbf{k}+2\kappa)} + \alpha^\dagger\ \alpha_{(\mathbf{k}-2\kappa)}]> \quad (64)$$

$$P_f = ½N<[\alpha^\dagger\ \alpha_{(\mathbf{k}+2\kappa)} - \alpha^\dagger\ \alpha_{(\mathbf{k}-2\kappa)}]> \quad (65)$$

Now use the promotion and demotion operators of section 4, but with a frequency of 2κ, instead of κ, to give:

$$U_f = ½N<[\alpha_{\mathbf{k}}{}^\dagger \mathbf{a}_{2\kappa}\ \mathbf{a}_{2\kappa}{}^\dagger\ \alpha_{\mathbf{k}} + \alpha_{\mathbf{k}}{}^\dagger \mathbf{a}_{2\kappa}{}^\dagger\ \mathbf{a}_{2\kappa}\ \alpha_{\mathbf{k}}]> \quad (66)$$

$$P_f = ½N<[\alpha_{\mathbf{k}}{}^\dagger \mathbf{a}_{2\kappa}\ \mathbf{a}_{2\kappa}{}^\dagger\ \alpha_{\mathbf{k}} - \alpha_{\mathbf{k}}{}^\dagger \mathbf{a}_{2\kappa}{}^\dagger\ \mathbf{a}_{2\kappa}\ \alpha_{\mathbf{k}}]> = U_o \quad (67)$$

Here U_o is the smallest unit of energy transfer associated with the frequency of modulation given from the excitation frequency 2κ. The *observable* promotion and demotion operators come as $\mathbf{a}_{2\kappa}{}^\dagger$ and $\mathbf{a}_{2\kappa}$ and it is only these that have to be evaluated.

The two distinct stages, excitation and detection, are proposed as essential features of this new theory drawing on GA. The minimum energy transferred per interaction is the energy associated with a double vortex loop (eg energy in one whole wavelength of the 2κ modulation). This final energy *transfer* U_o is given from P connected with the forward waves with a group velocity *c*. The detection process cannot succeed until the energy has arrived, localised by a group of k-vectors. There is no violation of causality in spite of non-local fields.

6.3 Quantisation

Assume that equations 66/67 hold for all k-vectors accessible by promotion, demotion and annihilation using the operators $\mathbf{a}_{2\kappa}$. Then these operator equations are identical to those for the quantised harmonic oscillator with a frequency 2κ. The standard operator algebra leads to the same ladder of quantised frequencies, wave-vectors and modes as in the harmonic oscillator with $\mathbf{k} = \mathbf{k}_N = (2N+1)\boldsymbol{\kappa}$. However it is only the algebra that is used and the route to quantisation here is through: (i) the properties of analytic signals; (ii) a concept of conservation of vortex loops ; (iii) that energy is transferred by the retarded mode alone. Re-calling the operator algebra , one then finds:

$$< \alpha(\mathbf{k}_N)^\dagger\ \mathbf{a}_{2\kappa}\ \mathbf{a}_{2\kappa}{}^\dagger \alpha(\mathbf{k}_N) > = (N+1)U_o\ ; \quad (68)$$

$$< \alpha(\mathbf{k}_N)^\dagger\ \mathbf{a}_{\kappa}{}^\dagger\ \mathbf{a}_{\kappa}\ \alpha(\mathbf{k}_N)\ > = NU_o \quad (69)$$

Consistency requires that $\mathbf{k}_0 = \boldsymbol{\kappa}$ is the ground state before annihilation with $U_o \propto 2\kappa$ so that, as already noted, the energy per loop is linked with frequency: the 'modulator' frequency.

Another point of consistency concerns the ground state (wave-vector **κ)** which is never actually observed. In a *lossless* resonant e.m. cavity, it is argued that the fields have been excited but not detected. Any 'detection' of the standing waves is only by minor sampling of the fields. The vortex scheme then argues that the ground state mode $\alpha(\boldsymbol{\kappa})$ is modulated by the excitation component $\mathbf{a}_{\kappa}{}^\dagger$ changing $\alpha(\boldsymbol{\kappa})$ to $\alpha(2\boldsymbol{\kappa})$. The second modulation for detection does not apply because the cavity is lossless. The longest wavelength of any such trapped observable α mode is then correctly given from $\lambda = 2\pi/2\kappa$, as measured in a resonant cavity.

7. CHIRALITY

Temporal chirality, mentioned in Sec.5.4, is now discussed more fully. The implicit assumption is that γ_{t1},γ_{t2} and γ_{t3} form a 'right handed' set of temporal axes along with a right handed set of spatial axes γ_{s1},γ_{s2} and γ_{s3}. Interchanging $\gamma_{t1}$and γ_{t2} changes $i = \gamma_{t1}\gamma_{t2}$ into $-i$ and changes right handed temporal chirality into left handed temporal chirality. These left and right handed chiral systems are independent. For example if a 'right handed' system has fields $\mathbf{E}_+ - iS\,\mathbf{B}_+ = [(\mathbf{E}_r+i\mathbf{E}_i) - iS(\mathbf{B}_r+i\mathbf{B}_i)]$ and 'left handed' fields have $\mathbf{E}_- + iS\,\mathbf{B}_- = [(\mathbf{E}'_r - i\mathbf{E}'_i) - iS(\mathbf{B}'_r - i\mathbf{B}'_i)]$ then these fields may be combined to give a new mixed chirality field:

$$\mathbf{E} + iS\,\mathbf{B} = (\mathbf{E}_r+i\mathbf{E}_i+\mathbf{E}'_r-i\mathbf{E}'_i) - iS\,(\mathbf{B}_r+i\mathbf{B}_i-\mathbf{B}'_r+i\mathbf{B}'_i) \qquad (70)$$

Although $(\mathbf{E}_+ - iS\,\mathbf{B}_+)^*$ has left handed chirality, it is not necessarily equal to $(\mathbf{E}_- + iS\mathbf{B}_-)$. With *classical* Maxwell equations there are only real fields **E** and **B**. The additional freedom caused by temporal chirality has important consequences.

Positive frequency modes with 'positive' temporal chirality are now written with a subscript '+' as $\boldsymbol{\alpha}_+ = (k\ \mathbf{E}_+ + i\ \mathrm{curl}\,\mathbf{B}_+)$. If nothing was done, changing the chirality would make $\boldsymbol{\alpha}_-$ have a negative frequency. To keep the notation that all α-modes have positive frequencies, the α and β modes are simultaneously simply re-labelled on changing temporal chirality. 'Negative' temporal chirality is thus written as $\boldsymbol{\alpha}_- = (k\,\mathbf{E}_- + i\ \mathrm{curl}\,\mathbf{B}_-)$. The whole system of promotion, demotion, annihilation and quantisation works for the these independent $\boldsymbol{\alpha}_-$ (and $\boldsymbol{\beta}_-$) modes in exactly the same way as for $\boldsymbol{\alpha}_+$ (and $\boldsymbol{\beta}_+$).

The independence of $\boldsymbol{\alpha}_+$ and $\boldsymbol{\alpha}_-$ can be further emphasised by requiring that

$$\boldsymbol{\alpha}_+(\mathbf{k})^\dagger\,\boldsymbol{\alpha}_-(\mathbf{k}) = \boldsymbol{\alpha}_-^\dagger(\mathbf{k})\,\boldsymbol{\alpha}_+(\mathbf{k}) = 0\,. \qquad (71)$$

then

$$\boldsymbol{\alpha}^\dagger(\mathbf{k})\,\boldsymbol{\alpha}(\mathbf{k}) = \boldsymbol{\alpha}_+^\dagger(\mathbf{k})\,\boldsymbol{\alpha}_+(\mathbf{k}) + \boldsymbol{\alpha}_-^\dagger(\mathbf{k})\,\boldsymbol{\alpha}_-(\mathbf{k}).$$

and similarly

$$\boldsymbol{\beta}^\dagger(\mathbf{k})\,\boldsymbol{\beta}(\mathbf{k}) = \boldsymbol{\beta}_+^\dagger(\mathbf{k})\,\boldsymbol{\beta}_+(\mathbf{k}) + \boldsymbol{\beta}_-^\dagger(\mathbf{k})\,\boldsymbol{\beta}_-(\mathbf{k}).$$

The energies of the two temporal chiralities of the electromagnetic field have to be added together: these energies are independent. Equ. 71 is shown next to lead to correlated polarisations in the 2 independent modal chiralities.

8. POLARISATION

8.1 Stokes Parameters

With fields which are fundamentally complex there can be a lack of physical clarity as to what is meant by polarisation. Stokes parameters [**53**] can determine real physical polarisations from complex fields:

The parameters, below, can be interpreted unambiguously in terms of polarisations.

$$\left.\begin{aligned} K_0 &= \alpha_x{}^*\alpha_x + \alpha_y{}^*\alpha_y \\ K_1 &= \alpha_x{}^*\alpha_y + \alpha_y{}^*\alpha_x \\ K_2 &= i\alpha_y{}^*\alpha_x - i\alpha_x{}^*\alpha_y \\ K_3 &= \alpha_x{}^*\alpha_x - \alpha_y{}^*\alpha_y \end{aligned}\right\} \qquad (72)$$

Matrices $\boldsymbol{\pi} = \{\pi_1, \pi_2, \pi_3\}$ are defined :

$$\pi_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\ ;\ \pi_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\ ;\ \pi_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad (73)$$

These Pauli matrices are being used for their convenient algebra. Reduce the normal modes as before to their orthogonal components perpendicular to the vector direction **k**. Normalise so that $K_{+0} = \boldsymbol{\alpha}_+(\mathbf{k})^\dagger\boldsymbol{\alpha}_+(\mathbf{k}) = 1$. Define a 3-component array $\mathbf{K}_+ = \{K_{+n}\}$ with $\boldsymbol{\pi} = \{\pi_n\}$ so that for positive temporal chirality:

$$\boldsymbol{\alpha}_+(\mathbf{k})\,\boldsymbol{\alpha}_+(\mathbf{k})^\dagger = 1 + \mathbf{K}_+.\boldsymbol{\pi} \qquad (74)$$

and similarly for negative temporal chirality $\boldsymbol{\alpha}_-$ and $\mathbf{K}_-$. Then one notes that:

$$\boldsymbol{\alpha}_+(\mathbf{k})\,\boldsymbol{\alpha}_+(\mathbf{k})^\dagger\boldsymbol{\alpha}_-(\mathbf{k})\,\boldsymbol{\alpha}_-(\mathbf{k})^\dagger$$

$$= (1 + \mathbf{K}_+.\boldsymbol{\pi})(1 + \mathbf{K}_-.\boldsymbol{\pi})$$
$$= 1 + \mathbf{K}_+.\mathbf{K}_- + i\,\boldsymbol{\pi}.(\mathbf{K}_+ \times \mathbf{K}_-) \qquad (75)$$

Equation 75 has used the commutation properties of Pauli matrices. Equ. 71 now requires that equ. 75 is zero. This means that $\mathbf{K}_+$ is 'anti-parallel' to $\mathbf{K}_-$ making $\mathbf{K}_+.\mathbf{K}_- = -1$ and $\mathbf{K}_+ \times \mathbf{K}_- = 0$. In other words the two chiralities have correlated polarisations. Table 2 below interprets this correlation.

Table 2 Correlation of Polarisation

$\mathbf{K}_{1+} = 1$	$\alpha_x = 1/\sqrt{2}$	$\alpha_y = 1/\sqrt{2}$	linear: +45;
$K_{1-} = -1$	$\alpha_x = -1/\sqrt{2}$	$\alpha_y = 1/\sqrt{2}$	linear: –45;
$\mathbf{K}_{2+} = 1$	$\alpha_x = 1/\sqrt{2}$	$\alpha_y = i/\sqrt{2}$	circular; +;
$K_{2-} = -1$	$\alpha_x = 1/\sqrt{2}$	$\alpha_y = -i/\sqrt{2}$	circular: –;
$\mathbf{K}_{3+} = 1$	$\alpha_x = 1$	$\alpha_y = 0$	linear; vert;
$K_{3-} = -1$	$\alpha_x = 0$	$\alpha_y = 1$	linear; horiz.;

8.2 'Instantaneous' knowledge of polarisation around the vortex

Excitation of STVs creates a non-local web of vortices from normal modes with +/–**k** vectors and +/– k frequencies defined from $\boldsymbol{\alpha}_\mathbf{k}$ and $\boldsymbol{\beta}_\mathbf{k}$. Like Fourier analysis of local excitations, the spatial Fourier components are the same every-where. Again comparison may be made between an STV and a *lossless* resonant microwave cavity. Information about polarisation is known over the whole cavity and similarly over the whole STV. Causality is not violated by the instantaneous non–local knowledge within the STV because energy transfer requires the co-operation of adjacent k-vectors. Differences in these k-vectors have to travel with c_g , the group velocity of light. The proposed vortex system provides instantaneous knowledge about chirality and polarisation over space but energy extraction occurs locally and causally as determined by c_g.

9. ENTANGLEMENT

Einstein first highlighted the problems of entanglement [**5**]. 'Photon entanglement' covers experiments where measurements of polarisation on two (or more) photons are always correlated even though the photons are too far apart to be able to communicate at the speed of light in the time available between the measurements [**54**]. Mermin emphasises the role of correlations in QT [**55**]. An explanation using vortex concepts could be as follows.

Suppose that at the origin, vortices (photons) *'L'* and *'R'* are excited. The fields are waves with both +**k** and –**k**. So that after excitation energy propagates in both directions. Because two photons are produced together, the energy is split between the two temporal chiralities and hence the polarisations will be correlated as in table 2. Without any measurement there are too many degrees of freedom to determine which chirality and polarisation belongs to *'L'* or *'R'*. The boundary conditions for the detection in the +**k** direction ('*R*') will determine what polarisation can be detected at 'R' and with what chirality. The detection process cancels the advanced modal energy in that chirality. As soon as 'R' is detected, the only energy remaining is in the other chirality and hence the polarisation of 'L' is determined. There is no communication between L and R because the correlation was determined at the outset as part of the global space-time vortex excitation.

However no use can be made of this correlation of polarisations as the energy has not arrived at 'L'. Advanced waves in the +**k** direction now become retarded waves when viewed in the –**k** direction. The detection of 'L' via retarded waves then removes the energy and has to have the polarisation correlated with 'R'. There is no 'spooky action' at a distance.

It is envisaged that in a one dimensional spatial system, such as a 'lossless' optical fibre, the vortices extend over the whole of the fibre. With a non-focussed system in space, just taking +/– **k** with a single k-vector is no longer appropriate. The spherical waves require superpositions of k-vectors which will effectively weaken in their probability of interaction as one moves away from the source. This is not considered here. The role of source coherence is also a factor that may play a significant role.

10. CONCLUSIONS

A new interpretation of Maxwell's equations is explored where 3 dimensions are introduced into time but with a preferred 'transverse' temporal plane. Rest mass is interpreted from variations within this transverse plane. The concept is consistent with the Klein-Gordon and Dirac equations and also with the sero mass for photons in Maxwell's classic equations. The extra dimensions provide an explanation of charge and the apparent lack of magnetic sources for the B-fields.

Maxwell's equations remain intact except that there are additional degrees of freedom for the spatial fields through the fields being treated as complex. Promotion, demotion and annihilation emerge as features of the complex analysis and do not have to appeal to quantum theory of harmonic oscillators – other than to make convenient use of the algebra.

A concept of space-time vortex loops is advanced. With a 'single' k-vector, fields circulate back and forth in space and time. The energy in the fields is determined from the number of vortex loop. To remove/add energy at the boundaries requires careful modulation of the modes so as to preserve the numbers of vortex loops and ensure that global energy is not altered in an unphysical ways. Unidirectional energy flow and quantisation follow.

So much has been written on the foundations and philosophy of quantum theory [refs **56**, **57**, **58**, **59**, **60** give a *very* small sample] that to review these in relation to this preliminary account of space-time vortex theory would be too lengthy and premature. Although a review remains for future work, accounts by Cramer [**1**] and Mermin [**63**] must be acknowledged as key elements in shaping the author's views. It could also be claimed that there are elements of Everett's work [**68**] in having many different solutions that are compatible with the observable boundary conditions but there is no suggestion here of moving between different simultaneously existing worlds. Random electro-dynamics of Marshal and others is also relevant [**61,62,63,64**]. The additional degrees of freedom with complex Maxwell equations, combined with the possibility of only observing terms like E*E mean that there are a multiplicity of differently phased solutions for any given E*E. The interference of all these differently phased fields could lead to random fields and so to random electrodynamics.

The major difference between STV concepts and previous philosophical accounts of Quantum Theory is that STVs *derive* quantisation of the electromagnetic field: other work interprets quantum theory. STVs offer new routes for future exploration, notably: (i) calculation of particle masses from appropriate evaluation of the transverse temporal operators or perhaps finding charge/mass ratios; (ii) better understanding of entanglement with importance for quantum computing; (iii) higher order interactions that might even provide new if speculative ideas about unifying gravity and quantum theory. Acceptance of vortex theory could give a 'crisp definition' [**65**] of the difference between classical and quantum measurements. Quantum measurements reconcile the boundary conditions set by transverse time. Classical measurements have no knowledge of transverse time. Could this point towards a 'completion' that was sought by Einstein [**5**]?

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APPENDIX 1. FOURIER ANALYSIS AND ANNIHILATION OF ANALYTIC SIGNALS [48, 49]

Although there are two temporal chiralities for all the modes, the fact that these behave the same in the respect of topic in this appendix means that the subscripts +/– are omitted. Concentrating on α modes, combinations of fields $[E_1(\mathbf{k}, t)+iE_2(\mathbf{k}, t)]$ and $[B_1(\mathbf{k}, t)+i B_2(\mathbf{k}, t)]$ create complex functions of time t, $\alpha(\mathbf{k}, t)$. Concentrating only on the temporal variations, $\alpha(t)$ is synthesised from :-

$$\alpha(t) = \int_0^{\infty} \alpha^{\wedge}(k) \exp(-i\, k\, t)\, dk/2\pi \qquad \text{A.1}$$

This converges over the lower half of the complex plane $\mathbf{t} = t + i\tau$; $\tau < 0$ leaving $\alpha(t)$ analytic in $\mathbf{t}$. The spectral analysis can be determined from a contour integral over the upper half complex plane:-

$$\alpha^{\wedge}(k) = \oint_{\Gamma upper} \alpha(\mathbf{t}) \exp(i\, k\, \mathbf{t})\, \mathbf{dt} \qquad k > 0 \qquad \text{A.2}$$

With $\mathbf{t} = (t + i\,\tau)$, $\tau > 0$ and $k > 0$, Jordan's lemma allows A.2 to be written as:-

$$\alpha^{\wedge}(k) = \int_{-\infty}^{\infty} \alpha(t) \exp(i\, k\, t)\, dt \qquad k > 0 \qquad \text{A.3}$$

If $k < 0$, then the lower half complex plane has to be used

$$\alpha^{\wedge}(k) = \oint_{\Gamma lower} \alpha(\mathbf{t}) \exp(i\, k\, \mathbf{t})\, \mathbf{dt} \qquad k < 0 \qquad \text{A.4}$$

$\alpha(\mathbf{t})$ is analytic within Γ_{lower} so there are no poles. Hence $\alpha^{\wedge}(k) = 0$ for $k < 0$.

Now modulate $\alpha(t)$ by $\exp(-i\, K\, t)$ to form $\alpha_P(t) = [\exp(-i\, K\, t)\ \alpha(t)]$ with $K > 0$

$$\alpha^{\wedge}{}_P(k') = \oint_{\Gamma upper} \alpha_P(\mathbf{t}) \exp(i\, k'\, \mathbf{t})\, \mathbf{dt} \qquad \text{A.5}$$

For $k' > K > 0$, A.5 (like A.3) can be limited to the real time axis to show:-

$$\alpha^{\wedge}{}_P(k') = \alpha^{\wedge}(k'-K) \ ; \qquad \text{A.6}$$

Given an upper frequency $k_o = k' - K$ in $\alpha^{\wedge}$, the highest frequency in the promoted frequency $\alpha^{\wedge}{}_P$ is $k' = k_o + K$.

If $\alpha(t)$ is now modulated to form $\alpha_D(t) = [\exp(i\, K\, t)\ \alpha(t)]$ with $K > 0$, the frequency is demoted. The spatial spectrum is pushed down: i.e. given an upper frequency k_o in the initial system, the highest frequency in the demoted frequency is $k' = k_o - K$. If $K > k_o$ then there are no positive frequencies components and the signal is annihilated. This analysis of annihilation can be put in terms of contour integrals but for brevity a pictorial view is given in Figure 1. Given a limited range of frequencies in the positive spectrum for the initial signal, *demotion that attempts to reverse the sign of the frequency in all these components will annihilate the analytic signal.*

The same arguments apply, interchanging the upper and lower contours in the discussion, for $\beta(t)$. However preserving the clockwise path of the contour integration, the integration along the real time axis for the β modes is in the opposite sense for that part of the integration with α modes. The β modes can be said to have an opposite sense of frequency to the α modes . The rule however remains that any temporal modulation forcing frequency reversal, annihilates the mode.

Conjugation also changes the sign of the frequency. However if α is analytic in the lower half plane, α^* is analytic in the upper half plane and the change of sign in the frequency is fully consistent with the analytic regions of complex time. Frequency reversal by conjugation does not annihilate the signal.

Finally because α is analytic over the lower half of the complex t plane while β is analytic over the upper half, a linear combination such as $a\,\alpha + b\,\beta$ is not in general analytic. Hence the fields E(**r,** t) and B(**r,** t) are not analytic.

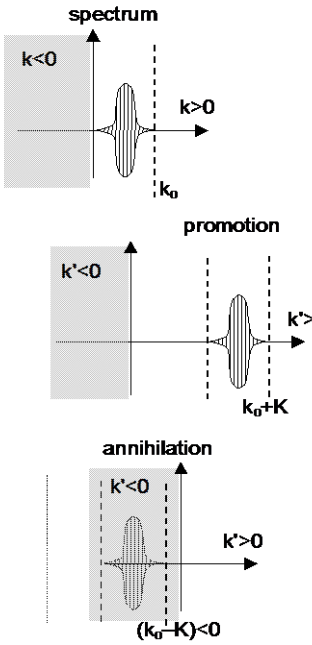

Figure 1 Promotion/Annihilation (for fields with only positive k)

APPENDIX 2. LORENTZ TRANSFORMATIONS OF E AND B FIELDS

Let the boost parameter be α and the velocity be along the spatial direction **n** (|**n**| = 1).
Define ch = cosh ½α ; sh = sinh ½α
Ch = cosh α ; Sh = sinh α

$R_L = \exp(\tfrac{1}{2}\alpha\gamma_{t3}\mathbf{n}.\boldsymbol{\gamma}_s) = \mathrm{ch} + \gamma_{t3}\,\mathbf{n}.\boldsymbol{\gamma}_s\,\mathrm{sh}$;

$\tilde{R}_L = \exp(\tfrac{1}{2}\alpha\mathbf{n}.\boldsymbol{\gamma}_s\gamma_{t3}) = \mathrm{ch} + \mathbf{n}.\boldsymbol{\gamma}_s\,\gamma_{t3}\,\mathrm{sh}$

Write F= $\mathbf{E} - iS\mathbf{B}$, then $R_L(\gamma_{t3}\mathbf{F}.\boldsymbol{\gamma}_s)\,\tilde{R}_L = \mathbf{F}_L$ gives the general Lorentz transformation. Evaluating terms gives, after some manipulation,:

$$\mathbf{F}_L = \mathrm{Ch}\,(\gamma_{t3}\,\mathbf{F}.\boldsymbol{\gamma}_s) - \mathrm{Sh}\;iS\,\gamma_{t3}\boldsymbol{\gamma}_s.(\mathbf{n}\times\mathbf{F})$$

$$-\;\gamma_{t3}\,(\mathbf{n}.\mathbf{F})\,\mathbf{n}.\boldsymbol{\gamma}_s\,(\mathrm{Ch}-1)$$

Simplifying with the special case $(\mathbf{n}.\mathbf{F}) = 0$ and equating terms with and without iS

$\mathbf{E}_L = \mathbf{E}\,\mathrm{Ch} + (\mathbf{n}\times\mathbf{B})\,\mathrm{Sh}$
$\mathbf{B}_L = \mathbf{B}\,\mathrm{Ch} - (\mathbf{n}\times\mathbf{E})\,\mathrm{Sh}$

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