Verification of the ‘essential’ GRT experiments in a scalar Lorentz-covariant gravitation.

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Abstract

An elementary model of scalar Lorentz-covariant gravitation is obtained by adequately introducing a ‘rod and clock’ affectation function into a differential Lorentz Transformation (GMLT or Gravitationally Modified Lorentz Transformation). The space-time GMLT, preserves the local invariance of the velocity of light, while the energy-momentum GMLT satisfies Einstein form covariance and the invariance of Planck’s constant. The latter transformation allows to obtain, starting from the rest energy as observed by a momentary coincident LIF (Local Inertial Frame), a Hamiltonian dynamics for a test mass or (scalar optical) light in a static gravitational field. The calculations of the basic experiments for testing GRT (Einstein’s General Relativity Theory) are done in this scalar model. For these experiments, we obtain identical results as in GRT (*i.* the gravitational redshift of spectral lines, *ii.* the deflection of light by the sun, *iii.* the precession of orbital perihelia. *iv.* the gravitational delay of radar echo.) The scalar model is reminiscent to a recent Lorentz-Poincaré physical-vacuum model as developed by Torgny Sjödin, and the precursor gravitational theories of Gunnar Nordström. I.e. based on gravitational and kinematical Lorentz contraction and dilation of space-time observations, and completed with a Hamiltonian mechanics according a modified gravitational force. We discuss some limitations of the scalar model concerning Eötvös experiments and the status of the WEP (Weak Equivalence Principle), and present possible further developments.

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1 Introduction

The ‘G+P’ issue has been raised many times (Poincaré 1905, Einstein 1921, Reichenbach 1927, Duhem 1954, Grünbaum 1973): Poincaré’s conventionalism of geometry (‘G’) seems untenable with regard to observed minute effects in gravitational physics (‘P’). The successful intrinsically Riemannian geometry of gravitation has found no equivalent of a physical gravitation with a materially affected, ‘measured’ geometry (sometimes called ‘non-geometrical’ gravitation theories). From a conservative point of view, space and time *an sich* being affected by the presence of matter is conceptually not straightforward. Evidently we can obtain a measurement of space and time only through the means of material objects or light, which themselves are prone to variability under the effect of e.g. pressure, temperature, electromagnetism, diffraction and alike.In effect all of them could lead to departure from Euclidean or Minkowskian ‘measured’ geometry, but the effects are differentiated and therefore extractable and can be cast in a separate physical theory. Attempts of gravitation in a Minkowski space — with e.g. a Lorentz-Covariant, massless, spin-two field — are only partially successful, and lead upon correction to GRT (cf. overview in (Misner, Thorne, Wheeler 1973, ch. 7)). Intrinsically mutable space-time, instead of their material receptors, is considered, in order to obtain a consistent and accurate theory of gravitation, a proven necessity (e.g. Misner, Thorne, Wheeler 1970, ch. 39), or a severe constraint on ‘alternative’ metric theories (Will, 1993, ch. 5).

Still, we will try to conceive a scalar Lorentz-covariant gravitational model, with a limited and static scope for now, in which the presence of matter influences the physical manifestation of space and time and which satisfies the four basic GRT experiments. Scalar Lorentz-covariant theories need to procure explicitly an effect of rod contraction and clock slowing, due to a gravitational agent. The ‘universality’ of the space-time effect of gravitation (Reichenbach,1927, Grünbaum, 1973) applies locally: a displacement in a gravitational potential gives a different modification of all ‘rod-clock’ materila systems effected by, in this case, a single scalar function. The program of a scalar Lorentz-covariant theory is therefore more elaborate than a metric theory; i. a model of gravitational and relativistic kinematic microphysical effects on the space-time measurement tools, in extenso, of matter, ii. local observer transformations conserving particular physical constants, iii. gravitational force and its minute effects.

Various attempts in that sense have been made (cf. references in Pocci and

Sjödin, 1980), most often invoking VSL (or variable speed of light), even the precursor theories of gravitation to GRT where conceived — more or less — along these lines (e.g. Einstein 1912, Abraham 1912, Nordström 1913 a, b). Our present approach is linked to a Lorentz-Poincaré physical vacuum model by Torgny Sjödin and relates to Nordström's scalar gravitation theory (1913 a, b). Sjödin's model allows the calculation of the 'essential' GRT experiments in not unambiguous manner: light bending and delay of radar echo by applying Fermat's principle in a field of refractive index (Sjödin 1982), the gravitational redshift of spectral lines can indirectly be deduced from a particle model using de Broglie waves (Podlaha and Sjödin 1984), while the perihelion shift of orbitals is calculated using an *ad hoc* Lagrangian containing affectation functions (Sjödin 1990). The affectation by mass, in our model, is reminiscent to Nordström's exponential dependence of mass on potential energy in his first theory (Nordström 1913 a), and somewhat less with his material space-time affectation of his second theory (Nordström 1913 b)[1]. The gravitational dependency of mass causes gravitational acceleration to depend on velocity, or the inner structure of the object (Nordström 1913 a, Brans 1997) : the WEP is badly broken, and its repercussions in Eötvös experiments should be scrutinized.

In this presentation we can not go into all elements needed in scalar Lorentz-covariant model: we skip *i.*, ramifications of gravitation on quantum structure of particles, and we will only briefly sketch *ii.*, the construction of GMLT (Gravitationally Modified Lorentz Transformation). We mainly develop *iii.*, the four basic experiments verifying GRT.

2 Lorentz Transformations with gravitational 'rod and clock' affectation

We proceed by adequately introducing a static gravitational affectation function $\Phi(\mathbf{r})$, $0 \leq \Phi(\mathbf{r}) \leq 1$, in a differential relation connecting a gravitationally affected observer S' (with 'natural' geometry) and a gravitationally unaffected observer S_0 (with 'coordinate' geometry). Adopting a *principle of gravitational affectation* in a static configuration, we let rod intervals and

[1] A clear historical critique of Nordström's theory is given by J. D. Norton (Norton 1992)

clock intervals, and the speed of light for the same event be non-directionally affected — relative to the field Φ — according:

$$d\mathbf{x} = d\mathbf{x}'\Phi(\mathbf{r}) \quad , \quad dt = \frac{dt'}{\Phi(\mathbf{r})} \quad , \quad c(\mathbf{r}) = c'\Phi(\mathbf{r})^2 \tag{1}$$

Where c' is the observed local velocity of light 'in vacuum', the usual universal constant. We emphasize that relative to the gravitationally unaffected observer S_0 we have an explicitly variable speed of light, and explicit clock dilation and rod contraction. We must take care of the interpretation of affected intervals. Let two non-coincident observers measure an identical unaffected length interval $d\mathbf{x}_1 = d\mathbf{x}_2$, then these are observer related as:

$$d\mathbf{x}'_1 = d\mathbf{x}'_2 \frac{\Phi(\mathbf{r})_2}{\Phi(\mathbf{r})_1} \tag{2}$$

while an identical rod at S'_1, with length $d\mathbf{x}'_1$, and at S'_2, with length $d\mathbf{x}'_2 = d\mathbf{x}'_1$ are, according to S_0, proportioned as:

$$d\mathbf{x}_1 = d\mathbf{x}_2 \frac{\Phi(\mathbf{r})_1}{\Phi(\mathbf{r})_2} \tag{3}$$

Suppose that S'_2 is able to free himself of gravitational influence, presumptively by asymptotic recession from the source, becoming S'_∞ with $\Phi_\infty = 1$. Then the intervals relate in the respective cases as:

$$\text{Identical coordinate length :} \quad d\mathbf{x}'_1 = d\mathbf{x}'_\infty \frac{1}{\Phi(\mathbf{r})_1} \quad \rightarrow \quad d\mathbf{x}'_1 \geq d\mathbf{x}'_\infty \tag{4}$$

$$\text{Identical natural length :} \quad d\mathbf{x}_1 = d\mathbf{x}_\infty \Phi(\mathbf{r})_1 \quad \rightarrow \quad d\mathbf{x}_1 \leq d\mathbf{x}_\infty \tag{5}$$

Which gives a consistent interpretation of the effect of gravitational rod contraction. A similar reasoning can be done for gravitational clock dilation. We remark that these static affectation principles should be among the constraints we put on microphysical behavior of matter in a gravitational field. Subsequently we introduce the effect of kinematics on rods and clocks in a given static field $\Phi(\mathbf{r})$, as a gravitationally modified Lorentz transformation (GMLT).

2.1 Space-time GMLT

We adopt the *principles of kinematic affectation* consisting of *i.*, physical Lorentz longitudinal contraction by velocity relative to the field Φ, *ii.*, physical Lorentz clock dilation by velocity relative to the field Φ. Together with the convention of *standard synchronization*, or equivalently *Einstein synchronization* (Mansouri and Sexl 1977, Sjödin 1979)[2], between momentary local affected observers, these principles lead directly to a gravitationally modified Lorentz transformation: the GMLT of S' to S_0:

$$\begin{aligned} d\mathbf{x} &= \left((d\mathbf{x}'_{\parallel} - \mathbf{u}'\ dt')\gamma\left(u'\right) + d\mathbf{x}'_{\perp}\right)\Phi\left(\mathbf{r}\right) && (6) \\ dt &= \left(dt' - \frac{\mathbf{u}'.d\mathbf{x}'}{c'^2}\right)\frac{\gamma\left(u'\right)}{\Phi\left(\mathbf{r}\right)} && (7) \end{aligned}$$

From which the inverse GMLT S_0 to S' is obtained[3]:

$$\begin{aligned} d\mathbf{x}' &= \left((d\mathbf{x}_{\parallel} - \mathbf{u}dt)\gamma\left(u\right) + d\mathbf{x}_{\perp}\right)\frac{1}{\Phi\left(\mathbf{r}\right)} && (8) \\ dt' &= \left(dt - \frac{\mathbf{u}.d\mathbf{x}}{c(\mathbf{r})^2}\right)\gamma\left(u\right)\Phi\left(\mathbf{r}\right) && (9) \end{aligned}$$

with $c(\mathbf{r})$ as in (1,c), and $\gamma\left(u\right)$ and $\mathbf{u}$ satisfying:

$$\begin{aligned} \mathbf{u} &= -\mathbf{u}'\Phi\left(r\right)^2 \\ \gamma\left(u\right) &= \gamma\left(u'\right) = \frac{1}{\sqrt{1-\frac{u'^2}{c'^2}}} = \frac{1}{\sqrt{1-\frac{u^2}{c(\mathbf{r})^2}}} \end{aligned}$$

From which we observe that the unaffected observer S_0 considers the γ factor a combined kinematic-gravitational term, due to appearance of $c = c(\mathbf{r})$. The velocity relation, angle conversion[4], and γ relations are obtained in the usual way, S_0 to S':

$$\mathbf{v}' = \frac{\mathbf{v}_{\parallel} - \mathbf{u} + \frac{\mathbf{v}_{\perp}}{\gamma(u)}}{\left(1-\frac{\mathbf{u}.\mathbf{v}}{c(\mathbf{r})^2}\right)\Phi\left(\mathbf{r}\right)^2} \quad , \quad \mathbf{v} = \frac{d\mathbf{x}}{dt}\ ,\ \mathbf{v}' = \frac{d\mathbf{x}'}{dt'} \qquad (10)$$

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[2]Synchronization decides the factors to the space expression in the time transformation. Standard synchronization and Einstein synchronization lead to the time relation of a Lorentz transformation. In directional VSL models these synchronizations lead to a frame-independent velocity of light (Sjödin 1979).

[3]The corresponding metric would be $ds'^2 = d\mathbf{x}'^2 - c'^2 dt'^2 = \Phi\left(r\right)^{-2}\left(d\mathbf{x}^2 - c(\mathbf{r})^2 dt^2\right)$. But the space-time metric is not a sufficient characterization of our scalar model, also a 'momentum-energy metric' is required (cf. the energy-momentum GMLT).

[4]We remark the absence of gravitational effect, only the SRT kinematical effect is present.

$$\tan\theta' = -\frac{\tan\theta}{\gamma(u)\left(1-\frac{u}{v_{\parallel}}\right)} \quad , \quad \theta = \angle(\mathbf{u}, d\mathbf{x}) \ , \ \theta' = \angle(\mathbf{u}', d\mathbf{x}') \tag{11}$$

$$\gamma(v') = \gamma(v)\gamma(u)\left(1-\frac{\mathbf{u}.\mathbf{v}}{c(r)^2}\right) \tag{12}$$

Finally we note that by elimination of the S_0 perspective, two or more locally coincident affected observers are related according a standard Lorentz transformation. Their composition is again an element of the Lorentz Group of symmetry. Such is not the case for composed transformations between non-local affected observers with different Φ value: in the general case, $\mathbf{u}_1 \not\parallel \mathbf{u}_2$ and $\mathbf{u'}_1 \neq 0 \neq \mathbf{u'}_2$, a counter posed scaling factor $\frac{\Phi_1^2}{\Phi_2^2}$ in the, S_1' to S_2', relative frame velocity spoils the Lorentz symmetry.
While a non-local composition is in general not in the Lorentz Group, it is still a legitimate affected observer relation. We remark however that in many cases there is no need to combine GMLT's non-locally, as most calculations can be done conveniently relative to S_0, and transformed afterwards to any affected observer S'.

2.2 Energy-momentum GMLT

Following a common procedure to obtain momentum-energy LT from space-time LT in SRT, we multiply both members of the space-time GMLT with

$$m_0'\gamma(v')\frac{1}{dt'} = m_0'\frac{\gamma(v)}{\Phi(\mathbf{r})}\frac{1}{dt} \tag{13}$$

as obtained from the time equation of the GMLT and eq. (12). With a multiplication of c'^2 on both members of the time equation GMLT and using eq. (1,c), this results after some reordering in :

$$\mathbf{p}' = \left(\left(\frac{m_0'\gamma(v)}{\Phi(\mathbf{r})^3}\mathbf{v}_{\parallel} - \frac{\frac{m_0'\gamma(v)c(\mathbf{r})^2}{\Phi(\mathbf{r})^3}}{c(\mathbf{r})^2}\mathbf{u}\right)\gamma(u) + \frac{m_0'\gamma(v)}{\Phi(\mathbf{r})^3}\mathbf{v}_{\perp}\right)\Phi(\mathbf{r}) \tag{14}$$

$$E' = \left(\frac{m_0'\gamma(v)\,c(\mathbf{r})^2}{\Phi(\mathbf{r})^3} - \left(\mathbf{u}.\frac{m_0'\gamma(v)}{\Phi(\mathbf{r})^3}\mathbf{v}\right)\right)\gamma(u)\frac{1}{\Phi(\mathbf{r})} \tag{15}$$

Where $\mathbf{p}' \equiv m_0'\gamma(v')\,\mathbf{v}'$ and $E' \equiv m_0'\gamma(v')\,c'^2$, as consistently interpreted by S', for a locally coincident test body. We immediately see that the reordering was made in order to expose the kinematical part of a Lorentz transformation, and leads to a counterposed gravitational affectation function Φ as compared to the space-time GMLT. We identify E, $\mathbf{p}$ and mass m according:

$$\mathbf{p} \equiv m(\mathbf{r})\mathbf{v} \tag{16}$$
$$E \equiv m(\mathbf{r})c(\mathbf{r})^2 \tag{17}$$
$$m(\mathbf{r}) \equiv m'\frac{1}{\Phi(\mathbf{r})^3} \equiv m_0\gamma(v) \equiv m'_0\frac{\gamma(v)}{\Phi(\mathbf{r})^3} \tag{18}$$

The dependence (18) of mass on gravitation endorses Mach's principle of mass induction[5]. We will discuss repercussions on WEP in section (3.1). In return we obtain an energy expression which is the 'formal' equivalent of SRT[6]:

$$\text{Matter} \qquad E^2 - c(\mathbf{r})^2p^2 = m_0(\mathbf{r})^2c^4 \tag{19}$$
$$\text{Light, } (\mathrm{m}'_0 \equiv 0) \qquad E = pc(\mathbf{r}) \tag{20}$$

The momentum-energy GMLT S_0 to S' becomes:

$$\mathbf{p}' = \left(\left(\mathbf{p}_\parallel - \frac{E}{c(\mathbf{r})^2}\mathbf{u}\right)\gamma(u) + \mathbf{p}_\perp\right)\Phi(\mathbf{r}) \tag{21}$$
$$E' = (E - \mathbf{p}.\mathbf{u})\frac{\gamma(u)}{\Phi(\mathbf{r})} \tag{22}$$

while the inverse transformation S' to S_0 is given by:

$$\mathbf{p} = \left(\left(\mathbf{p}'_\parallel - \frac{E'}{c'^2}\mathbf{u}'\right)\gamma(u') + \mathbf{p}'_\perp\right)\frac{1}{\Phi(\mathbf{r})} \tag{23}$$
$$E = (E' - \mathbf{p}'.\mathbf{u}')\gamma(u')\Phi(\mathbf{r}) \tag{24}$$

We do retain Lorentz covariance of momentum-energy for locally coincident affected observers. For non-local observers the $(\mathbf{p}, E)$-GMLT has the same kinematical non-Lorentzian structure as the $(\mathbf{x}, t)$-GMLT in that case, only the isolated gravitational affectation function is counterposed.

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[5] In the static case, $\mathbf{v} = 0$, we have $m(\mathbf{r}) = m'_0\Phi(r)^{-3}$, i.e. $m(\mathbf{r})$ increases when approaching the source but remains finite on asymptotic recession, $m_\infty = m'_0$

[6] Local Lorentz Group symmetry of energy-momentum lead us to mass expression (18) not form invariance. A historical critique on Einstein's form covariance is done by J.D. Norton (Norton, 1993)

The combination of quantities covarying with $(\mathbf{p}, E)$-GMLT and $(\mathbf{x}, t)$-GMLT respectively, simplify due to the contraposition of the statical Φ factor, e.g.:

$$\mathbf{p}'.d\mathbf{x}' - E'dt' \quad = \quad \mathbf{p}.d\mathbf{x} - Edt \tag{25}$$

Taking into account the Einstein-Compton relations for corpuscular light[7], and supposing validity of Einstein form covariance in S_0 and S' :

$$E = h\nu \ , \ p = \frac{h}{\lambda} \quad \text{and} \quad E' = h'\nu' \ , \ p' = \frac{h'}{\lambda'} \tag{26}$$

we have, with $\mathbf{k} = \frac{\boldsymbol{\lambda}}{\lambda^2}$, $\mathbf{k}' = \frac{\boldsymbol{\lambda}'}{\lambda'^2}$

$$h'(\mathbf{k}'.d\mathbf{x}' - \nu' dt') \quad = \quad h(\mathbf{k}.d\mathbf{x} - \nu dt) \tag{27}$$

Given the trivial invariance of a dimensionless phase, we obtain in our model the 'universal' invariance of Planck's constant:

$$h \quad = \quad h' \tag{28}$$

In order to solve some mechanical problems we must now identify Φ.

2.3 Newtonian fit or elementary field equation

A Newtonian fit is done according the following procedure. Let S'_w be an observer at rest relative to the background field Φ, and S'_e be the eigen observer of a test mass at rest in the field, i.e. $\mathbf{p}'_e = 0$ and $E'_e = m'_0 c'^2$. Then the energy GMLT between S'_e and S'_w is reduced to:

$$E'_w \Phi_w \quad = \quad E'_e \Phi_e \tag{29}$$

Relative to S'_w a displacement of the test body leads to a change in static energy, while S'_e attributes invariantly its proper energy, according:

$$\Phi_w dE'_w \quad = \quad E'_e d\Phi_e \tag{30}$$

Let this displacement bring S'_e locally coincident to S'_w , i.e. $\Phi_w = \Phi_e$, then S'_w can identify the change of static energy, as a shift in Newtonian potential

[7] The validity of the Einstein-Compton relation depends on the variation of Φ over the wavelength of light.

energy[8];

$$\frac{d\Phi_e}{\Phi_e} = \frac{1}{m'_0 c'^2} d\left(-Gm'_0 \int_{\text{Source}} \frac{\rho(\mathbf{r}^*)}{|\mathbf{r}' - \mathbf{r}'^*|} d^3 r'^*\right) \tag{31}$$

this solves to:

$$\Phi = e^{-\frac{G}{c'^2}\int_S \frac{\rho(\mathbf{r}'^*)}{|\mathbf{r}'-\mathbf{r}'^*|} d^3 r'^*} \quad \text{(in general)} \tag{32}$$

$$= e^{-\frac{\kappa'}{r'}} = e^{-\frac{\kappa}{r}} \quad \text{(outside spherically symmetric source)} \tag{33}$$

where $\kappa \equiv -\frac{GM}{c'^2}$, is half the Schwarzschild radius, and in the last equation was taken into account the 'invariant' nature of a fraction of two extensive lengths. The expression for Φ in coordinate length earns primacy, for it is independent of practical measuring protocols for extensive lengths. We notice no cause for singular behaviour in either metric or mechanics, $0 \leq \Phi \leq 1$.

Subsequently we verify the basic field equation for Φ in coordinate expression (32). We find, given the $|\mathbf{r}|^{-1}$ dependence of the exponent of Φ:

$$\Delta \ln \Phi = 4\pi \frac{G}{c'^2} \rho(\mathbf{r}) \tag{34}$$

from which follows

$$\Delta\Phi = \frac{4\pi G}{c'^2}\rho(\mathbf{r})\Phi + \frac{(\nabla\Phi)^2}{\Phi} \tag{35}$$

The second term suggests a contribution to gravitation by the field itself.[9]

3 The 'Essential' experiments

We most conveniently describe physical processes in the S_0 perspective, but are actually always physically bound to bring back the results of calculation into the affected S' perspective, which we will do when essential. In order to

[8] In the present scope this conservative fit appears sufficiently accurate, the trace of the stress tensor $T_{\mu\mu}$ as source is not required.

[9] Based on invariance requirements, a similar equation for a point source was derived by Torgny Sjödin (private communication). Eq. (34) resembles eq. (3b) of (Einstein 1912, see also Norton, 1992): $c\Delta c - \frac{1}{2}(\nabla c)^2 = kc^2\rho$

calculate the experiments, we use the Hamiltonian[10] expression for energy (19) of mass and (20) for light ($m_0' = 0$):

$$H \;=\; \sqrt{{m_0}^2 c(\mathbf{r})^4 + c(\mathbf{r})^2 p^2} \;=\; \Phi\left(r\right)\sqrt{{m_0'}^2 c'^4 + p^2 c'^2 \Phi\left(r\right)^2} \qquad (36)$$

Then the Hamilton equations give in general:

$$\dot{H} = \partial_t H \quad , \quad \dot{\mathbf{r}} = \nabla_p H \quad , \quad \dot{\mathbf{p}} = -\nabla_r H \qquad (37)$$

In the case of a source with spherical symmetry the mechanical problem is simplified. The central force leads to conservation of the angular momentum. With $\theta = \pi/2$ the motion is constrained to the (r, φ) plane, and Hamilton's equations read:

$$H = E_0 \qquad (38)$$

$$p_\theta = p_{\theta_0} = 0 \;\;,\;\; \dot{\theta} \;=\; 0 \qquad (39)$$

$$p_\varphi = p_{\varphi 0} \;\;,\;\; \dot{\varphi} \;=\; \tfrac{c(r)^2}{E_0 r^2} p_{\varphi 0} \qquad (40)$$

$$\dot{p_r} = -\left(E_0 + (p_r^2 + \tfrac{p_{\varphi 0}^2}{r^2})\tfrac{c(r)^2}{E_0}\right)\tfrac{\partial_r \Phi(r)}{\Phi(r)} + \tfrac{c(r)^2}{E_0}\tfrac{p_{\varphi 0}^2}{r^3} \;\;,\;\; \dot{r} \;=\; \tfrac{c(r)^2}{E_0} p_r \qquad (41)$$

The orbit of a test body in the spherical symmetrical field, with conserved quantities $E_0, p_{\theta_0}, p_{\varphi_0}$, is given by the equation:

$$\frac{dr}{d\varphi} \;=\; \frac{\dot{r}}{\dot{\varphi}} \;=\; r\sqrt{\frac{r^2}{c(r)^2}\frac{\left(E_0^2 - {m_0}^2 c(r)^4\right)}{p_{\varphi 0}^2} - 1} \qquad (42)$$

3.1 Acceleration, Force, Weak Equivalence Principle

From the Hamiltonian equations (37) we obtain the acceleration and force expressions for a test mass in the gravitational field

$$\mathbf{g} \;=\; 4\mathbf{v}\frac{\mathbf{v}.\nabla\Phi\left(r\right)}{\Phi\left(r\right)} - \left(c(\mathbf{r})^2 + v^2\right)\frac{\nabla\Phi\left(r\right)}{\Phi\left(r\right)} \qquad (43)$$

While for the force $\mathbf{f}$

$$\mathbf{f} \;=\; -E_0\left(2 - \frac{1}{\gamma\left(v\right)^2}\right)\frac{\nabla\Phi\left(r\right)}{\Phi\left(r\right)} \qquad (44)$$

[10]The corresponding Lagrangian for mass is $L = -m_0' c'^2 \Phi\gamma$, i.e. the Lagrangian used by Sjödin (Sjödin, 1990), and the Lagrangian $-m\sqrt{c^2 - q^2}$ proposed by Einstein in his early Lorentz-covariant model (Einstein, 1912, *Nachtrag zur Korrektur*)

We notice therefor that the WEP is lost. All material bodies do not fall according the same acceleration. The velocity dependence of the acceleration is evident according to S_0 , and remains explicit relative to S' , e.g. for fixed S' we obtain from eq. (10), eq. (12), and eq. (13) all with $\mathbf{u} \equiv 0$, the acceleration relation:

$$\mathbf{a}' = \left(\mathbf{a} - 2\mathbf{v}\frac{\mathbf{v}.\nabla\Phi(\mathbf{r})}{\Phi(\mathbf{r})}\right)\frac{1}{\Phi(\mathbf{r})^3} \tag{45}$$

In particular for gravitational acceleration relative to a fixed affected observer S' , with gradient transformation[11], this gives:

$$\mathbf{g}' = 2\mathbf{v}'\frac{\mathbf{v}'.\nabla'\Phi}{\Phi} - \left(c'^2 + v'^2\right)\frac{\nabla'\Phi}{\Phi} \tag{46}$$

The problem is common to theories with dependence of mass on gravitational potential (Nordström 1913, Brans 1997). The implications need to be studied in detail. For now we can only remark that Eötvös experiments based on variable torque due to different inner composition of contrasting masses seem not to be sensitive to average velocity orientation. The solid angle average of the velocity orientation dependent part, supposing no preferred direction of velocity, vanishes:

$$\langle \mathbf{g}'_{\mathbf{v}'}\rangle_\Omega = \frac{1}{4\pi}\int d\Omega'\left(2\mathbf{v}'\frac{\mathbf{v}'.\nabla'\Phi}{\Phi}\right) = 0 \tag{47}$$

The internal composition of the bodies does act differently for contrasting masses with different average amplitude v^2. Therefor temperature differences could contribute to torque along the gradient of Φ, a result very dissimilar to GRT.

3.2 Deflection of light orbitals

The unbound states are characterized by a single flexion point ($\dot{p}_r = 0$) in the orbital at the point of minimal approach. The latter and the impact

[11] The gradient operator transforms according the contra-variant space-time GMLT, S' to S_0 :

$$\nabla = \mathbf{u}'\frac{1}{\Phi(\mathbf{r})}\left((\gamma(u')-1)\frac{\mathbf{u}'.\nabla'}{u'^2} + \frac{1}{c'^2}\gamma(u')\,\partial_{t'}\right) + \frac{1}{\Phi(\mathbf{r})}\nabla'$$

$$\partial_t = \gamma(u')\,\Omega(\mathbf{r})\,(\partial_{t'} + \mathbf{u}'.\nabla')$$

parameter are used to characterize the orbital. The angle α is defined between the radial vector $\mathbf{1}_r$ and the tangent vector $\mathbf{1}_t$ of the orbital, then: $\tan\alpha = \frac{rd\varphi}{dr}$. The impact parameter is obtained from asymptotic orbital conditions: $r_{\rm in}\sin\alpha_{\rm in}|_{r_{\rm in}\to\infty} = b$. Setting the observation of deflection again asymptotically, $r_{\rm out}\to\infty$, the deflection angle $\alpha_{\rm D}$ is defined by:

$$\alpha_{\rm D} \;=\; 2|\varphi_\infty - \varphi_{r_-}| - \pi \tag{48}$$

The orbital equation is obtained by putting $m_0 = 0$ in (42), and using the condition at the point of shortest approach, $\dot{p}_{r_-} = 0$, then $\frac{E_0}{p_{\varphi_0}} = \frac{c'}{b} = \frac{c_-}{r_-}$:

$$\frac{dr}{d\varphi} \;=\; r\sqrt{\frac{{c_-}^2}{c(r)^2}\frac{r^2}{{r_-}^2} - 1} \tag{49}$$

the integration is evaluated in the approximation $O(\kappa^2/{r_-}^2)$

$$\varphi_\infty - \varphi_r \;\approx\; \int_r^\infty \left(1 + 2\kappa\frac{r}{r_-}\frac{1}{r+r_-}\right)\frac{dr}{r\sqrt{\frac{r^2}{{r_-}^2} - 1}} \tag{50}$$

$$\approx\; \arcsin\frac{r_-}{r} + 2\kappa\left(\frac{1}{r_-} - \frac{\sqrt{r^2 - {r_-}^2}}{r_-(r+r_-)}\right) \tag{51}$$

Then the deflection angle $\alpha_{\rm D}$ in $O(\kappa^2/{r_-}^2)$ is given by:

$$\alpha_{\rm D} \;=\; 2\left|\frac{\pi}{2} + 2\frac{\kappa}{r_-}\right| - \pi \tag{52}$$

in correspondence with GRT in coordinate space-time.

$$\alpha_{\rm D} \;=\; 4\frac{GM}{c'^2 r_-} \tag{53}$$

3.3 Perihelion precession

The bound state has two flexion points in eq. (42); the perihelion $\frac{dr}{d\varphi}|_{r=r_-} = 0$ and aphelion $\frac{dr}{d\varphi}|_{r=r_+} = 0$. The initial values E_0 and p_{φ_0} can be expressed using the values at the extrema of the bounded orbits:

$$E_0 \;=\; m_0'c'^2\sqrt{\frac{\frac{{r_+}^2}{\Phi_+^2} - \frac{{r_-}^2}{\Phi_-^2}}{\frac{{r_+}^2}{\Phi_+^4} - \frac{{r_-}^2}{\Phi_-^4}}} \quad , \quad p_{\varphi_0} \;=\; m_0'c'\sqrt{-\frac{\Phi_+^2 - \Phi_-^2}{\frac{\Phi_+^4}{{r_+}^2} - \frac{\Phi_-^4}{{r_-}^2}}}$$

If $r_+ > r_-$ then $\Phi_+ > \Phi_-$, $\frac{{r_+}^2}{\Phi_+^4} > \frac{{r_-}^2}{\Phi_-^4}$, $\frac{r_+}{\Phi_+} > \frac{r_-}{\Phi_-}$.
The orbital equation (42) can now be expressed using the extrema values:

$$\frac{dr}{d\varphi} = r^2\sqrt{\frac{1}{\Phi^4}\left(\frac{\Phi_-^4}{{r_-}^2}\left(\frac{\Phi_+^2-\Phi^2}{\Phi_+^2-\Phi_-^2}\right)+\frac{\Phi_+^4}{{r_+}^2}\left(\frac{\Phi_-^2-\Phi^2}{\Phi_-^2-\Phi_+^2}\right)\right)-\frac{1}{r^2}} \quad (54)$$

The angular perihelion shift $\Delta\varphi$ is defined by:

$$\Delta\varphi = 2|\varphi(r_+)-\varphi(r_-)|-2\pi$$

The integration of the equation requires some standard approximations and substitutions. We approximate till $O(\kappa^3/r^3)$ the integrand according a quadratic form as integrand has zero points at $r=r_-$ and $r=r_+$:

$$\frac{1}{\Phi\left(r\right)^4}\left(\frac{f_-}{r_-^2}+\frac{f_+}{r_+^2}\right)-\frac{1}{r^2} \approx -\alpha\left(\frac{1}{r_-}-\frac{1}{r}\right)\left(\frac{1}{r_+}-\frac{1}{r}\right) \quad (55)$$

with α a constant that can be evaluated by considering the value of the integrand for $r\to\infty$:

$$\alpha = -\frac{r_-r_+}{\Phi_\infty^4}\left(\frac{f_{-\infty}}{r_-^2}+\frac{f_{+\infty}}{r_+^2}\right) \quad (56)$$

With;

$$\Phi_\infty = 1\ ,\ f_{-\infty}=\Phi_-^4\left(\frac{\Phi_+^2-1}{\Phi_+^2-\Phi_-^2}\right)\ ,\ f_{+\infty}=\Phi_+^4\left(\frac{\Phi_-^2-1}{\Phi_-^2-\Phi_+^2}\right) \quad (57)$$

Then

$$\alpha = \frac{r_+\Phi_-^4}{r_-}\left(\frac{1-\Phi_+^2}{\Phi_+^2-\Phi_-^2}\right)-\frac{r_-\Phi_+^4}{r_+}\left(\frac{1-\Phi_-^2}{\Phi_+^2-\Phi_-^2}\right) \quad (58)$$

We consider next the integral of the orbital angle:

$$\varphi(r)-\varphi(r_-) \approx \frac{1}{\sqrt{\alpha}}\int_{r_-}^{r}\frac{dr}{r^2\sqrt{\left(\frac{1}{r_-}-\frac{1}{r}\right)\left(\frac{1}{r}-\frac{1}{r_+}\right)}} \quad (59)$$

This part of the integration is identical to (Weinberg 1972, sec. 8.6):

$$\varphi(r_+)-\varphi(r_-) \approx -\frac{\pi}{\sqrt{\alpha}} \quad (60)$$

The perihelion shift angle is then given by:

$$\Delta\varphi \;\approx\; 2\pi\left(\frac{1}{\sqrt{\alpha}}-1\right) \tag{61}$$

Approximation of $\alpha^{-\frac{1}{2}}$ to order $O(\kappa^2/r^2)$, and $\Phi = e^{-\frac{\kappa}{r}}$ till $O(\kappa^3/r^3)$ gives:

$$\alpha^{-\frac{1}{2}} \;=\; 1+\frac{3}{2}\kappa\left(\frac{1}{r_+}+\frac{1}{r_-}\right)+O\left(\frac{\kappa^2}{r^2}\right) \tag{62}$$

Then the perihelion shift angle, up to $O\left(\frac{\kappa^2}{r^2}\right)$, is given by,

$$\Delta\varphi \;\approx\; \frac{6\pi\kappa}{L} \tag{63}$$

with $L \equiv \frac{1}{2}\left(\frac{1}{r_+}+\frac{1}{r_-}\right)$, i.e. the GRT result in coordinate space-time.

3.4 Radar echo delay

The travel time of light is obtained from the velocity of light at every point of its orbit between source and observer. From the Hamiltonian equation (41,b), with conserved energy $E_0 = pc$ (38) and angular momentum p_{φ_0} (40) we obtain:

$$\dot{r} \;=\; c(r)\sqrt{1-\frac{p_{\varphi_0}^2}{E_0^2}\frac{c(r)^2}{r^2}} \tag{64}$$

The radial velocity vanishes at the point of closest approach near the Sun, $r = r_-$, i.e. $\dot{r}|_{r=r_-} = 0$ giving $\frac{p_{\varphi_0}}{E_0} = \frac{r_-}{c_-}$. Then, light traveling over its orbit from r_i to r_f takes the time Δt:

$$\Delta t \;=\; \int_{r_i}^{r_f}\frac{dr}{c(r)\sqrt{1-\frac{r_-{}^2}{c_-{}^2}\frac{c(r)^2}{r^2}}} \tag{65}$$

The travel time $\Delta t(r, r_-)$, approximated till $O\left(\frac{\kappa^2}{r^2}\right)$, is found to be:

$$\Delta t(r,r_-) \;\approx\; \frac{1}{c'}\int_{r_-}^{r}\left(1+2\frac{\kappa}{r}+\frac{\kappa r_-}{r(r+r_-)}\right)\frac{1}{\sqrt{1-\frac{r_-{}^2}{r^2}}}dr \tag{66}$$

The three components of the integrand lead respectively to:

$$\Delta t(r, r_-) \approx \frac{1}{c'}\left(\sqrt{r^2 - {r_-}^2} + \kappa\left(2\ln\frac{r + \sqrt{r^2 - {r_-}^2}}{r_-} + \left(\frac{r - r_-}{r + r_-}\right)^{\frac{1}{2}}\right)\right)$$

i.e. the delay term of GRT in coordinate time (Weinberg 1972, eq. 8.7.4).

3.5 Gravitational shift of spectral lines

The effect of gravitation on the frequency of light over its orbit is calculated in the Hamiltonian description with the Planck-Compton relations. From the former we have the conservation of energy $E = E_0$, eq. (38), on the light orbital. With the invariance of Planck's constant, eq. (28), the invariance of the light frequency relative to S_0 is straightforward: $\dot{\nu} = 0$. In our gravitation model, the spectral shift of light is therefor due to comparison of measurements by affected observers only. The time GMLT for a static observer S', eq. (7) ($\mathbf{u} \equiv 0$), gives the frequency relation: $\nu = \nu'\Phi(r)$
Conserved energy E_0 expressed at two locations relative to S' gives:

$$\mathrm{h}\nu'\Phi(r) = \mathrm{h}\nu'_0\Phi(r_0) \tag{67}$$

Then, light travelling from a source at r_s out into a gravitational field will have an observed frequency ratio at r_o by S' equal to

$$\frac{\nu'_s}{\nu'_o} = \frac{\Phi(r_o)}{\Phi(r_s)} \tag{68}$$

$$= e^{-\kappa\left(\frac{1}{r_o} - \frac{1}{r_s}\right)} \tag{69}$$

till $O\left(\frac{\kappa^2}{r^2}\right)$, when $r_0 >> r_s$, the red shift at the observer is given by

$$\frac{\nu'_s - \nu'_o}{\nu'_o} \approx \frac{\kappa}{r_s} \tag{70}$$

The affected observer S' recovers a spectral shift in accordance with GRT. The pendant of this frequency relation is of course the non-conservation of the energy relative to S'.
The behaviour of the wave length of light is very different as compared to

frequency. From the Hamiltonian and Compton relation we have $\lambda = \frac{hc(r)}{E_0}$ Therefor S_0 will record a wave-length spectral shift:

$$\frac{\lambda_s}{\lambda_0} = \frac{\Phi(r_s)^2}{\Phi(r_o)^2} \tag{71}$$

till $O\left(\frac{\kappa^2}{r^2}\right)$, when $r_0 >> r_s$, S_0 observes a red shift on the wavelength of twice the magnitude an unaffected observer would measure:

$$\frac{\lambda_s - \lambda_0}{\lambda_0} \approx -2\frac{\kappa}{r_s} \tag{72}$$

Evidently the GLT for static observers S_0 to S' applied to wavelength: $\lambda = \lambda'\Phi(r)$, reduces the wave length redshift observed by S' to:

$$\frac{\lambda'_s}{\lambda'_0} = \frac{\Phi(r_s)}{\Phi(r_o)} \tag{73}$$

till $O\left(\frac{\kappa^2}{r^2}\right)$, when $r_0 >> r_s$, S' recovers the usual red shift on the wavelength:

$$\frac{\lambda'_s - \lambda'_0}{\lambda_0} \approx -\frac{\kappa}{r_s} \tag{74}$$

We conclude that the affected observers measure the required spectral shift on frequency and wavelength. The unaffected observer measures no frequency shift due to conservation of energy, while a wave length shift is observed due to changing momentum on the orbit in the gravitational field.

3.6 Gravitational clock slowing

We briefly mention gravitational time dilation using material clocks. We consider the readings of similar clocks, synchronized on departure, travelling along differing orbits and kinematics and compared on meeting. From the time GMLT-equations, for S_0 to the affected eigen frames of the clocks (7), we obtain:

$$\frac{\Delta t'_1}{\Delta t'_2} = \frac{\int_{\Gamma_1} \gamma(\mathbf{u}_1)\Phi_1 dt}{\int_{\Gamma_2} \gamma(\mathbf{u}_2)\Phi_2 dt}$$

The specific orbital kinematics decides on adequate approximations.

4 Conclusion

The present scalar Lorentz-covariant gravitation model succeeds in giving correctly the results for the basic experiments of general relativity theory, but it raises an acute problem by not satisfying the weak equivalence principle. In our model, mass dependends on gravitational potential following Mach's principle but it is not its origin. Although our model does not cover the problem of electromagnetism in a gravitational field, the approximation of corpuscular light and Plank-Compton relations, gives an unexpected explanation of gravitational red shift. The essence of which is the difference between 'natural' geometry and 'coordinate' geometry.

In the present scope the model was restricted to a static gravitation field. The lack of retardation effects makes it ineffective in issues concerning gravitation waves.

The model is very incomplete in explaining a subjacent model for gravitational and relativistic kinematic effects on matter, nor minute effects by extendedness and proper kinematics of source and test object.

Some of the latter problems could although be compatible with the present scheme of our scalar gravitation model.

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