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# ELEMENTARY DEVELOPMENT OF GRAND UNIFIED THEORY BASED ON FORMAL RELATIVISTIC APPROACH

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## 1.0 INTRODUCTION

To derive the general relativity theory, we may follow the tensor formulation set by Einstein [1] and this is the formal approach to treat the theory. After successfully developed the general relativity, Einstein has tried to develop the Grand Unified Theory to apply to other elementary forces like electromagnetism and nuclear interaction. Since then various approaches have been formulated to derive the Grand Unified Theory but met with limited success. The problem may not due to the missing or wanting of new theory, but rather certain basic principle like equivalent principle [2] that may need to be treated more strictly. There is one area in the development of relativity theory that is not much studied on, which is the formulation of equivalent principle. The equivalent principle is taken for granted and few studies are concentrated on its strict tensor presentation. In this, we are going to show that by deriving the equivalent principle in a strict tensor form, we can generalize it to potentials other than gravitation. What is interesting is that in this way it is not necessary for us to put forward any new theory for the development of Grand Unified Theory. The expression of the equivalent principle in strict tensor form does not change or affect the metric tensor of gravitation, so strictly speaking it is inline with the formal treatment of relativity theory and is not a new proposition. The strict study of the equivalent principle does shed more light into our understanding of relativity and it remains a fascinating fact that by using the conventional approach in the relativity theory, it may be possible for us to develop the fundamental basis of Grand Unified Theory [2].

## 2.0 THE EQUIVALENT PRINCIPLE

The equivalent principle states that the kinetic acceleration is equal to the potential acceleration, which conventionally [1] is expressed as

$$d^2r/dt^2 = -d\phi/dr \qquad (2.1)$$

where the L.H.S of the equation is the kinetic acceleration due to dynamic motion and the R.H.S of the equation is the potential acceleration caused by a potential field on the object or particle of concern. Equivalent principle is one of the key principles in the derivation of general relativity. Let us look into greater details of this principle to provide us the possibility to generalize the relativity theory into various potentials.

To express in tensor, we may write the kinetic acceleration as

$$a^1{}_{44} = d^2r/dt^2 \qquad (2.2)$$

where the superscript 1 represent the radial dimension and the subscript 4 represent the time dimension respectively. Similarly, the potential acceleration can be expressed as

$$\alpha_1 = -d\phi/dr \qquad (2.3)$$

In strict tensor calculus, equation ( 2.2 ) cannot be directly equated to equation ( 2.3 ), as the kinetic acceleration is a three degree mixed tensor, with one degree in contravariant and two degree in covariant indices while the potential acceleration tensor is only of one degree in covariant tensor. To overcome this problem, it is a common practice in tensor calculus to reduce the tensor by multiplication of the opposite metric. Once the tensors are equally reduced into zero state, we have no difficulty to equate them together. Now, let us reduce both the kinetic acceleration tensor and potential acceleration tensor with opposite metric tensor and equate them as

$$m^{44}{}_1\, a^1{}_{44} = m^1\alpha_1 \qquad (2.4)$$

where $m^{44}{}_{1}$ and $m^{1}$ are the respective opposite metric tensors. Rearranging the above equation, we get

$$a^{1}{}_{44} = m^{11}{}_{44}\alpha_{i} \qquad (2.5)$$

or

$$d^{2}r/dt^{2} = - m^{11}{}_{44}d\phi/dr \qquad (2.6)$$

This is the equivalent principle derived in strict tensor approach. The metric tensor $m^{11}{}_{44}$ in weak field like gravitation can be ignored and taken as unity. However, in strong fields of electromagnetism it can no longer be ignored. So if we are to generalize the equivalent principle to electromagnetic potentials, the metric must be taken seriously. In the later section of this paper it can be shown that after all, it does not affect or change the final results of general relativity. So equation (2.6) should be taken as a strict presentation rather than a new proposition of equivalent principle. In fact, it is precisely the above strict presentation of equivalent principle that the formal relativity theory can be extended to formulate other potentials than that of gravitation.

**3.0 THE UNIFORM AND NON-UNIFORM METRIC TENSOR**

The complete differential of a tensor $\chi$ may be expressed as

$$d\chi = m_{j}d\chi^{j} + \chi^{k}dm_{k} \qquad (3.1)$$

If the potential is uniform and the transformation is also uniform, then the variation of the metric tensor $dm_{k}$ vanished and the above equation is simplified to

$$d\chi = m_{j}d\chi^{j} \qquad (3.2)$$

The above is known as uniform transformation and consequently the metric tensor $m_{j}$ is also known as uniform metric tensor. This is equivalent to the partial derivative in the tensor calculus.

However, in the real physic world a uniform field seldom exist and the particles are in constant motion and always travel through different strength of potentials, the above equation is only true for certain specific and static state. In general, the variation of the metric tensor cannot be ignored. So equation (3.1) can be rearranged into

$$\begin{aligned} d\chi &= (1 + \chi^{k}dm_{k} / m_{j}d\chi^{j})\, m_{j}d\chi^{j} \\ &= (1 + \chi^{k}d\ln(m_{j})/ d\chi^{k})\, m_{j}d\chi^{j} \end{aligned} \qquad (3.3)$$

If $m_{j}$ is the function of $(\chi^{k})^{\eta}$, where $\eta$ is an unknown power, such that

$$m_{j} = f((\chi^{k})^{\eta}) \qquad (3.4)$$

Applying Euler theorem to equation (3.3), we finally get

$$d\chi = (1 + \eta\ln(m_{j}))\, m_{j}d\chi^{j} \qquad (3.5)$$

$$= g_{j}d\chi^{j} \qquad (3.6)$$

where

$$g_{j} = (1 + \eta\ln(m_{j}))\, m_{j} \qquad (3.7)$$

is the complete metric tensor. We can call it non-uniform metric tensor. Once again, we are expressing the metric tensor in a strict tensor formality. In weak field like gravitation the term $\ln(m_{k})$ is very negligible and can be neglected. The strength of gravitational field over a microscopic region is practically uniform and thus the non-uniform and the uniform metric can practically taken to be the same. In strong field like electromagnetic fields, even at the atomic level the variation of fields is great, the term $\ln(m_{k})$ cannot be ignored and neglected.

Actually equation (3.2) and (3.5) are the partial and complete derivatives respectively in tensor calculus. As components of a metric equation are partial derivatives, it is always the uniform metrics that are applied to the metric equation, not the complete metric. However, in real physical observations that involve translocation of tensor, partial derivative do not describe the condition completely, so the complete or non-uniform metric is the actual observation. So after we get the solution for the uniform metric tensor, we need to substitute it into the non-uniform tensor for real physical observations. This discussion is inline with the affinity concept of parallel translocation of tensor.

**4.0 DERIVATION OF UNIFORM AND NON-UNIFORM METRIC TENSOR FOR OPEN POTENTIAL**

The general metric equation [1][2][3] for a free particle is

$$d^2x^i/ds^2 + \{^i{}_{jk}\}dx^j dx^k/(dsds) = 0 \qquad (4.1)$$

Following the orthodox approach in the derivation of relativity, we replaces all ds with dt, and when a particle is momentarily stationary in a field, all time derivatives vanishes except the acceleration factor and the derivative of time component where $dx^4/dt= ic$. Assuming that the acceleration factor is pointed along the radial direction, we simplified the above equation to

$$d^2r/dt^2 - \{^1{}_{44}\}c^2 = 0 \qquad (4.2)$$

Expanding the above equation[1][2][3], we finally get

$$d^2r/ds^2 = -(c^2/2)m^{11}dm_{44}/dr \qquad (4.3)$$

note the usage of uniform metric, as the above are all components of partial derivative. Substitute the equation ( 2.6 ) from the equivalent principle, we get

$$-m^{11}{}_{44}d\phi/dr = -(c^2/2)m^{11}d(m_{44})/dr \qquad (4.4)$$

or

$$d(2\phi/c^2)/dr = (m_{44})^{-1}d(m_{44})/dr = d(\ln m_{44})/dr$$

Solving for $m_{44}$, we get

$$m_{44} = \exp(2\phi/c2 + k)$$

where k is the integration constant. When $\phi = 0$, $m_{44}=1$, therefor $k = 0$. Finally

$$m_{44} = \exp(2\phi/c^2) \qquad (4.5)$$

and

$$m_4 = (m_{44})^{1/2} = \exp(\phi/c^2) \qquad (4.6)$$

From the relativity theory, it can be proved that the radial metric tensor is the reciprocal of time metric tensor[1][2][3], that is

$$m_{11} = (m_{44})^{-1} = \exp(-2\phi/c^2) \qquad (4.6a)$$

and

$$m_1 = (m_{11})^{1/2} = \exp(-\phi/c^2) \qquad (4.6b)$$

Above are the uniform metric expressed in potential $\phi$. As the potential in equation (4.6) can be both positive and negative, it can only be applied to the potential that can be polarized into positive and negative polarities. Clearly, equation (4.6) is not suitable for gravitation, as it is always negative, cannot be polarized and universal in nature. For the times being, we take equation (4.6) to be suitable for electromagnetic potentials. This is an important first step taken to derive the Grand Unified Theory.

If $\phi$ is inversely proportional to the radius r as in electromagnetic potentials,

$$\phi \propto (r)^{-1} \qquad (4.7)$$

$\eta$ in equation ( 3.7 ) becomes –1, we get

$$g_k = (1 - \ln(m_k))\, m_k \qquad (4.8)$$

and after substitute equation ( 4.6 ) into equation ( 4.8 ), we get the non-uniform equation for the metric tensor for positive potential as

$$g_4 = (1 - \phi/c^2)\exp(\phi/c^2) \qquad (4.9)$$

and

$$g_1 = (g_4)^{-1} = (1 - \phi/c^2)^{-1} \exp(-\phi/c^2) \qquad (4.9a)$$

## 5.0 THE UNIVERSAL METRIC

In classical physics, a positively charged particle can cancel out the effect of a negatively charged particle of the same magnitude. However, in relativistic formulation, this is not so. Say there is a neutral particle, which is consisted of a pair of oppositely charged sub particles that are interacting with each other to form a stable neutral particle. The positive sub particle will exert a non-uniform transformation of

$$g_{4(+)} = (1 - \phi/c^2)\exp(\phi/c^2) \qquad (5.1)$$

where $\phi$ is the magnitude of the total potentials. The negative particle will exert an opposite transformation as

$$g_{4(-)} = (1 + \phi/c^2)\exp(-\phi/c^2) \qquad (5.2)$$

The resultant transformation is the superimpose of the above two equations, we get

$$\begin{aligned} g_{44(u)} &= g_{4(+)}g_{4(-)} \\ &= (1 - \phi^2/c^4) \qquad (5.3) \end{aligned}$$

The resultant equation shows that when two opposite potentials interacting with each other, there is a resultant secondary transformation which is universal in nature. It is universal because there is only one type of transformation and cannot be polarized into a positive and a negative transformation. The negative sign in the equation indicate that it is always an attraction.

In nature, an electrical potential $\phi_{(e)}$ of a particle is always accompanied by magnetic moment potential $\phi_{(m)}$, if these two potentials are equal in magnitude, from electromagnetic theory, the total potential can be expressed as

$$\begin{aligned} \phi^2 &= \phi_{(e)}^2 + \phi_{(m)}^2 \\ &= 2\phi_{(e)}^2 \qquad (5.4) \end{aligned}$$

Substitute the above expression into equation ( 5.3 ), we have

$$g_{44(u)} = (1 - 2\phi_{(e)}^2/c^4) \qquad (5.5)$$

From electromagnetic theory, we may express the electrical potential and electrical energy as

$$\phi_{(e)}^2 = \gamma E/r \qquad (5.6)$$

where $\gamma$ is the proportional constant and r is the distance from the source of the potential. Substitute the above equation into equation ( 5.5 ), we have

$$g_{44(u)} = (1 - 2\gamma E/(rc^4)) \qquad (5.7)$$

Substitute the Einstein energy equation $E = Mc^2$ into the above equation, we get

$$g_{44(u)} = (1 - 2\gamma M/(rc^2)) \qquad (5.8)$$

if we define the unit of the energy and the mass such that the constant $\gamma$ is equal to the gravitational constant G, we are arriving at the Einstein's gravitational metric tensor

$$g_{44(g)} = (1 - 2GM/(rc^2)) \qquad (5.9)$$

Comparing equation ( 5.9 ) and equation ( 5.5 ), the relationship of gravitation to electric or magnetic moment potential is

$$\begin{aligned} \phi_{(g)} &= -GM/r \qquad (5.10) \\ &= -\phi_{(e)}^2/c^2 \\ &= -\phi_{(m)}^2/c^2 \end{aligned}$$

If our formulation and reasoning is correct, we have just proved that gravitation is the secondary relativistic effect of electromagnetic transformation and also explained why it is always universal in nature. What is interesting, a pure gravitational field may not exist, but rather the resultant of the relativistic interaction of electromagnetic potentials which created the effect of gravitation. If electromagnetic fields are the primary fields, then gravitation is the secondary field generated by the primary fields. Based on this reasoning, the present of gravitation is the indirect proof that transformation of electromagnetic fields does exist.

## 6.0 THE UNIFORM AND NON-UNIFORM METRIC FOR BOUNDED POTENTIAL

After successfully associate electromagnetic fields to that of a universal field, which may be the gravitation, let us try to formulate the link between nuclear potentials to that of electromagnetic potentials. We shall take the same tensor approach as the above to formulate the nuclear transformation. As the electrical or magnetic potential are unbounded and disperse through out the space, while the nuclear potential is only existed over a very small region around the nucleus, there is a basic different in the approach to solve the metric tensor for nuclear forces than that of electromagnetism.

How the particles are bound together is poorly understood until today. Unfortunately, without the understanding of how nuclear potential is acting, it is not possible for us to derive the metric tensor. Under this circumstance, we have to make certain intelligent assumptions of how the nuclear potential is acting and what is the most probable structural model of elementary particles. That will form as a basis for the derivation of the metric tensor of nuclear force.

During pair production, a high-energy photon is breaking down into a pair or pairs of electrons and positrons. Here we assume that an electron is consisted of at least a pair of sub-photons interacting with each other to form a stable particle. As each sub-photon is basically electromagnetic in nature with its fields quantified [4][5] successively with opposite polarities, travelling in a close path will turn it into a harmonic oscillator that is constantly emitting away energy. Under normal circumstances the sub-photon will quickly disintegrated into secondary electromagnetic wave and decay away. However, if the pair of the sub-photons are identical or similar to each other, except one of them is having an extra negative electric field resulted in the negative charge, the energy emitted away is equal to the energy absorbed in. If both the sub-photons are in a resonance state then a stable system may be resulted. Both of them will become standing waves constantly exchanging energy. This means the close path they are travelling in is a simple multiple of their wavelengths. We assume that all the electromagnetic fields are quantified at velocity of light except the extra negative field that becomes an open field.

Even though changing energy may result in a stable system, this alone will not bind them together and will not bend the path of the sub-photons. In the above discussion sub photons are contributing the exchanging energy to each other, which means actually there are two equal parts of energy being exchanged all the time. If one part of the energy is emitted away and only the remaining energy is remained, a more stable and binding condition is resulted. Due to lacking of one part of the exchange energy, a negative potential is created among the two sub-photons to bend their path and bind them together. We assume that the negative potential is sufficiently strong to bend the path of the sub-photon into a close circuitry path. Without the negative potential, the path of the sub-photons will remain straight and no close circuitry of path will occur. The mechanism of energy exchange remains as it is except that while a sub-photon is emitting a positive energy, it is equivalent to receiving an equal amount of negative energy, while the other party is receiving a positive energy and emitting away a negative energy. Once the positive energy is emitted, the negative energy created on this side of the sub-photon is not only bending the path of the other sub-photon, it also bend the energy emitted by the other sub-photon towards itself. Thus making the interaction of energy in a small area possible. We are going to solve the metric tensor based on this energy exchange model.

If $\Delta E$ is the positive energy being exchanged and a potential energy P associated with the negative counterpart, we have

$$P = -\Delta E \qquad (6.1)$$

which resulted in a negative potential $-\phi$ such that

$$P = -m\phi \qquad (6.2)$$

where m is a proportional constant between the potential energy and its potential. In this case we may define the potential $\phi$ in such a way that m is the mass of the sub-photon.

Differentiate equation ( 6.2 ) with respect to r, we get

$$P_{,1} = -m\phi_{,1} \qquad (6.3)$$

or

$$\phi_{,1} = (-P/m)_{,1} \qquad (6.4)$$

From the strict equivalent principle, we get

$$\begin{aligned} d^2 r/dt^2 &= -m^{11}{}_{44}\phi_{,1} \\ &= m^{11}{}_{44}(P/m)_{,1} \qquad (6.5) \end{aligned}$$

Substitute the above equation into equation ( 4.3 ) and solving the metric tensor, we get

$$\begin{aligned} m_{44(-P)} &= \exp(-2P/mc^2) \qquad (6.6) \\ &= \exp(-2P/E) \end{aligned}$$

or

$$m_{4(-P)} = \exp(-P/E) \qquad (6.7)$$

where E is the total energy of the sub-photon.

Substitute the above equation into equation ( 4.8 ), we get the non-uniform metric tensor for bounded potential as

$$g_{4(-P)} = (1 + P/E)\exp(-P/E) \qquad (6.8)$$

This is a tremendous transformation. Comparing to the electromagnetic tensor, there is an absent of a factor division of $c^2$, this means that nuclear interaction is $10^{20}$ times stronger than an electromagnetic transformation. Equation ( 6.8 ) is successfully express the metric tensor of nuclear transformation in the total potential energy and also total energy which is electromagnetic in nature. In this way we have link the electromagnetic energy to that of nuclear transformation.

## 7.0 UNIVERSAL NUCLEAR INTERACTION

For the sub photon that is emitting the energy $\Delta E$, it experiences a non-uniform transformation as

$$g_{4(-)} = (1 + \Delta E/E)\exp(-\Delta E/E) \qquad (7.1)$$

For the sub photon which is absorbing energy, it experiences a positive potential and similarly the non-uniform transformation is

$$g_{4(+)} = (1 - \Delta E/E)\exp(\Delta E/E) \qquad (7.2)$$

As both sub-photon are interacting over a microscopic region, they will experience the superimpose of both transformations and the resultant metric is

$$g_{44(N)} = (1 - (\Delta E/E)^2) \qquad (7.3)$$

Comparing to the gravitational metric, this transformation is $c^4$ or $10^{40}$ time more powerful than gravitational transformation and concentrated in a small microscopic region, making the bending and binding of sub-photons possible. As the potential is existed by exchanging of energy, which is quantified to a certain region and is not an open potential but a bounded one, the influence of the nuclear force outside this region is depending on the probability of finding the exchanging energy beyond this region. Which is quickly fading away and expect to follow the general Gauss exponential law of probability. The same discussion on electron can be easily extended to other particles and also the atomic nucleus.

If our discussion is logical, we have closed the gap between nuclear interaction and electromagnetic fields. Like gravitation, no pure nuclear potentials exist in isolation and independent of the electromagnetic potentials.

## 8.0 DISCUSSION ON EQUIVALENT PRINCIPLE

As we can see from the above discussion, the strict tensor equation of the equivalent principle is the key factor in the development of the Grand Unified Theory. Unless this principle can be verified experimentally, otherwise all the formulations will remain unproven thesis. From equation ( 2.6 ), ( 4.5 ) and (4.6a), the metric tensor for the equivalent principle of a positive potential is

$$m^{11}{}_{44} = m_{4444} = \exp(4\phi/c^2) \quad > 1 \qquad (8.1)$$

of a negative potential is

$$m^{11}{}_{44} = m_{4444} = \exp(-4\phi/c^2) \quad < 1 \qquad (8.2)$$

This means that for a positive potential, the kinetic acceleration is greater than the potential acceleration in magnitude. For a negative potential, the reverse is true. This indicate that if a particle come closer to another particle of the same charge, it will be deflected away more than that predicted by classical physics. If a particle comes closer to an opposite charge, it will be attracted more towards the center than expected. This will form as a basis for us to verified the equivalent principle experimentally. If it can be verified experimentally, then it is the direct verification of the relativistic transformation of electromagnetic potentials.

The strict equivalent principle predicted that a true circular orbit is impossible. Say in a hydrogen model, if electron is revolving around the center of mass in circular orbit, then the outward acceleration, which is equivalent to the acceleration due to motion and thus a kinetic acceleration, is always smaller that the inward acceleration, which is caused by the attraction and thus a potential acceleration. As a result, the electron will collapse into the center. In order to stabilize the orbit, the whole circular orbit must have extra angular moment by revolving around the axis that passing through the center of mass, to create extra outward acceleration to balance out the higher potential acceleration and forming a stable orbiting. Due to high speed of electron, the orbit is practically revolves itself into a spherical shell.

A perfect elliptical orbit is also not possible. As the particle will also fall toward the center of mass more than expected, thus the elliptical orbit will process. This has been observed in the case of planet Mercury orbiting around the sun. In atomic level, due to great speed of motion and the transformation due to electromagnetism is much higher than gravity, the procession of the elliptical orbit should be very prominent and observable. As in the case of circular orbit, by procession of the elliptical orbit may not be sufficient to offset the greater potential acceleration due to attraction, then it may need to revolve to gain more kinetic acceleration for a stable orbit. The elliptical orbit will just develop into a fuzzy shell. If the orbit is like a figure of 8, it will develop into an hourglass like shell orbit.

## 9.0 DISCUSSION ON THE STRUCTURE OF ELECTRON

Coming back into our assumption that electron is made up of a pair of standing waves due to sub-photons. This may explain the wave-particle nature of electron. Actually, from quantum mechanics [4][5] we learn that if two infinite monochromatic waves of wave factor k and angular frequency $\omega$ superimpose on each other to form a wave pocket, the group velocity of the wave packet is

$$v_g = d\omega/dk \qquad (9.1)$$

where dω and dk is the difference in their angular frequency and wave factor respectively. The phase velocity of the infinite wave is

$$v_p = \omega/k \qquad (9.2)$$

which is equal to the velocity of light. This phase velocity is always ignored in quantum mechanic. However, it fits well into our sub-photon model for electron. As sub-photon travels in a closed circuitry as a standing wave, it behaves as an infinite monochromatic wave travels at the velocity of light. The differential in the energy of sub-photons will create variation in the angular velocity and also the wave factor and resulted in the physical velocity of electron. If there is no differential in the energy of sub-photons, the group velocity is zero and the electron is 'at rest'. The orbiting of sub-photon may explain the spin phenomenon of electron in quantum mechanics. Also, our electron model explains why a photon is not contracted to a point as predicted by special relativity. It is clear that the velocity related to the Special Relativity is the physical velocity of electron that is expressed in equation (9.1), which do not have any possibility of attaining to velocity of light. While equation (9.2) express the velocity of electromagnetic wave, which is always invariant in free space, follows Maxwell electromagnetic formalities which does not predict that the wave is contracted to a point. Equation (9.2) is not the physical velocity mentioned in Special Relativity and thus does not subject to the condition predict by the theory. So it is possible for the sub-photon to be of certain length and not contracted to a point. When talking about the structure of electron and fundamental particles, we are not distracting ourselves from relativity. In fact, once the sub-photon model of electron can be verified experimentally, then the metric tensor of nuclear interaction can be automatically verified.

## 10.0 FORMAL RELATIVISTIC DERIVATION OF GRAVITATION IS A WEAK FIELD APPROACH

At first it may seem that we are using a different approach in the formulation of equivalent principle and metric tensors but eventually arrived at Einstein's gravitational metric. In fact, we are employing exactly the same methodology put forward by Einstein. Only that Einstein using weak field approach, while we are from the very beginning bearing in mind of electromagnetic fields which are much stronger than gravitation, using strong field approach.

It is a normal approach in calculus to ignore or neglect the second and higher orders infinitesimal components and still getting a right solution, as long as the same approach is applied thoroughly through out the derivation. Let us make a study of how weak field approach can get a right solution for gravitational metric.

Einstein making the proposition that in a weak field, the metric tensor can be expanded and neglecting the components of second and higher orders, we get

$$g_{ij} = m_{ij} = 1 + h_{ij} \qquad (10.1)$$

where

$$h_{ij} << 1 \qquad (10.2)$$

The metric tensor in equivalent principle

$$\begin{aligned} m^{11}{}_{44} &= 1 + h^{11}{}_{44} \qquad (10.3) \\ &= 1 + h_{4444} \\ &= 1 + (h_{44})^2 \\ &= 1 \end{aligned}$$

where second order of $h_{44}$ is ignored.

From equation (4.4)

$$\begin{aligned} d\phi/dr &= (c^2/2)(1 + h^{11})d(1+h_{44})/dr \qquad (10.4) \\ &= (c^2/2)d(h_{44})/dr + (c^2/2)(h^{11})d(h_{44})/dr \\ &= (c^2/2)d(h_{44})/dr \end{aligned}$$

where $(h^{11})d(h_{44})$ is second order and can be ignored. Putting the gravitational potential as

$$\phi = -GM/r \qquad (10.5)$$

and solving equation ( 10.4 ), we get

$$h_{44} = -2GM/(c^2 r) \qquad (10.6)$$

hence the metric is

$$g_{44} = 1 - 2GM/(c^2 r) \qquad (10.7)$$

Which is the familiar Einstein's metric tensor for gravitation. Apparently, this approach is not applicable for electromagnetism that is much stronger than gravitation, where the assumption in equation (10.1) is not suitable. Strong field approach is strict and may be more general than the weak field approach and getting more solutions. The solution by weak field approach is more specific and getting a limited solution. Actually, when we say weak field approach, it does not mean that the solution is only applicable in weak field condition. It can be applied to any strength of field after integrating equation (10.4) to solve the metric. However, it is a specific approach and getting only a specific type of solution. Without prejudice, this may be the main reason why Einstein did not arrived at the Grand Unified Theory.

## 11.0 GRAVITATIONAL TRAVELLING

Equation (5.10) predicted that interaction of positive and negative electromagnetic potentials can produce gravitational effect. If it can be verified experimentally, then it is possible in future for mankind to develop gravitational spaceship. Where with the production of strong electromagnetic fields, the spaceship will created an artificial gravitational fields in the direction where it need to travel into. Then the ship will 'fall' into the gravitational field created. As all the crewmembers and instruments are at free fall, no matter how great is the acceleration of the ship and the velocity, it does not exert any strain on them. Hence theoretically a gravitational spaceship can accelerate in whatever magnitude it can afford. More drastically, it may make abrupt stop and sharp turn at high velocity without causing any harm to the crews and instruments, provided the direction and the concentration of electromagnetic potentials can be changed at will. Also, the air around the ship will experience the same free fall and there is no much air friction in the travelling. The maneuverability and safety of this type of spaceship will be much better than that propelled by rocket. Further more, the ship can be easily operated by nuclear energy. If there is any similarity between our gravitational spaceship with that of UFO, it is only incidental**.**

## 12.0 DISCUSSION ON SCHWARZSCHILD SOLUTION

If $F_{ij}$ and $T_{ij}$ are the force tensor and the kinetic tensor respectively, we may equate them as

$$F_{ij} = T_{ij} \qquad (12.1)$$

where the force tensor $F_{ij}$ may be expressed generally as

$$F_{ij} = \alpha R_{ij} + \beta g_{ij} R + \gamma g_{ij} \qquad (12.2)$$

Einstein has arrived at the formulation [3] that

$$R_{ij} = -\kappa(T_{ij} - 1/2 g_{ij} T) \qquad (12.3)$$

It is assumed that at a space that is away from the material source,

$$T_{ij} = 0 \qquad (12.4)$$

therefore from equation (12.3)

$$R_{ij} = 0 \qquad (12.5)$$

for a space free from material.

Schwarzschild has derived the metric equation in a spherical symmetric field as

$$ds^2 = \exp(u)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + \exp(v)(icdt)^2 \qquad (12.6)$$

Based on conditions specified by equation (12.5), three equations expressed in the derivatives of u and v can be written as

$$v''/2 + v'/r - (v'/4)(u'-v') = 0 \qquad (12.7)$$
$$v''/2 - u'/r - (v'/4)(u'-v') = 0 \qquad (12.8)$$
$$1 - \exp(u) + (r/2)(v'-u') = 0 \qquad (12.9)$$

Solving the above equation, we get

$$g_{44} = \exp(v) = (1-2k/r) \quad \text{where k is an integration constant} \qquad (12.10)$$
$$g_{11} = \exp(u) = (1-2k/r)^{-1} \qquad (12.11)$$

After comparing with the Einstein gravitational metric tensor in equation ( 10.7 ), Schwarzschild deduced that

$$k = GM/c^2. \qquad (12.12)$$

This is the basis for the Schwarzschild solution of the metric tensor. However, if we substitute equation (12.10), (12.11) and (12.12) back into equation (12.7), (12.8) and (12.9), non of the later equations vanish. They only vanish when

$$k=0 \qquad (12.13)$$

and consequently

$$g_{44}=g_{11}=1. \qquad (12.14)$$

Hence condition in equation (12.12) may not be correct. As we have shown in the previous sections, we can derive the Einstein gravitational metric equation without making the assumption that $R_{ij}$ is zero and null. The comparison between equation (12.10) and equation (10.7) may not be done on the same boundary conditions. Equation (12.14) indicates that the assumption $R_{ij} = 0$ is only true for a free space, where it is a vacuum and completely free from the influence of any potential. Indeed, once the space is under the influence of certain potential, both the kinetic and potential acceleration tensors are not null tensors as expressed in the equivalent principle of equation (2.2) and (2.3), the metric equation (4.3) and (4.4). If the acceleration tensors are not null tensors, then the force tensor cannot be a null tensor. Therefore, $R_{ij}$ and $T_{ij}$ in equation (12.2) and (12.3) may not be void and null in a space under the influence of a potential, even the space under consideration contain no material in it and away from the material source of potential.

Deriving the metric tensors by making the assumption that $R_{ij}=0$ is a very complicated process and the logic may not be correct. Relativity and even the Grand Unified Theory are basic theories in physics. Most of the basic theories in physics can be derived in a straightforward manner, with the exception of relativity. To simplify the derivation is not only to make the process easier, but to gain better understanding into the theory. Indeed, the derivation of Grand Unified Theory is possible due to better understanding on the nature of transformation. In order to simplify the derivation of the theory, to by-pass the assumption that $R_{ij}=0$ and many other complicated tensor analysis are necessary. To derive the whole relativity theory without using the tedious tensor formulations, it is necessary for us to make some modification on the tensor calculus, to study closely the nature of space and the velocity of light.

## 13 MODES OF COORDINATES

In tensor calculus, it is generally accepted that the coordinate is contravariant in nature. If a coordinate is expressed in its specific state, the mode of the coordinate is not marked. For example, if $x^i$ is a general polar coordinate, then r, θ, ϕ, t are their specific coordinates. Generally, it is understood that r is in the first dimension, θ is the second dimension and so forth. Only the dimension of the general coordinates are marked as $x^1$ , $x^2$, $x^3$, and $x^4$ and not on the specific coordinate as $r^1$, $\theta^2$, $\phi^3$, and $t^4$. We are having the opinion that even in specific coordinates, the mode of the coordinates must be clearly indicated. The mode is defined as the indices that indicate the dimensions, the tensor state, whether it is covariant, contravariant or invariant, of a coordinate. When the mode is not indicated, there may be some confusion of what is the final expression of a tensor. Previously the equivalent principle is not expressed in a strict tensor form because the mode of the specific radial and time coordinates are not indicated. For example, the kinetic acceleration is expressed as $a = d^2r/dt^2$ and the potential acceleration as $\alpha = - d\phi/dr$. Then they are simply equated as $a = \alpha$ , which is not the strict tensor presentation. If the kinetic acceleration is indicated as $a^1{}_{44} = d^2x^1/(dt^4)^2$ and the potential acceleration as $\alpha_1 = -d\phi/dx^1$, then very clearly $a^1{}_{44} = m^{11}{}_{44}\alpha_1$. In the coming section when we are to differentiate the local and third party observations, where the time in third party observation is contravariant in nature and first party is covariant in nature, indication of mode is absolutely necessary.

**14 SPACE POINTS ARE INVARIANT**

If a vacuum space is free from the influence of any potential or external forces, it is defined as free space. If there is a point P in free space, we assume that all observers agree to the same point in the space about the location of P. As a result, a point in free space is an invariant to all observers. Now we are going to find out, after introducing a strong potential to the space, will the point be dislocated to a new position? If it is dislocated to a new position, then it is no longer an invariant after transformation, as different transformations will result in different locations. Different observers under the influence of different transformations will then not agree to the same point in the space about the location of P. However, if it is not dislocated, then it is an invariant and all observers agree to the same point in the space.

Say there are four space point P,Q,R, and S situated in space which divide the circumference of origin O into four equal quarters. After introducing an object which is the source of a strong potential at origin O, assuming that the field is polar symmetry, will the point P,Q,R, and S be dislocated from their previous position?

If before transformation, the polar coordinate of the points in free space are defined generally as

$$x1 = r1$$
$$x2 = r1\theta2$$
$$x3 = r1\text{sine}\theta2(\phi3)$$
$$x4 = icdt4 \qquad \text{where } i = (-1)^{1/2}, \text{ c is the velocity of light in free space}$$

Note that the mode of the coordinate is always indicated. Number of the mode corresponding to the dimension of the coordinate, even though the coordinates are specifically specified as r, θ, ϕ, and t. When the mode is indicated as a superscript, then the coordinate becomes a contravariant coordinate. If the mode is indicated as subscript, then the coordinate is a covariant coordinate. If it is just indicated as a normal indices like that in the above, then it is an invariant coordinate. If no mode is indicated, then the quantity is a scalar quantity. So there are four dimensions (1,2,3, and 4), three tensor states (contravariant, covariant, invariant) and a scalar condition which are associated with the modes of the coordinates. This is the modification we made to the tensor calculus.

Assuming that a strong potential originated from O is introduced and the potential is polar symmetry. Say after transformation, the coordinates are transformed and dislocated to contravariant coordinates as $r^1 + \Delta r^1$, $\theta^2 + \Delta\theta^2$, $\phi^3 + \Delta\phi^3$, $t^4 + \Delta t^4$. To say that the coordinates are transformed to contravariant state is quite arbitrary, the discussion remains unchanged if all of them are transformed to covariant coordinates. So the polar coordinates are transformed to

$$x1 = m_1(r^1 + \Delta r^1)$$
$$x2 = m_1(r^1 + \Delta r^1)m_2(\theta^2 + \Delta\theta^2)$$
$$x2 = m_1(r^1 + \Delta r^1)\text{sine}(m_2(\theta^2 + \Delta\theta^2))(m_3(\phi^3 + \Delta\phi^3))$$
$$x4 = m_4(t^4 + \Delta t^4)$$

Now, due to polar symmetry of the field, the angular dislocation $\Delta\theta^2$ and $\Delta\phi^3$ are not possible. As it will indicate that the field is experienced a torque and distorted to the direction of $\Delta\theta^2$ and $\Delta\phi^3$, all points

P, Q, R, and S will experience a slight clockwise or anti-clockwise rotation. This will destroy the symmetry of the polar field. So

$$\Delta\theta^2 = \Delta\phi^3 = 0$$

The radial increment $\Delta r^1$ is also null, due to the fact that the arc length r1θ2 and r1ϕ3 which are normal to the line of force of the potential, and are not affected by transformation. This fact is discussed in the following section. Any dislocation along the radial line will transform the length of the circumference. This can be proved does not happen.

If the space point is time independent, then the incremental change of time $\Delta t^4$ is zero. Finally, the transformation of space point can be simplified as

$$x_1 = r_1 = m_1 r^1$$
$$x_2 = r_1\theta_2 = m_1 r^1 m_2 \theta^2$$
$$x_3 = r_1 \text{sine}(\theta_2)\phi_3 = m_1(r^1)\text{sine}(m_2(\theta^2))m_3(\phi^3)$$
$$x_4 = t_4 = m_4 t^4$$

As will be proved in section (15) that angular metric tensor of $m_2$ and $m_3$ are not affected by transformation, we can further simplified the above polar coordinates to

$$x_1 = r_1 = m_1 r^1$$
$$x_2 = r_1\theta_2 = r_1\theta_2$$
$$x_3 = r_1\text{sine}(\theta_2)\phi_3 = r_1\text{sine}(\theta_2)\phi_3$$
$$x_4 = t_4 = m_4 t^4$$

If points P, Q, R, and S are random points, we just need to describe each circle for each point and the discussion remain unchanged. This proves that space points after transformation remains invariant to all observers.

## 15 DISCUSSION ON VELOCITY OF LIGHT

Light velocity is invariant is an important experimental fact that forming the basis for the derivation of relativity theory. Let us study more details into the velocity of light.

When we talk about transformations, we must first ask who is the observer that is making the observation. We define the local observer, who observes the physical phenomena in his immediate surrounding, as the first observer. The other observer, who is far away from him and practically is not influenced by the potentials around the observation, is known as global observer or the third observer. If we observe something happening at the level of an atom, we are the third observers. Someone who is travelling along with the electron or stay at the nucleus of the atom is the first observer. Clearly, the first observer may not realize that some transformations have happen around him, so the transformation of a metric equation is always the observations that are observed by a third observer.

If the time metric tensor is $m_4$, the third observer finds that the reading of local time is transformed by a factor $m^4$ compared to his own clock, as the observed time is inversely proportional to the transformation of unit time. The longer (greater) the unit length of time, the shorter (smaller) is the record of time registered by a clock. So if $m_4$ is the time metric tensor for the transformation of time interval, $m^4$ is the transformation factor for the time reading registered by the clock of the third observer.

Due to transformation of time, if the clock at the local observer is running more slowly than the clock of third observer, then the clock of the local observer is registered lesser reading than that of the third observer. To express this phenomenon mathematically, if $m^4$ is the transformation of the time reading as observed by third observer, then $m_4$ is the transformation of the local time reading as observed by the local observer. The transformation factor for time reading registered by local clock is reciprocal to that of the clock of the third observer. If the reading of time registered by the clock of a third observer is $\Delta T^4$, then the reading of local clock is $\Delta T_4$, with their transformation factor reciprocal to each other as discussed.

Now, if $dx^i$ is the unit length observed by local observer, and $dt_4$ is the time taken for the light signal to travel over the length as observed by the local observer, then the local velocity of light can be expressed as

$$dx^i/dt_4 = c \qquad (15.1)$$

where c is the velocity of light which is always an invariant to the local observer. Now, the velocity of light in a free space is also an invariant, mathematically this is expressed as

$$dx_i/dt_4 = c \qquad (15.2)$$

Reducing the tensor in equation (15.1) to invariant tensor, we get

$$(m^i/m_4)\, dx_i/dt_4 = c \qquad (15.3)$$

Comparing with (15.2), we get

$$(m^i/m_4) = 1 \qquad (15.4)$$

or $$m^i = m_4 \qquad (15.5)$$

So from the experimental fact that local velocity of light is an invariant, we can deduce that spatial metric tensor is the reciprocal of time metric tensor. If we expressed the coordinates in polar coordinate, the radial velocity of light as observed by the local observer is

$$\begin{aligned} V^{14} &= dr^1/dt_4 \\ &= (m^1/m_4)c \qquad (15.6) \\ &= c \end{aligned}$$

we get $$m^1 = m_4 \qquad (15.7)$$

The tangential velocity of light in the direction of θ is

$$\begin{aligned} V^{24} &= r^1\, d\theta^2/dt_4 \\ &= (m^1/m_4)m_2 r1d\theta2/dt4 \\ &= c \qquad (15.8) \end{aligned}$$

as $(m^1/m_4) = 1$, we get

$$V^{24} = m_2 c$$

therefore $$m_2 = 1 \qquad (15.9)$$

Similarly, we can also deduce that

$$m_3 = 1 \qquad (15.10)$$

There is no transformation of the angular coordinate in a spherically symmetric field. Normally, during the formulation of the metric tensors in spherical field, invariant of angular transformation is an assumption. Now it can be derived mathematically based on the fact that local velocity of light as observed by the first observer is always an invariant.

Now, the radial velocity of light as observed by the measuring units of a third observer is slightly different from that of the local observer. The transformation of time is reciprocal to that of local observer. We may express the velocity of light as observed by the third observer as

$$\begin{aligned} V^1{}_4 &= dr^1/dt^4 \\ &= (m^1/m^4)(dr1/dt4) \\ &= m_{44}c \qquad (15.11) \end{aligned}$$

which is not an invariant. The observed velocity can be slower or faster than the velocity of light in free space, depending on the nature of the time metric tensor. We have just arrived at an important conclusion, that even the velocity of light is an invariant to the local observer, it may not be an invariant to a third observer. Mathematically, the metric equation for light is a null metric. Expressing in polar coordinates, we get

$$0 = m_{11}(dr^1)^2 + (r_1)^2 (d\theta_2)^2 + (r_1)^2 \sin^2(\theta_2)(d\phi_3)^2 - c^2 m_{44}(dt^4)^2 \qquad (15.12)$$

If the light is only travelling along the radius, then all other time derivatives are null except

$$m_{11}(dr^1)^2 / m_{44}(dt^4)^2 = c^2$$

or

$$(dr^1)^2 / (dt^4)^2 = (m_{44}/ m_{11})c^2 \qquad (15.13)$$

which is identical to equation (15.11). Similarly, the tangential velocities of light as observed by the third observer are expressed as

$$(v2)_4 = m_4 c \qquad (15.14)$$

and

$$(v3)_4 = m_4 c \qquad (15.15)$$

both are not invariant. When a high energy photon is passing through a strong electric field, the quantified fields of the same electric polarity will travel faster than the opposite polarity, this may resulted in the splitting of photon and lead to pair production.

Similar condition may be applied to the universal gravitation. When a photon is passing by a heavy mass, its overall velocity is slowed down. The upper crests of the wave, which are further away from the source of potentials, will travel slightly faster than that of the lower crests, resulted in the bending of the light path. In nucleus level, if the universal nuclear transformation is very strong, the velocity of the outer crests of the sub-photon are faster than the inner crests and may bend the path of sub-photon into a closed path. In a sense, we can explain repulsion and attraction as due to differential of light velocity under the influence of similar potential or opposite potential. When a charged particle is coming close to a particle of the same charged, the extra field that give rise to the electrical charge of a particle will travel faster in the lower crest than the upper crest, that leads to repulsion. When coming to an opposite charge, the reverse is happening and lead to attraction.

As a point of interest, let us go back to our gravitational spaceship in section (11). Say the technology of mankind is already so advance and we can produce any strength of electromagnetic potentials as will, and we produce an electric potential of magnitude

$$\phi_{(e)}^2 = c^4 + \varphi^2 \qquad (15.16)$$

Substitute into equation ( 5.5 ), we get

$$g_{44(u)} = -(1 + 2\varphi^2 / c^4) \qquad (15.17)$$

Mathematically, the negative sign of the second order metric tensor does not change the result of transformation. This is positive gravitation and the spaceship will be propelled by it. This will make the ‘conventional’ gravitational spaceship in section (11) very ‘low-tech’. As there is no upper limit to the speed of the positive gravitational spaceship. A positive gravitation will make the light travel faster, theoretically the ship can travel to any speed where the technology can sustain, as the new limit to the velocity of the ship is increased due to increment of the velocity of light. As long as there is no limit to the factor $\varphi$ in equation (15.17), there is no limit to the velocity of light and thus the limit to the velocity of the ship. The ship may travels thousands of time faster than the velocity of light in free space, but still under the new limit of velocity of light in positive gravitation. As the travelling is free fall travelling, no matter how great is the acceleration and speed of the ship, there is no strain and harm to all the crews and the instruments.

## 16 THE LAGRANGIAN FORMULATION

To by-pass the complicated tensor analysis and derive the whole theory of relativity in a simple and straightforward manner, it is useful to use the Lagrangian for the derivation of the metric equation.

Say there is a Lagrangian L such that

$$L = (1/2)m_{ij}(dx^i /ds)(dx^j /ds) \qquad (16.1)$$

$$= (1/2)m_{ij}(dx'^{i})(dx'^{j})$$

and the least value of the Lagrangian can be derived from the Lagrange equation

$$d(dL/dx'^{i})/ds - dL/dx^{k} = 0 \qquad (16.2)$$

As $$dL/dx'^{i} = m_{ij}dx^{j}/ds \qquad (16.3)$$

Replacing ds with $dt^4$, for a time independent condition, all time derivative vanished except the radial acceleration factor, we get

$$d(dL/dx'^{i})/ds = d(dL/dx'^{i})/dt^{4}$$
$$= m_{11}d^{2}x^{1}/(dt^{4})^{2} \qquad (16.4)$$

and $$dL/dx^{k} = (1/2)d(m_{ij})/dx^{k}(dx'^{i})(dx'^{j}) \qquad (16.5)$$
$$= -(c^{2}/2)\, d(m_{44})/dx^{1}$$

where all time derivatives vanished except $dx'^4 = dx^4/dt^4 = ic$, and assumed that the metric equation is a function of the radial factor.

Substitute (16.4) and (16.5) into equation (16.2), we get

$$m_{11}d^{2}x^{1}/(dt^{4})^{2} = -(c^{2}/2)\, d(m_{44})/dx^{1} \qquad (16.6)$$

Rearranging the above equation, we arrived at equation (4.3)

$$d^{2}x^{1}/(dt^{4})^{2} = -(c^{2}/2)\, m^{11}\, d(m_{44})/dx^{1} \qquad (16.6)$$

The derivations of the rest of the metrics are then the same as that in the previous sections. We have just shown that it is possible to by-pass completely the assumption that $R_{ij} = 0$ and still derive the relativity theory by strict tensor formalities. We also by-pass many complicated tensor analysis and making the derivation of relativity a lot more easy.

**17 NATURE OF SPACE**

Riemannian calculus is useful for the study of non-uniform Gaussian surface. When it is employed to derive the metric tensors of relativity, naturally the concept of a curve Gaussian surface is extended to the concept of a curve space-time. However, we must point out that the study of transformation of space-time is conceptually different from the study of Gaussian surface. Generally, you cannot construct an orthogonal frame on a Gaussian surface but it is always possible to do so in a space even under transformation. The study of Gaussian surface is always involved inter-frames transformation, where it is the transformation between two different frames. But the study of space-time can be an intra-frame observation, transformation within the same frame, say from invariant sub-frame to contravariant sub-frame of the same frame structure, without involving another third party frame. As we can explain the attraction and repulsion due to differential of light velocity under transformation, it may not be necessary for us to introduce the concept that space is curved.

Our understanding of space is more and more towards the theory that it is an inelastic ether. When we say it is inelastic, we do not mean that it is non-transformable. We mean that the points you defined in space always stay as they are, irrespective of whatsoever transformation on them. A space point is always an invariant. If the local observer makes enough explorations to his surroundings and making comparison to the reading in near and far away environment, eventually he will realized the transformation of his coordinates and synchronized all the readings with other observers. He may even find out his velocity with respect to the stationary frame in the space. So, in a sense, space can acted as an invariant frame. With the understanding of space-time transformation, our concept of an invariant frame is definitely different from that of Newtonian absolute frame where the coordinates are assumed absolute and their transformation is not possible. In a Newtonian or Cartesian frame there is only one mode of presentation of coordinate. However, we need at least four modes and four sub-frames super impose on each other to construct a total space frame. The frame is invariant in a sense that all observers may agree to the definition of axis and all invariant points to grid the space and mark the spacing of coordinates, and using the frame as reference to compare their reading with all the other frames. Due to different modes of presentations, the coordinates remain transformable under the influence of potential.

A space can be flat or curved depending on your choice of reference frame. If you chose an orthogonal frame, it is always flat. If you chose an arbitrary Gaussian frame, where the axis of the frame may not even straight and normal to each other, of course the space can be very arbitrary in nature. It is your choice of frame that matters, not the nature of space.

In the study of the solution put forward by Einstein to solve for the gravitational metric tensor, it is always state that the potential equation of gravitation $\phi_{(g)} = - GM/r$ is a Newtonian approximation, as though there is a stricter representation of gravitational potential. However, our equation in ( 5.9 ) is a strict derivation and a complete representation, not an approximation. This actually supports our argument that space is an invariant frame. Eventually, all observers will agree to the same value of $\phi_{(g)}$ , G, M, and r. This may not be possible if they do not have a common and an invariant frame.

Different strength of potentials may lead to different physical transformations. In order to compare all the transformations in different frames, it is necessary to reduce all tensors to zero state, to make them invariant to all observers and making the comparison possible and meaningful. The most widely use metric equation $ds^2=m_{ij}dx^i dx^j$ is a zero degree tensor which is invariant to all observers. So knowingly or unknowingly, we have already made use of the concept of invariant frame.

## 18 FUTURE DEVELOPMENT OF THE GRAND UNIFIED THEORY

The naming of Grand Unified Theory in itself may give people a sense that it is the final theory for relativity. Actually there are still many theoretical treatments of relativity that need to be formulated, like the nature of time, why it is a scalar and not a vector like space and having only one dimension. When the gravity turn positive, will space becomes a scalar and time become dimensional so that time travelling is possible? Also, we have a strong feeling that many phenomena in quantum mechanic which are ‘unimaginable’ like the quantification of light energy, structure of elementary particles, spin of the electron, quantification of angular momentum, wave-particle duality, Bohr’s standing wave theory for hydrogen atom, the structure of the nucleus of atom and many physical phenomena can be explained if our understanding of the theory is further improved. In future, the study of Relativity or the Grand Unified Theory and Quantum Mechanics may be merged into a unified science.

From our discussion, all matters may made up of sub-photons and actually are just the vibration of space. Due to invariant of light velocity in free space, we are only dealing with one velocity or just one type of vibration in space. In nature, there are always multiple harmonies associated with any type of vibration. Is there any world where their vibrations are different from that of ours? Does heaven or hell so widely believed by many religions exist in different vibrations? Are we the superimposition of various vibrations where our physical bodies and our souls (if they exist) co-exist in different vibration? Can we change the nature of our vibration and go into other dimension in this universe?

With the generalization of relativity, it is possible for us to observe the theory in the laboratory. Any experimental advancement will bring about the improvement of the concern theory. Apart from our discussion of heaven and hell in metaphysics, establishment of experimental formalities will be the most important direction for the future understanding of relativity. Comparing with quantum mechanics, the experimental facts of relativity are few and inadequate. The observation of strong gravitation in the universe is difficult and costly. If our formulation of the theory is right, we may be able to observe the relativistic effect of other elementary fields, which can be generated at will in the laboratory. If this can be done, we have just open up a new chapter in the study of relativity.

## Bibliographical references

**[1] W. PAULI, Theory of relativity, Pergamon Press (1959)**

**[2] P.G. BERGMANN, Introduction to the theory of relativity, Chinese translation, People Educational Publisher, Beijing (1979)**

**[3] YU YIN QIANG, Introduction to General Relativity, in Chinese Language, Xinhua, Beijing (1987)**

**[4] A. K. GHATAK, S. LOKANATHAN, Quantum Mechanics, The MacMillan Company of India Limited, New Delhi (1981)**

**[5] P.M. MATHEWS, K. VENKATESAN, A textbook of quantum mechanics, Tata McGraw-Hill Publishing Limited, New Delhi (1982)**