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\title{The definition of a Dirac monopole in QED and its implications to gravitation}
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\begin{abstract}
Dirac monopoles have been often described by the so called Dirac string which has been presented as a gauge artifact. Recently however it has been shown that if the monopole charge is different from zero in QED, then the Dirac string must be a physical observable. In the present work we discuss the Dirac string related to monopoles leads naturally to a space-time distortion which can be identified to the one a single point - mass generates in its sourroundings. It is then a physical observable, i.e., the gravitation generated by a monopole particle. It is also mentioned the same results are also valid for any QED particle. The main goal of the work is to study the definition of a particle in QED and its consequences for the space-time.
\end{abstract}

\section{Introduction}

The goal of this work is to show that the proper definition of the Dirac monopole leads to some consequences for the spacetime sorrouding it. This letter's objective is to present \textit{only the basic ideas} of a model in which the definition of a singular electromagnetic potential is related to space-time distortions.

The monopole proposed by Dirac in 1931 [1] is the result of a generalization of the Maxwell's classical theory by the dual symmetry, being described by an Abelian potential with singularities (semi-infinite strings in space). This description resulted in the electric charge quantization.

It became clear in the years after the proposal, [2], that in order to be possible to derive the motion equations for particles in QED that electric charges must never touch a string of singularities (nor even its end, the monopole). Dirac imposed such condition and it was precisely this that became the main point of criticism towards the theory. It was not until 1974 that it became generally accepted that the string is a non - observable physical entity as proposed by Wu and Yang, [3]. In their theory the field of a magnetic charge may be properly defined in many regions or subspaces (sub - regions of the physical space), which overlap. In the overlapping regions there is a gauge transformation relating the quantum phases of two of such subspaces. If this transformation leads to a magnetic flux - that is related to a phase which is a multiple of 2$\pi $ by the Dirac quantization condition, then there will be no consequence due to the topological structure of space-time into subspaces, i.e., no consequences related to strings (which the subspace's business try to avoid for any electric charge's path).

Besides this work on the monopole issue, which became unchanged through twenty years, in 1994, He, Qiu and Tze demonstrated in a very clear way that if the monopole charge (in QED) is nonzero, then a physical interpretation must be given to the strings related to it, [4]. The main point presented by these authors relies on the fact that Stokes theorem has been wrongly applied in the early works on the subject. This conclusion is in direct contradiction to the idea that strings are non - observable entities. Their striking result is that a Dirac string cannot be removed from the path of a charged particle without giving it some physical interpretation.

In the present work we are studying a physical interpretation for the Dirac string. It is discussed that in order to properly define a Dirac monopole it is necessary to assume no particle or field is defined at the Dirac string's region, making the space-time accupied by all particles and fields a reduced one (regarding the whole space which contains the monopole's string). In the sequence it is argued there is in fact no need of any assumption since this space-time shrinking is a natural consequence of having a strink - like singular vector potential defined to describe a monopole, i.e., it is the result of solving the Schr\"{o}dinger wave equation for all particles.

To conclude the work it is discussed that not only Dirac monopoles are responsable for space-time distortions, but also electrons, positrons and photons as well. The electron is \textit{described} as composed of two spinless Dirac monopoles of opposite sign plus a spinless electric charge. If the space-time distortion is always related to a Dirac monopole, then it is directly related to fermions as well. Photons may be \textit{described} in terms of fermions according to M. de Broglie proposition, making it also a space-time distortion cause.

\section{The definition of a Dirac monopole and its consequences for the space-time}

When one defines a source of field in Electrodynamics some hypothesis are taken in order to the interaction energies to be finite. It would be interesting if such hypothesis would be part of the theory (between the boundaries of its possibilities and definitions), i.e., if could exist a physical reason for it to be seriously considered in order to have a good explanation for the finiteness of the interaction energies derived from the theory.

Take the case of an elementary charge in Classical Electrodynamics as an example: It is necessary to define a minimum radius (different from zero) for the elementary source of electric field. At the time of the definition of an elementary electric charge, Poincar\'{e} proposed that such structure should be sustained by a tension of some sort, but did not find the physical agent of such, being even today an open question.

In the problem we are interested in this work, i.e., the monopole as proposed by Dirac, the same kind of question is considered. The difference is that now the structure is no longer a single small sphere, but a string which is defined over the space, starting at the particle's position and going up to the infinite. It would be possible that, doing the same steps as done in the single electric charge case, we can find some physical motivation to the hypothesis (for finite energy results) in the monopole case?

In order to recall the general procedure to treat sources in Electrodynamics, let us consider two charged particles in the classical approach. Defining it by index ''1'' and ''2'', the scalar potentials related to each particle (both at rest in the laboratory) are V_{1} and $%
V_{2}$, such that the resulting electric fields are $\mathbf{E}_{1}=-\mathbf{%
\nabla }V_{1}$ and $\mathbf{E}_{2}=-\mathbf{\nabla }V_{2}$. The energy of the system is:
\begin{equation}
\frac{\varepsilon _{0}}{2}\int \left(\mathbf{E}_{1}\cdot \mathbf{E}_{1}+%
\mathbf{E}_{2}\cdot \mathbf{E}_{2}+2\mathbf{E}_{1}\cdot \mathbf{E}%
_{2}\right) dv
\end{equation}
where dv is the differential volume of the observable Universe. The first two terms can be easily calculated and the result is: $%
(e_{1}^{2}/a_{1}+e_{2}^{2}/a_{2})/\left(8\pi \varepsilon _{0}\right) $
where $e_{1,2}$is the charge of the particle 1, 2 and $a_{1,2}$ the radius. In applying the Green's theorem to the third term, we define a closed surface as an external shell S^{e} at the infinite and two inner shells $S_{1,2}^{i}$ around each particle, just over the radii, so that:
\begin{eqnarray}
&&\varepsilon _{0}\int \mathbf{\nabla }V_{1}\cdot \mathbf{\nabla }V_{2}dv \\
&=&\frac{\varepsilon _{0}}{2}\left(\oint V_{1}\mathbf{\nabla }V_{2}\cdot \mathbf{dS}^{e}-\oint V_{1}\mathbf{\nabla }V_{2}\cdot \mathbf{dS}%
_{2}^{i}-\int V_{1}\nabla ^{2}V_{2}dv\right)   \nonumber \\
&&+\frac{\varepsilon _{0}}{2}\left(\oint V_{2}\mathbf{\nabla }V_{1}\cdot \mathbf{dS}^{e}-\oint V_{2}\mathbf{\nabla }V_{1}\cdot \mathbf{dS}%
_{1}^{i}-\int V_{2}\nabla ^{2}V_{1}dv\right) . \nonumber
\end{eqnarray}

In the first parenthesis the first term vanishes with the external surface at the infinite, as well as the third term since in the considered volume between the two charges there is no field divergences. The second term is nonzero and the scalar potential due to particle 1 may be approximated by its value at the particle 2 position (if particle's 2 radius is a vanishing quantity), so being treated as a constant in the integral. Therefore, taking the same considerations for the second parenthesis we get:
\begin{eqnarray}
&&\frac{\varepsilon _{0}}{2}\left(V_{1}\left(\mathbf{r}_{2}\right) \oint E_{1}\cdot dS_{2}^{i}\right) +\frac{\varepsilon _{0}}{2}\left(V_{2}\left(\mathbf{r}_{1}\right) \oint E_{2}\cdot dS_{1}^{i}\right) \\
&=&\frac{V_{1}\left(\mathbf{r}_{2}\right) e_{2}+V_{2}\left(\mathbf{r}%
_{1}\right) e_{1}}{2} \nonumber
\end{eqnarray}
It is now clear that in order to obtain this result it is necessary to consider all fields are non - existent at the region inside the spheres defined as structure for the particles. The same hypothesis is necessary in the complete theory of the electron, i.e., even taking into consideration the quantum aspects of the fields, [5]. As mentioned before, Poincar\'{e} tried to find a physical description for such sphere - like stability with no success, [6].

Now let us consider the monopole proposed by Dirac applying the same reasoning with the goal to find the consequences of avoiding divergences. The simplest monopole we can define is described by the vector potential:
\begin{equation}
\mathbf{A}=g\frac{1+\cos \theta }{r\sin \theta step(\theta -\delta)}\mathbf{%
\varphi }
\end{equation}
where the position vector is described by the spherical coordinates, $%
\mathbf{r}=(r,\theta ,\varphi)$. The step function is defined in a way that, $step(x-x_{0})=0$ for , $x<x_{0}$, =1/2 for $x=x_{0}$ and =1 for $%
x>x_{0}$. The parameter $\delta $ is an arbitrary angle. The resulting magnetic field may be derived as:
\begin{equation}
\mathbf{B}=2\pi g(1+\cos \delta)\mathbf{r}/r^{3}
\end{equation}
for $\delta <\theta \leq \pi $, any $\varphi $, and it is not defined in any other region. When $\delta =0$ it is said the vector potential remains singular on a semi - infinite line, while the magnetic field is well defined everywhere. Let us consider the general case where $\delta \neq 0$. If there is a system of two particles (as two monopoles or an electric charge and a monopole) the energy integral $\int \mathbf{B}_{i}\cdot \mathbf{B}_{j}dv$ (and the momentum integral $\int \mathbf{E}_{i}\times \mathbf{B}_{j}dv$), with i, j the particle's indices, have no meaning, i.e., the integrals are not defined if the fields at the string's position are taken into consideration. In order to the energy and momentum above described to be finite, it is necessary to consider no field to be defined in the

structure's region (the string region) - the same procedure as used in the elementary charge case.

Is there a possible physical motivation to be associated to such a restriction (in the same philosophy taken by Poincar\'{e} for the electron case)? The simplest way of finding out the existence of these strings is to consider the necessary conditions to properly define a Dirac monopole and then search for physical consequences.

In defining the monopole charge is finite (and different from zero), one is assuming the flux measure (at a spherical surface area with the monopole at the center) is finite. It means the surface integral goes all over the spherical surface but the region where the string crosses it. It means that for a given time, the measure of the surface area taken at a given fixed radius R from the particle is smaller than $4\pi R^{2}$, i.e., $\sqrt{area/4\pi }<R$, any R . The actual available space has been reduced by the presence of the string. In doing this, we are performing a space-time distortion.

It is very well known that for a flat three - dimensional world it is possible to define an average curvature by means of a defect from $4\pi R^{2}$ of the measured area of some surface of radius R. The connection of this idea to the theory of gravitation comes when one identifies the G_{4}^{4} component of the stress - energy tensor. It is the average curvature $R_{12}^{12}+R_{23}^{23}+R_{13}^{13}$ of the three - space, perpendicular to the time, [7]. Consider a three - dimensional sphere of a given surface area. It actual radius exceeds the radius calculated by Euclidean geometry ($\sqrt{area/4\pi }$) by an amount proportional to the matter inside the sphere, as interpreted from usual General Theory of Relativity.

So taking the usual procedure of making the flat three - dimensional space smaller by taking out of it the space occupied by the structure of the fields (the source - string related to the monopoles), one gets space-time distortions, exactly as shown by a mass concentrated in a small region. In this work we derive the complete expression for the string and its association to the space-time distortion related to a single mass concentrated in a small region of the space (with a suitable choice of the vector potential, with a different, i.e., more elaborated potential then the example given above in Eq. (4)).

An important question then arrives: Are we speculating that to a singular potential it is associated a space-time distortion just because of the hypothesis made for the interaction energy to be finite, or it is a \textit{%

natural consequence} of the fact we are defining a singular electromagnetic vector potential in a string - like form?

Some arguments can be drawn to show this is more than a speculation. The singular string is, by first principles, responsable for such space-time distortions just because it is singular, i.e., divergent ($\mathbf{A} \rightarrow \infty $ at that region). Consider the particles described by the Schrödinger wave function $\psi $ are subjected to the vector potential (take the one of Eq. (4) mentioned above as an example) which describes the monopole, i.e., $(-i\mathbf{\nabla }-e\mathbf{A})^{2}\psi =2m\epsilon \psi $. In the general case the vector potential is associated to a nonzero volume's string which implies $\psi =0$ at the string's position (all the space-time occupied by the string). Ir order to see this let us remind the reader that the Hamiltonian

$$
\frac{1}{2m}\int \psi ^{\dagger }\left(-i\mathbf{\nabla }-e\mathbf{A} \right) ^{2}\psi dv
$$

results in an interaction term (with an electric charge) like:

$$
\frac{-e}{im}\int \psi ^{\dagger }\mathbf{A}\cdot \mathbf{\nabla }\psi dv.
$$

If $\psi $ is the wave function of an electron, it must vanish at the string's region since the vector potential is singular. It implies that the solution of the Schrödinger equation points towards the fact that electrons and positrons are forbidden to be found at the string's region.

In solving the Schrödinger equation for a test - magnetic charge under the action of another one, we have two possibilities, i.e., the interaction energy may be positive or negative. It is necessary to solve the equation for a particle under the action of a radial potential with the complication of having a semi - infinite singular string over the space. We are particularly interested in the behaviour of the test particle at the string region. In order to simplify the analysis (as the divergent part of the field is much more relevant than the finite part) we are going to consider a free - test particle under the action of a potential barrier in two cases: The barrier is positive or negative in signal (depending on the of the sign of the interaction). The simplest situation we may use to describe the system is of a energy $\varepsilon $ particle in one - dimension subject to $\phi $, a step - like potential function which has zero value for $x<0$ and a constant value, $\phi _{0}$, for $x>0$. The Schrödinger equation has the solution: $\psi =\alpha e^{ikx}+\beta e^{-ikx}$ for ($x<0$), and $\psi =\gamma e^{-\widetilde{k}x}$ for ($x>0$), where $k=\sqrt{2m\varepsilon }$ and $\widetilde{k}=\sqrt{2m(\phi _{0}-\varepsilon)}$. By matching the wave

function and its slope at the discontinuity of the potential (at $x=0$), we
have:
\begin{eqnarray}
\frac{\beta }{\alpha } &=&\frac{ik+\widetilde{k}}{ik-\widetilde{k}} \\
\frac{\gamma }{\alpha } &=&\frac{2ik}{ik-\widetilde{k}} \nonumber
\end{eqnarray}
which shows that, if $\phi _{0}\rightarrow \infty $, $\gamma \rightarrow 0$,
demonstrating that $\psi =0$ at the places where $\phi \neq 0$ (and
infinite). The other case, when $\phi _{0}\rightarrow -\infty $ results $%
\psi =\gamma e^{i\widetilde{k}x}$ for ($x>0$), so that
\begin{eqnarray}
\frac{\beta }{\alpha } &=&\frac{k-\widetilde{k}}{k+\widetilde{k}} \\
\frac{\gamma }{\alpha } &=&\frac{2k}{k+\widetilde{k}} \nonumber
\end{eqnarray}
which for $\widetilde{k}=\sqrt{2m(\varepsilon -\phi _{0})},$ results $\gamma
\rightarrow 0$, demonstrating that $\psi =0$ as in the first case, i.e., no
monopole is allowed to go over the string region (as the fermions). The
remaining doubt is about the photon and the static (electric and magnetic)
fields. It is very important to observe that not only the quantized fields
of monopoles and fermions are subjected to such restriction, but the static
fields (electric or magnetic) are also forbidden at the Dirac string's
region: \textit{If the Schr\"{o}dinger's wave function of an electric charge
is zero at the string, its static electric field has also no presence at the
string}. This is due to an intrinsic property of the particles, which can be
explained in simple terms: The interaction energy (or momentum) of a system
composed by two monopoles (or a monopole and an electric charge) is not
infinite as seen by any of the particles since the potential it interacts
with at any avaliable point is always finite and well behaved since $\psi =0$
for any particle at the strings regions. As we have seen, this quantity, the
interaction energy (or momentum of the system), may also be derived from the
field itself from $\int \mathbf{B}_{i}\cdot \mathbf{B}_{j}dv$ (or $\int
\mathbf{E}_{i}\times \mathbf{B}_{j}dv$). As the results must be equivalent,
i.e., by calculating the charge times the local potential at the point it is
located, or via the integration of the energy density of the system over
space, one concludes no divergence will appear since in the first
calculation the result is finite. This means the static fields (electric or
magnetic) are also not defined at the string's position.

This conclusion is also valid for photons. The interaction energy between
two monopoles (or a monopole and an electric charge) in a dynamic situation
is finite if the test particle never touches the string of the other

particle (even in the case the potentials include radiation terms). By the principle that the situation is \textit{equivalent} as if one takes the calculation via the energy - density of the strengh - fields, no radiation field is allowed to be defined over the Dirac string's region.

The conclusion is so that the space-time reduction is a real consequence if a singular string of an electromagnetic vector potential is defined in the flat three - dimensional space any time a Dirac monopole is considered, i.e., no particle or field is allowed to be defined at the string's region as result of the solution of the Schr\"{o}dinger equation in the undistorted world.

When a Dirac monopole is defined, a space-time distortion is an immediate consequence; it is the observable consequence of having the string (which cannot be an artifact as He, Qiu and Tze, [4], have shown). The non-volumetric string (in the vector potential equation of the given example, Eq. (4), with $\delta =0$) has no physical definition since in this case it is not possible to find a physical consequence associated to the string.

\section{The Dirac string and the associated space-time distortion}

In this section we attempt to show the basic features related to the fact there is a singular vector potential in a string - like form over space. As it has been discussed in the previous section, the solution of the Schr\"{o}dinger equation (for the wave function $\psi $) for all QED particles is $\psi =0$ at the string's region in the flat three - dimensional world, ${\cal R}^{3}$. It was discussed that all the static fields are also not defined at this region, making it not visited by any kind of physical object. Nor particle or field is defined at the string region, so that it does not belong to the Universe avaliable for any physical entity. The test particle used to measure the magnetic flux generated by a Dirac monopole cannot go over the so called forbidden region (the string region), in a way that the magnetic flux is finite. It results in a covered area that is smaller than the one in which such a forbidden region is not present (if the surface where the measure is taken is spherical, $\sqrt{area/4\pi }<R$, any R), making the space distorted (shrinked) regarding the flat ${\cal R}^{3}$, i.e., it is no longer Euclidean.

In order to see the effects due to the Dirac's monopole presence we can use a geometrodynamic clock as designed by Marzke and Wheeler (a clock which is based only on the basic physical constants of Nature, being then a ruler to measure variations in the bahaviour of the particles around a Dirac monopole, [8]). The clock is defined as a pair of mirrors separated by a fixed distance with a photon bouncing back and forth between them at a constant velocity c. Any variations in the time rate (as the result of the bouncing movement of the light between the mirrors) is related to the properties of the local space-time at the clock's position. With the aim to illustrate the effects due to a vector potential like the one in Eq.(4), let us consider the string passing through the clock. For the photon field there will be no motion over the forbidden region (the string) so that, from the usual distance between the mirrors, we must subtract the distance $2\delta r$ (if the distance from the monopole is r). If the natural frequence of the clock is $\omega ^{0}=c/L$, with L the distance between mirrors, the frequence $\omega $ when the monopole is present is:

$$
\omega =\frac{c}{L(1-\sigma r)}
$$

where $\sigma =2\delta /L$. If $\delta \rightarrow 0$ the clock's frequence can be written:

$$
\omega \simeq \omega ^{0}(1+\sigma r).
$$

If the clock is in movement at some velocity $\mathbf{v}$, the relativistic (special) correction to its frequence relative to the laboratory (where the monopole is at rest) is:

$$
\omega =\omega ^{0}\sqrt{1-v^{2}/c^{2}},
$$

where $v=|\mathbf{v}|$, and for $v<<c$ the last expression can be written as:

$$
\omega \simeq \omega ^{0}(1-v^{2}/2c^{2}).
$$

If the time given by the clock when no monopole is present is dt, the corrected time will be (up to first order approximation) then:

$$
dt\left\{ 1+\sigma r-v^{2}/2c^{2}\right\} .
$$

The total time delay over the clock's trajectory is the integral of the additional term on dt,

$$
\int \left(\sigma r-v^{2}/2c^{2}\right) dt,
$$

and multiplying this difference by the constant $-mc^{2}$ (where m is the inertial mass of some test particle) one gets:

$$
\int \left(-c^{2}\sigma mr+\frac{mv^{2}}{2}\right) dt.
$$

Requiring it to be a minimum (by the minimum action principle), the test particle will behave like it is under the action of a potential proportional to the distance from the monopole, i.e., when free of external forces, it will perform a parabola - like path in space.

It is a simple \textit{illustration} of the consequence of the fact a
singular string - like vector potential is introduced to form a monopole.
The constant $\delta $ is related to the properties of the Dirac's string
associated to a monopole and the constant L of the clock is related to
a
physical constant of Nature when defining the geometrodynamic clock in
terms
of natural units.

In the next section we are going to introduce a \textit{physical} example
of
a space-time distortion made up by a monopole.

\section{The connection to the General Theory of Relativity}

The basic idea related to a Dirac string, i.e., its \textit{physical
image},
is of a long, semi - infinite solenoid with flowing electric current.
This
picture helps to understand the variations one can choose for the string
in
terms of volume in space as well as the reason it is as singular as the
center of a single electric charge. The possibilities for its shape over
space are infinite. We may invent some string of any shape as a set of
elementary strings and the vector potential will be the result of the
integration on such elementary constributions. In the present stage of
the
theory we can only say that some physical space-time distortions may be
described as due to the action of a Dirac string (we do not know the
reason
that make the other non - physical possibilities for the string's shape
to
be avoided). On the other hand we may say that, surprisingly, the effect
on
the space-time predicted by the General Theory of Relativity of a point -
like mass presents a semi - infinite, string - like structure of
forbidden
regions as seen from the flat ${\cal R}^{3}$.

The curved spacetime interval outside the region of some matter
distribution
can be written as:
\begin{equation}
ds^{2}=A(r)dt^{2}-B(r)dr^{2}-C(r)r^{2}d\theta ^{2}-C(r)r^{2}\sin
^{2}\theta
d\phi ^{2}
\end{equation}
where r, $\theta $, $\phi $ are regarded as spherical coordinates and
$%
A(r) $, $B(r)$, $C(r)$ are given functions of r. It is possible to show
that for $r=R$ the corresponding physical area $\Delta $ for R fixed
is:
\begin{equation}
{\Delta }=4\pi R^{2}C(R)
\end{equation}
and that the physical distance $\Lambda $ between the points $r=R_{0}$
and $%
r=R$ on a given radial line is:
\begin{equation}

\Lambda =\int_{R_{0}}^{R}\sqrt{B(r)}dr.
\end{equation}
Consider a spherical mass m at the center of the coordinate system with some given radial matter distribution function. In this case the interval for $r>2m$ is:
\begin{equation}
ds^{2}=(1-\frac{2m}{r})dt^{2}-(1-\frac{2m}{r})^{-1}dr^{2}-r^{2}d\theta
^{2}-r^{2}\sin ^{2}\theta d\phi ^{2}
\end{equation}
according to Schwarzschild. In this particular case the function $C(r)$ is
equal to one. We are interested in to study this metric as seen in the three - dimensional flat world. For the parameter $r=R,$
the physical area ${\Delta }$ is $4\pi R^{2}.$ The physical radius $\Lambda $
will be:
\begin{equation}
\Lambda =\int_{R_{0}}^{R}r/\sqrt{r^{2}-2mr}dr=\left[\sqrt{\left(
r^{2}-2mr\right) }+m\ln \left(r-m+\sqrt{\left(r^{2}-2mr\right) }\right)
\right] \allowbreak _{R_{0}}^{R},
\end{equation}
where R_{0} is some internal radius ($R_{0}<R$) in the region of the matter distribution. We assume that the resulting value of the expression above for the constant parameter R_{0} is vanishingly small (of the order
of $2m$) compared to the one for R, so that the physical radius is:
\begin{equation}
{\Lambda }=\sqrt{\left(R^{2}-2mR\right) }+m\ln \left(R-m+\sqrt{\left(
R^{2}-2mR\right) }\right)
\end{equation}
and for $R>>2m$,
\begin{equation}
{\Lambda }=R+m\ln \left(2R-2m\right)
\end{equation}
i.e., the physical distance ${\Lambda }$ is larger than the parameter R.

The conclusion is that for some physical distance ${\Lambda }$, the area avaliable to cover a sphere of this radius is $4\pi R^{2}$, with $R<{\Lambda
}$, in the flat world. In the three - dimensional flat
world it is then impossible to close the surface of radius ${\Lambda }$. For
a particle in the physical avaliable world no hole occurs, the space-time is
all continuous, but in the ${\cal R}^{3}$ world there is a region
where the spacetime is not defined. As each spherical surface has this non -
physical region, it performs a volume of forbidden places for all particles
and fields in ${\cal R}^{3}$. It is already very well known that for
a flat three - dimensional world it is possible to define an average
curvature by means of a defect from $4{\pi} r^{2}$ of the measured area of
some surface of radius r. The connection of this idea to the theory of
gravitation is via a conceptual significance of the G_{4}^{4} component of
the stress-energy tensor. It is the average curvature $%
R_{12}^{12}+R_{23}^{23}+R_{13}^{13}$ of the three-space, which is
perpendicular to the time. This is a known valid interpretation of the
theory of gravitation, [7].

As we have discussed in the last sections a singular vector potential in a
string - like form reduces space-time. This is a direct result of solving
Schr\"{o}dinger equation for all particles in the Universe. We also have the
possibility to describe any Dirac string's shape by composing elementary
Dirac strings as we have mentioned at the begining of the present
section.

It is then a direct consequence of the present discussion that the proper
definition of a Dirac monopole induces a space-time distortion that may be
identified to the one a point - like mass generates around itself in the
Universe with a suitable Dirac singular - string's shape.

A natural question arises: In defining a mass we are also defining a
magnetic charge? (it does not seems to be correct). The answer is \textit{no}%
. In the spirit of the association between monopoles and space-time
distortions a neutral mass can be imagined as composed by two monopoles of
opposite sign with the same magnitude, having no net magnetic charge at all.

\section{The definition of electrons and photons cause the same effect on the space-time}

>From Dirac's electron's theory it is possible to calculate the electric
charge and spin distributions associated to a particle. The electric charge
distribution in the neighborhood of an electron described by the vector
position $\mathbf{r}$ can be determined as:
\begin{equation}
D_{c}\left(\mathbf{x}\right) =e\int \left[\Psi ^{*}\left(\mathbf{r}-%
\mathbf{x}/2\right) \Psi \left(\mathbf{r}-\mathbf{x}/2\right) \right]
\left[\Psi ^{*}\left(\mathbf{r}+\mathbf{x}/2\right) \Psi \left(\mathbf{r}+%
\mathbf{x}/2\right) \right] d\mathbf{r}
\end{equation}
where $\Psi \left(\mathbf{r}\right) $ is a spinor so that $\left[\Psi
^{*}\Psi \right] =\sum_{\mu =0}^{3}\Psi _{\mu }^{*}\Psi _{\mu }$. Any
expansion of $\Psi $ in the second - quantized form will lead to $\Psi
\left(\mathbf{r}\right) =\sum_{\eta }a_{\eta }\psi _{\eta }\left(\mathbf{r}%
\right) $ with $a_{\eta }^{*}a_{\eta }=N_{\eta }$, the occupation number for
state $\eta $. For a single electron, $\eta =\widetilde{\eta }$ and the
expectation value of $D_{c}\left(\mathbf{x}\right) $ will be:
\begin{equation}
e\sum_{\eta }\int \left[\psi _{\widetilde{\eta }}^{*}\left(\mathbf{r}-%
\mathbf{x}/2\right) \psi _{\eta }\left(\mathbf{r}-\mathbf{x}/2\right)
\right] \left[\psi _{\eta }^{*}\left(\mathbf{r}+\mathbf{x}/2\right) \psi _{%
\widetilde{\eta }}\left(\mathbf{r}+\mathbf{x}/2\right) \right] d\mathbf{r.}
\end{equation}
Inserting the wave functions of a free electron with $\widetilde{\eta }$ the
state at rest, we get from the last expression:
\begin{equation}

\frac{e^{2}}{8\pi ^{3}\hbar ^{3}}\int d\mathbf{p}e^{i\mathbf{x}\cdot
\mathbf{%
p}/\hbar }=e^{2}\delta \left(\mathbf{x}\right)
\end{equation}
if the sum over $\eta $ is written in terms of an integral over the
momenta $%
\mathbf{p}$ of the states. The distribution of charge in one - electron
theory is equal to the Dirac - delta distribution. If the single electron
has some radius, it vanishes (but the electron's self - energy will be
infinite in such case).

The magnetic polarization density is $\mathbf{M}$,
\begin{equation}
M_{x}=\frac{e\hbar }{2mc}\left[\Psi ^{*}\beta \alpha _{y}\alpha _{z}\Psi
\right]
\end{equation}
where $\alpha _{i}$ are the Dirac matrices. The distribution of the spin
in
the electron will be:
\begin{equation}
D_{s}\left(\mathbf{x}\right) =\int \mathbf{M}\left(\mathbf{r}-
\mathbf{x}%
/2\right) \cdot \mathbf{M}\left(\mathbf{r+x}/2\right) d\mathbf{r.}
\end{equation}
>From this expression it is possible then to get: $D_{s}\left(
\mathbf{x}%
\right) =(3/4)(\hbar ^{2}/m^{2}c^{2})D_{c}\left(\mathbf{x}\right) $,
which
means that the spin is distributed in exactly the same way as the charge.
This expression is valid in one - electron or one - electron - plus -
vacuum fluctuation cases.

The self - energy of a single electron at rest will be the sum of the
electric and magnetic parts. The eletrostatic field energy can be
calculated
as
\begin{equation}
(1/2)\int D_{c}\left(\mathbf{x}\right) /|\mathbf{x}|d\mathbf{x=\lim_{%
\mathit{a}\rightarrow 0}e}^{2}/a
\end{equation}
(a the one - electron's defined radius). The spin movement does not
give
rise to any radiation (the time average of the Poynting vector is zero),
which in terms of fields may be demonstrated since $(1/8\pi)\int \left(
\mathbf{\nabla }\times \mathbf{A}\right) ^{2}d\mathbf{r=}$ $\mathbf{-}%
(1/8\pi c^{2})\int \left(\partial \mathbf{A}/\partial t\right)
^{2}d\mathbf{%
r}$ ($\mathbf{A}$ is the vector potential related to the spin), i.e., the
energy contribution due to the magnetic part cancels the electric one in
the
whole effect due to spin. The only contribution for the electron's self -
energy comes from the electrostatic part of the field as written in
equation
(29).

We can calculate the magnetic field produced by a single electron from $%
\mathbf{B}={\nabla }\times \mathbf{A}$,
\begin{equation}
\mathbf{A}\left(\mathbf{r},t\right) =\frac{1}{c}\int
\frac{\mathbf{i}\left(

\mathbf{R},t-|\mathbf{r}-\mathbf{R}|/c\right) }{|\mathbf{r}-
\mathbf{R}|}d%
\mathbf{R}
\end{equation}
with $i_{j}=ec{\Psi }^{*}{\alpha }_{j}{\Psi }$ (${\alpha }_{j}$ the Dirac
matrices) and $\Psi $ the solutions of the wave equation of a free -
electron. According to the expansion defined above,
\begin{equation}
A_{j}\left(\mathbf{r},t\right) =e\int \frac{\mathbf{\sum_{\eta
}}\sum_{\eta
^{^{\prime }}}a_{\eta }^{*}a_{\eta ^{^{\prime }}}\left(\psi _{\eta
}^{*}\left(R,T\right) \alpha _{j}\psi _{\eta ^{^{\prime }}}\left(
R,T\right) \right) }{|\mathbf{r}-\mathbf{R}|}d\mathbf{R}
\end{equation}
where $T=t-|\mathbf{r}-\mathbf{R}|/c$. If $\psi _{\eta }=u_{\eta }\exp
[i(%
\mathbf{p}_{\eta }\cdot \mathbf{r}-E_{\eta }t)/\hbar]$ with $p_{\eta }$
and
$E_{\eta }$ the momentum and energy of the state $\eta $, and $u_{\eta }$
the normalized spinor, the expression for the vector potential can be
written as:
\begin{equation}
A_{j}\left(\mathbf{r},t\right) =2\pi e\hbar ^{2}c^{2}\mathbf{\sum_{\eta
}}%
\sum_{\eta ^{^{\prime }}}\frac{a_{\eta }^{*}a_{\eta ^{^{\prime }}}\left(
u_{\eta }^{*}\alpha _{j}u_{\eta ^{^{\prime }}}\right) }{E_{\eta }E_{\eta
^{^{\prime }}}-m^{2}c^{4}-c^{2}\mathbf{p}_{\eta }\cdot \mathbf{p}_{\eta
^{^{\prime }}}}e^{i\left[\left(\mathbf{p}_{\eta }-\mathbf{p}_{\eta
^{^{\prime }}}\right) \cdot \mathbf{r-}\left(E_{\eta }-E_{\eta
^{^{\prime
}}}\right) t\right] /\hbar }.
\end{equation}
The physical interpretation of this field is straightforward if we
calculate
$(1/8\pi)\int \left(\mathbf{\nabla }\times \mathbf{A}\right)
^{2}d\mathbf{r%
}$, i.e., the magnetic field energy, which is equal to:
\begin{equation}
\frac{\pi }{2}e^{2}\hbar ^{2}\sum_{\eta }p_{\eta
}^{2}\frac{[u_{\widetilde{%
\eta }}^{*}\alpha _{s}u_{\eta }][u_{\eta }^{*}\alpha
_{s}u_{\widetilde{\eta }%
}]}{m^{2}\left(E_{\eta }^{2}-m^{2}c^{4}\right) ^{2}}.
\end{equation}
Averaging over spin directions and replacing the sum by an integral, it
is:
\begin{equation}
\frac{1}{2\pi }\lim_{a\rightarrow 0}\frac{e^{2}\hbar
^{2}}{3m^{2}c^{2}a^{3}}
\end{equation}
in one - electron case, which physically corresponds to the field energy
of
a magnetic dipole density concentrated in a sphere of radius a.

If we demand the single electron self - energy, e^{2}/a to be finite
and
equal to mc^{2}, we obtain a radius $a=e^{2}/mc^{2}$ for a net charge
e.
The spin is $\hbar /2$ and the magnetic moment $e\hbar /2mc$. The
elementary
particle (or single electron) in Dirac's theory is thus this object which

contains the characteristics above.

The physical image is then of a small sphere of radius a of some electric charge, magnetic moment and angular momentum. The minimum physical distance in one - electron case must be defined to be e^{2}/mc^{2}, which means any field's flux measure is made by the use of a surface which contains the sphere of radius a completely. Since the particle's internal region is not accessible it is not possible to identify the origin of the magnetic dipole field as due to some internal current or due to a pair of Dirac monopoles of opposite sign. The net electric flux will always be equal to e and the magnetic one identically zero since the magnetic poles (which perform a dipole in the present view) are defined within the particle's internal region.

We now must define the general equations for charges and monopoles as single entities as well as the conditions involved on the definition of a variational principle from which field and charge equations of motion can be derived properly. After this introduction about monopoles, we analyse the specific case of the single electron and attempt to describe this elementary particle by charge and monopoles' distributions within its internal region.

Let us consider the general case of having electric charges and Dirac poles in a system. The $g>0$ monopole is defined as a source of magnetic field while the $g<0$ monopole as a sink for this field. This assumption permits one to construct a magnetic dipole as a combination of a positive and a negative monopole so that it would be indistinguishable from that produced by a suitable electric current, [9]. By this definition the generalized Maxwell equations, including electric and magnetic currents, are:
\begin{eqnarray}
\partial _{\mu }F^{\mu \nu } &=&-j_{e}^{\nu } \\
\partial _{\mu }F_{*}^{\mu \nu } &=&j_{g}^{\nu } \nonumber
\end{eqnarray}
where $F^{\mu \nu }$ is the electromagnetic field tensor connected to the four - vector $j_{e}^{\nu }$ formed by the electric charge and current densities and $F_{*}^{\mu \nu }$ is the dual of $F^{\mu \nu }$ (which is connected to the four -vector $j_{g}^{\nu }$ formed by the magnetic charge and current densities):
\begin{equation}
F_{*}^{\mu \nu }=\frac{1}{2}\varepsilon ^{\mu \nu \sigma \delta }F_{\sigma \delta }
\end{equation}
and $\varepsilon ^{\mu \nu \sigma \delta }$ is the four - dimensional Levi - Civita tensor ($\varepsilon _{0123}=-\varepsilon ^{0123}=1$, completely antisymmetric).

Each charge and monopole can be described by a corresponding Lorentz's equation,
\begin{eqnarray}
m_{e}\frac{d^{2}z^{\mu }}{ds^{2}} &=&e\frac{dz_{\nu }}{ds}F^{\mu \nu }(z) \\
m_{g}\frac{d^{2}x^{\mu }}{ds^{\prime 2}} &=&-g\frac{dx_{\nu }}{ds^{\prime }}%
F_{*}^{\mu \nu }(x) \nonumber
\end{eqnarray}
where m_{e} and m_{g} are the electric and magnetic particles's inertial
masses, $z_{\mu }(s)$ is the charge's world - line as function of the proper
time s and $x_{\mu }(s^{\prime })$ the corresponding four - coordinate which account for the monopole's position.

The question about the possibility for a local action principle is based on
the fundamental interaction between an electric charge and a magnetic monopole. Considering a system composed of just one charge and one monopole,
the fields that appear in the equations of motion represent radiative reaction terms and the retarded fields due to their mutual interaction.

It is possible to define two four-vectors, $W^{\mu }$ and $V^{\mu }$ such that $F^{\mu \nu }=W^{\mu \nu }-V_{*}^{\mu \nu }$; $F_{*}^{\mu \nu
}=W_{*}^{\mu \nu }+V^{\mu \nu }$, with $W^{\mu \nu }=\partial ^{\mu }W^{\nu
}-\partial ^{\nu }W^{\mu }$, $V^{\mu \nu }=\partial ^{\mu }V^{\nu }-\partial
^{\nu }V^{\mu }$. By the use of the Lorentz condition, $\partial _{\mu
}W^{\mu }=0$, $\partial _{\mu }V^{\mu }=0$, we get:
\begin{equation}
{\partial}^{2} W^{\mu }=-j_{e}^{\mu };{\partial}^{2} V^{\mu }=j_{g}^{\mu }.
\end{equation}
In order to obtain a term $-eV_{*}^{\mu \nu }dz_{\nu }/ds$ on the right side
of the particle equation for the charge it is necessary to assume the action
integral has a term like:
\begin{equation}
-e\int \frac{dz_{\mu }}{d\tau }V_{*}^{\mu }(z)d\tau ,
\end{equation}
which is only possible if $\partial ^{\mu }V_{*}^{\nu }-\partial ^{\nu
}V_{*}^{\mu }=V_{*}^{\mu \nu }$. In order to see the conditions the last equation follows, lets consider the nonlocal potential as a consequence of $%
j_{g}^{\mu }\neq 0$:
\begin{equation}
V_{*}^{\mu }(z)=\int_{-\infty }^{0}V_{*}^{\alpha \beta }(\xi)\frac{\partial
\xi _{\beta }}{\partial z^{\mu }}\frac{\partial \xi _{\alpha }}{\partial \tau }d\tau
\end{equation}

where the four- vector $\xi ^{\mu }(z,\tau)$ is defined by
\begin{eqnarray}
\xi^{\mu }(z,0) &=& z^{\mu } \\
\lim_{\tau \rightarrow -\infty }\xi^{\mu }(z,\tau) &=&
space-like-infinity \nonumber

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\end{eqnarray}
where ``space-like infinity'' means relative to the monopole's position,
the
charge's position at the infinite past. The path from the distant point
to $z
$ is traversed by $\xi $ as $\tau $ varies from $-\infty $ to $0$ and
then,
from equation (17) one finds:
\begin{equation}
\partial ^{\mu }V_{*}^{\nu }-\partial ^{\nu }V_{*}^{\mu }=V_{*}^{\mu \nu
}+\int_{-\infty }^{0}\frac{\partial \xi _{\beta }}{\partial z^{\nu
}}\frac{%
\partial \xi _{\gamma }}{\partial z^{\mu }}\frac{\partial \xi _{\alpha
}}{%
\partial \tau }(\partial ^{\gamma }V_{*}^{\alpha \beta }+\partial
^{\alpha
}V_{*}^{\beta \gamma }+\partial ^{\beta }V_{*}^{\gamma \alpha })d\tau
\end{equation}
with $(\partial ^{\gamma }V_{*}^{\alpha \beta }+\partial ^{\alpha
}V_{*}^{\beta \gamma }+\partial ^{\beta }V_{*}^{\gamma \alpha
})=\varepsilon
^{\alpha \beta \gamma \sigma }\left[ j_{g}\right] _{\sigma }(\xi )$. The
action will be nonlocal as long as, in order to get $\partial ^{\mu
}V_{*}^{\nu }-\partial ^{\nu }V_{*}^{\mu }=V_{*}^{\mu \nu }$ from the
last
equation the paths of the charge and monopole are previously set to not
cross, so that a charge's world - line never crosses a monopole's world -
line at the same spacetime point.

Considering the general case of a charge and a monopole that meet at some
spacetime point, the contribution for $F^{\mu \nu }$ as given by the
second
term on the right of equation (42) will be:
\begin{equation}
\lim_{\tau \rightarrow 0}-\left[ \frac{\partial \xi _{\alpha }}{\partial
\tau }\right] _{\tau =0}\varepsilon ^{\alpha \mu \nu \sigma }\int_{\tau
}^{0}\left[ j_{g}\right] _{\sigma }(\xi )e\frac{dz_{\nu }}{ds}d\tau .
\end{equation}

This contribution will vanish provided ($[j_{g}{]}_{\sigma }$ written in
terms of $dx_{\sigma }/ds$)
\begin{equation}
\frac{dz_{\nu }}{ds}=\frac{dx_{\nu }}{ds}
\end{equation}
as in this case $\varepsilon ^{\alpha \nu \mu \sigma }(dz_{\sigma
}/ds)(dx_{\nu }/ds)=0$ with ${\nu }={\sigma }$. Therefore the local
character of the action principle, from which particles equations of
motion
can be derived, is assured as long as the world-line's paths of charges
and
monopoles never cross. When at the same spacetime point, the charge and
the
monopole must have the same four - velocities.

Futhermore, in order to have a general variational principle from which
both
particle as well as field equations can be derived simultaneously for a
system of distinct particles, it is necessary to have $j_{g}^{\mu }=0$
everywhere, [9].

The physical image from Dirac electron's theory is of a small sphere of
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radius a with some electric charge, magnetic moment and angular momentum.
Since the particle's internal region (the region understood within the radius a) is not accessible, it is not possible to identify the origin of the magnetic dipole field as due to some internal current or due to a pair of Dirac monopoles of opposite sign. In fact, if we define the magnetic poles such that $g>0$ monopole is a source of magnetic field and the $g<0$ monopole a sink for this field, it is possible to construct a magnetic dipole (as a combination of a positive and a negative monopole) so that it would be indistinguishable from the one produced by a suitable electric current.

The parameters involved on the description of the elementary particle are the mass, the charge and the distance a, i.e., the particle's radius. It is possible, as we will show now, to describe the particle's characteristics in terms of magnetic flux g because it may be written in terms of such parameters. It is assumed there is a rigid charge distribution over the surface of radius a in Dirac theory if we demand the particle's self - energy is mc^{2}. We now describe the particle's main characteristics by the use of poles (electric and magnetic) distributed over some distance parameter as well.

Now we intend to use spinless electric and magnetic poles to describe the charge, the spin and magnetic moment of an elementary particle, so instead of volumetric density of poles distributions we will consider point - like poles distributed over some distance parameter (once it is enough to discuss such features). The simplest charge and monopole distribution is that of point particles separated by some distance. Lets define an elementary particle as composed of a point electric charge and a magnetic dipole: In a Cartesian coordinate system, consider a positive monopole g at $(0,0,z_{0})$ with ($z_{0}>0$, z_{0} a real number), a negative monopole $-g$ at $(0,0,-z_{0})$ and an electric charge e at the origin. Now z_{0} is to be considered as the particle's radius.

Based on the last discussion we are able to ask if the particle defined in the present way has a possible description by a variational principle. By the present definitions $j_{g}=0$ everywhere since no net magnetic poles can be found (any field's flux measure must be made by the use of a surface which contains the entire particle according to the usual procedure about elementary particles [5]), so a general action principle can be defined in order to derive field as well particle equations for a system of particles like this. No world-lines' crossing will result in additional terms since any external charge interacting with one such particle never met a single monopole when their world - lines cross, but two of opposite sign, both in the same particle.

Considering the poles *inside* the the particle's structure, one could argue about their mutual interaction. Since these poles are rigidly fixed by definition, there will be no world-line crossing between charge and monopoles, but the $z_{0}\rightarrow 0$ limit must be considered in order to verify the validity of the description for any z_{0}. In the $z_{0}\rightarrow 0$ case the monopoles are at the same spacetime point with the electric charge. As by construction all the structure particles have the same four - velocity, contributions terms from the interaction between constituints are identically null according to eqautions (43) and (44).

Now let us calculate the angular and magnetic momenta of the system. The angular momentum of the electromagnetic field, $\mathbf{L}_{em}$, is:

$$\mathbf{L}_{em}=\frac{1}{4\pi c}\int \mathbf{r\times }\left(\mathbf{E}\times \mathbf{B}\right) dv$$

where $\mathbf{r}$ is the three - vector distance, $\mathbf{E}$ and $\mathbf{B}$ the electric and magnetic fields produced by the system of an electric charge plus a magnetic dipole (all at rest in the laboratory frame), and dv the volume integration element. It results:

$$\mathbf{L}_{em}=\frac{\left(2g\right) e}{c}\mathbf{\zeta }$$

with $\mathbf{\zeta }$ the versor on the positive direction on z axis. It is independent of z_{0}, so for some given e and g, the monopoles can be defined *arbitrarily* close to the charge for the same resulting particle's angular momentum.

Since any charge - monopole pair has an angular momentum naturally associated, the simple quantization of the momenta (as pointed out by Saha and Wilson [10]) leads to the quantization condition $\mathbf{L}_{em}=n\hbar /2$ or $n\hbar$ with n an integer. By the quantization condition one gets $\left(2g\right) e/c=n\hbar /2$ (or $n\hbar$) for the system.

If the magnetic moment of a particle is $e\hbar /(2mc)$, (in terms of the present description, $2gz_{0}$), by the quantization condition $2ge/c=n\hbar /2$, one gets from $e\hbar /(2mc)=2gz_{0}$:

$$z_{0}=\frac{e^{2}}{mc^{2}}\left(\frac{1}{n}\right) .$$

For an arbitrary large value of the angular momentum, the size of a particle

as composed of one electric charge and two monopoles of opposite charge
may
be arbitrarily small (for $n\rightarrow \infty $, the spin $n\hbar
/2\rightarrow \infty $ and $z_{0}\rightarrow 0$) for the same value of
the
magnetic moment $e\hbar /(2mc)$.

Now lets take $n=1$ in order to compare the Weisskopf result [5] on the
main
characteristics of a single electron with the ones derived by the present
description in which the angular momentum is $\hbar /2$, the distance
parameter defined for the magnetic dipole is $z_{0}=e^{2}/mc^{2}$ (which
is
the same particle's radius as in Weisskopf's) and the magnetic moment $%
e\hbar /2mc$ for a net electric charge e. It results the \textit{%
description of the particle's characteristics} as defined above
\textit{is
equivalent to those get from Dirac's theory in the one-electron case. }

It is now possible to suggest that the same conclusions about space-time
distortions related to Dirac monopoles are also valid for an electron,
positron or any other QED fermion. The above discussion about the
electron's
description has shown that it is possible to \textit{describe} this
particle's main features by the use of the Dirac monopole's idea. As a
consequence, it is possible to mathematically identify a Dirac monopole
related to any fermion, being plausible to afirm that it distorts the
space-time as predicted for the monopole.

The photon, another QED particle can also be said to make the same effect
on
the space-time. The simplet way to present this is by remembering it may
be
described (the photon) as composed by two fermions of vanishing inertial
mass (as pointed out by M. de Broglie, [11]). By the same reasoning, once
it
may be described by QED fermions (which in their turn may be described by
the composition of Dirac monopoles and electric charge) it may be said
the
photon generates a space-time distortion as any other particle in QED.

\section{Conclusions}

In this letter an introductory discussion has been opened towards the
connection between Electrodynamics and Gravitation. The main driving
reason
is the Dirac monopole which lacks of a well - defined theoretical
definition
if the string of singularities (of the associated vector potential) has
no
physical interpretation. The model is still very uncomplete since no
mathematical expression relating the magnitude of the magnetic charges
and
the gravitational mass has been given, or the reason some possibilities
for
the string's shape to be not physically observed in Nature. The main goal
was to get attention to the fact it is possible to make a coherent idea
of
Electromagnetism and Gravitation in a combined model. We also address to
a
future work the possible developments of the He, et. al, [4], work on the

monopole's subject which has shown the longitudinal part of the Electromagnetic theory must be an observable if the Dirac monopole's charge is nonzero.

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