Special relativity with light signals but from the Galilean postulate

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1. Introduction

Galileo, who knew that: "The true laws of physics are the same in all inertial reference frames and so it is impossible to detect the uniform motion of a reference frame by performing experiments which are confined to that frame" (Galilean postulate), proposed a very simple way to measure the velocity of light [1]. The proposed experiment is performed on Earth and failed because the involved clock was not precise enough in order to measure very short time intervals. If Galileo had had a precise clock, he would have obtained for it the well known value *c* and we give him that chance ! Based on his postulate, he states that: <u>If inertial observers are equipped with identical guns and light sources in all such frames, then measuring the velocity of the emitted pellet (*u*) or light signal (*c*), each of them obtains the same value as long as his gun or light source is in a state of rest relative to him, because otherwise that measurement can bring information about his state of uniform motion.</u>

He can also state that: Measuring distances perpendicular to the direction of relative motion of two inertial reference frames, all the inertial observers obtain the same value. The proof of that statement involves paint brushes rather than light signals [2]. At that level of knowledge, Galileo knows that inventing scenarios in which the involved inertial observers are not obliged to measure the velocity of a light signal emitted by a moving source, all the results obtained that way can be considered to be a direct consequence of his postulate. This postulate is still today considered to be something which cannot be proven, which has not yet been disproved, physicists finding it to be something that can be believed [3].

Deriving the formula which describes the addition law of parallel velocities, Sartori [4] mentions that special relativity is involved in it via the time dilatation effect. We investigate now how much special relativity is involved in the derivation of the formula which describes the time dilatation effect.

We propose the following scenario: Observers A and A′ located at the origin O and O′ of the reference frames S and S′ (S′ moving with constant velocity $v=\beta c$ relative to S in the positive direction of the common OX(O′X′) axes) are equipped each with identical light sources L and L′, with identical guns G and G′ and with identical clocks C and C′. When the origins O and O′ are located at the same point in space, the clocks C and C′ read t=t′=0 and the sources L and L′ emit light signals in all directions in space. One of the light signals emitted by L′ in the positive direction of the O′Y′ axis (θ'=90°) starts a stopped clock $C'_i(x',y') = C'_i(0,r')$ reading r'/c, creating the event $C'_i(x',y',t') = C'_i(0,r',r'/c)$ and we say that clocks C′ and C'_i are synchronised following a procedure proposed by Einstein [5] (Fig.1.a). As detected from S, the clock C'_i is located, when it reads r'/c, in front of a clock $C_i(x,y) = C_i(r\cos\theta, r\sin\theta)$, synchronised using the same procedure with clock C, using a light signal emitted by L when clock C reads t = 0 (Fig.1.b). Taking into account the fact that during the synchronisation of clock C_i, the clock C_i' has advanced with *vr/c* in the positive direction of the OX axis and the invariance of the distances perpendicular to the direction of relative motion, Pythagoras´ theorem enables us to state that

$$r^2 = r'^2 + v^2c^{-2}r^2 \tag{1}$$

all the lengths present in (1) being measured in the S frame. From (1) we obtain

$$r = r'\left(1 - v^2c^{-2}\right)^{-1/2} \tag{2}$$

$$\cos\theta = vc^{-1} \tag{3}$$

$$t = t'\left(1 - v^2c^{-2}\right)^{-1/2} \tag{4}$$

establishing a relationship between the space-time coordinates of the events $\mathrm{M}(x,y,t) = \mathrm{M}(r\cos\theta, r\sin\theta, rc^{-1})$ and $\mathrm{M}'(x', y', t') = \mathrm{M}'(0, r', r'c^{-1})$ which take place at the same point in space. Equations (2), (3) and (4) represent a particular case of the Lorentz-Einstein transformations, holding only in the case when one of the events is characterised by a x´=0 space coordinate. Taking into account the fact that at the moment when the synchronising light signals are emitted by L and L´ all the clocks C_i and C_i' read a zero time, we can consider that

$$t = t'\left(1 - v^2c^{-2}\right)^{-1/2} \tag{5}$$

t representing the difference between the readings of the clocks $C_{i,0}(0, r')$ and $C_i(r\cos\theta, r\sin\theta)$ when the clock C_i' is in front of them, whereas t´ represents the difference between the readings of the same clock C_i' at the two of its positions mentioned above.

We underline that the scenario which leads to equations (2), (3), (4) and (5) did not oblige the involved observers to measure the velocity of light signals emitted by moving sources and so we consider them as a consequence of Galileo´s postulate.

2. Measuring the velocity of a bullet fired by a moving gun and the velocity of a light signal emitted by a moving source

The scenario we propose is associated in the S frame with the following events:

1(x ,y, t) = 1(0, 0, 0) - "the gun G fires a first bullet at a velocity u_x in the positive direction of the OX axis which hits instantly a target located in front of it, moving at a constant velocity v in the same direction";

2(x, y, t) = 2(0, 0, T_e) - "the gun fires a second bullet at the same velocity and in the same direction when clock C reads T_e";

3(x, y, t)=3($u_x(T_r$-$T_e)$=vT_r, 0, T_r) - "the second bullet hits the target at a moment T_r which equates the reading of a clock Ci located where the second bullet hits the target".

From the obvious equation

$$u_x\left(T_r - T_e\right) = vT_r \tag{6}$$

which equates the space travelled by the target with the space travelled by the second bullet, we obtain

$$T_r = \frac{T_e}{1-\frac{v}{u_x}} \qquad \textbf{(7)}$$

The clock C′ comoving with the target measures between the reception of the two successive bullets a proper time interval T_r' related to T_r by

$$T_r = T_r'\left(1-v^2c^{-2}\right)^{-1/2} \qquad \textbf{(8)}$$

with which equation (7) becomes

$$T_r' = T_e\frac{\left(1-v^2c^{-2}\right)^{1/2}}{1-\frac{v}{u_x}} \qquad (9)$$

As detected from S′ the gun moves at a constant velocity -*v*, observers from S′ being obliged to measure the velocity of a bullet fired by a moving gun and let u_x' be the value they obtain for it. The experiment described above is associated in S′ with the following events:

1′(x', y', t') = 1′(0, 0, 0) - "the gun fires the first bullet when clock C′ reads t′=0";

2′(x', y', t') = 2′(0, 0, T_r') - "the second bullet hits the target when clock C′ reads T_r' ";

3′(x', y', t') = 3′($-vT_e = -u_x'(T_r' - T_e')$, 0, T_r') - "the second bullet is fired at a moment T_e' which equates the reading of a clock C_i' located where the second bullet is fired".

We have now

$$vT_e' = u_x'\left(T_r' - T_e'\right) \qquad (10)$$

equating the space travelled by the gun between the emission of the two successive bullets with the space travelled by the second bullet. The clock C comoving with the gun, measures between the emission of the successive bullets a proper time interval T_e related to T_e' by

$$T_e' = T_e\left(1-v^2c^{-2}\right)^{-1/2} \qquad (11)$$

From (10) and (11) we obtain

$$T_e = T_r'\frac{\left(1-v^2c^{-2}\right)^{1/2}}{1+\frac{v}{u_x'}} \qquad (12)$$

We mention that *c* in (9) represents the velocity of the light signal emitted by L which performs the synchronisation of the clocks C_i in S, whereas *c* in (12) represents the velocity of the light signal emitted by L′ performing the synchronisation of the clocks C_i' in S′.

Eliminating T_e and T_r' between (9) and (12) we obtain

$$u_x = \frac{u_x' + v}{1 + u_x' v c^{-2}} \qquad (13)$$

or

$$u'_x = \frac{u_x - v}{1 - u_x v c^{-2}} \tag{14}$$

Replacing the gun G with the source of light L, observers from S´ are obliged to measure the velocity of a light signal emitted by a moving source of light, and let c' be the value they obtain for it. In accordance with (14) we should have ($u'_x = c'$; $u_x = c$)

$$c' = c \tag{15}$$

and so Galileo´s postulate can be restated as:

<u>The velocity of a light signal emitted by a source has the same value *c* for all inertial observers in relative motion not only if the source is at rest but even if it moves with constant velocity relative to them</u>, recovering Einstein´s second postulate of special relativity.

3. The Lorentz-Einstein transformations

Galileo´s postulate and Einstein´s clock synchronisation procedure convinced us so far that the readings of two clocks, each synchronised in its rest frame and instantly located at the same point in space, are not equal to each other and that the addition law of velocities proposed by Galileo is not valid.

Galileo makes a net distinction between experiments performed below or above the decks of his "gedanken ship" which is an inertial reference frame at rest or in a state of uniform motion [5]. In a below decks experiment we establish a relationship between physical quantities measured in the same reference frame, whereas in the above decks experiment we establish a relationship between physical quantities measured in two different inertial reference frames in relative motion. Such a relationship represents a transformation equation and we have derived some of them, i.e. (2), (3) and (4), having a limited field of application, due to the fact that one of the involved clocks is located at the origin of the corresponding frame of reference. In order to derive transformation equations with general validity, consider a clock C_0 moving with constant velocity u_x relative to S but at a velocity u_x' relative to S´. After a given time of motion, it reads t_0, being located instantly in front of the clocks C_i and C_i', the first clock reading t, the second t'. What we have to compare are the space-time coordinates of the events $C_i(x=u_x t, 0, t)$ and $C_i'(x'=u_x' t', 0, t')$. In accordance with (3) we should have

$$t = \frac{t_0}{\left(1 - u_x^2 c^{-2}\right)^{1/2}} \tag{16}$$

$$t' = \frac{t_0}{\left(1 - u'^2_x c^{-2}\right)^{1/2}} \tag{17}$$

Eliminating t_0 between (16) and (17) we obtain

$$t = t' \frac{1 + u'_x c^{-2}}{\left(1 - v^2 c^{-2}\right)^{1/2}} = \frac{t' + v c^{-2} x'}{\left(1 - v^2 c^{-2}\right)^{1/2}} \tag{18}$$

Multiplying both sides of equation (18) with u_x it leads to

$$x = u_x t = t' \frac{u'_x + v}{\left(1 - v^2 c^{-2}\right)^{1/2}} = \frac{x' + vt'}{\left(1 - v^2 c^{-2}\right)^{1/2}} \tag{19}$$

We have obtained that way the Lorentz-Einstein transformations for the space-time coordinates of the same event which takes place somewhere on the OX(O´X´) axis when the clocks instantly located there read t and t´ respectively, still from a scenario in which no measurement of the velocity of a light signal emitted by a moving source is performed.

4. Conclusions

Using scenarios in which the involved inertial observers are not obliged to measure the velocity of light signals emitted by moving sources, we show that Einstein´s relativistic postulate is self-contained in Galileo´s relativistic postulate. Our approach shows that Einstein´s fundamental contribution to special relativity consists in his synchronisation procedure of distant clocks and that "time dilatation" is an observable relativistic effect, consisting in the comparison of readings of two clocks, instantly located at the same point in space, each synchronised in its rest frame.

References

[1] L.Sartori, Understanding Relativity (University of California Press) London 1996, p.45
[2] Reference 1, p.93-94
[3] Th.A. Moore, A Travellers Guide to Spacetime (McGraw-Hill, Inc.) New York 1995, p.3
[4] L.Sartori, "Elementary derivation of the relativistic velocity addition law", Am.J.Phys. **63** (1995), p.81-82
[5] Reference 1, p.60-67

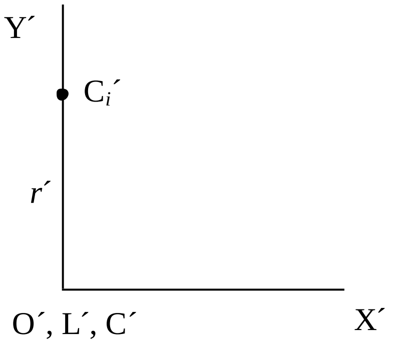

Fig.1.a. Synchronisation of clocks C´ and C_i´ as detected from the S´ frame.

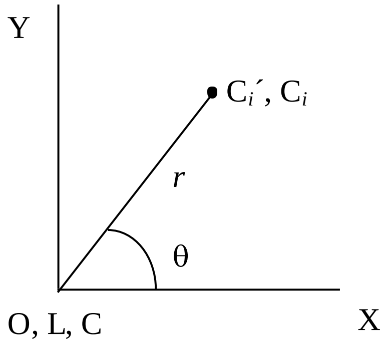

Fig.1.b. Synchronisation of clocks C´ and C_i´ as detected from the S frame.