THE PRIMORDIAL UNIVERSE IN A NEW TIME SCALE

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1 - INTRODUCTION

Using some reasonable hypothesis, General Relativity leads to a cosmology characterised by a universe having a singularity in the past (the Big Bang) with regard to a variable called cosmic time.

This singularity, inherent in Friedmann's equations, should be purely mathematical. From the physical point of view, quantum mechanics contains a concept known as Planck's time (of the order of 10^{-43} seconds). Beyond (the word is inadequate) that point, it would seem that time and space have no meaning and other quantities should be used to describe the physical universe. For the time being, however, we do not know what these quantities might be. The space-time should lose its status as a framework in physics due to inadequacy in describing the overall universe.

Under these conditions, the difficulties are linked to the choice of the time-scale. Our definition of time, which used to be linked to the rotation of the Earth, is at present based on atomic transition. What was the situation when neither the Earth nor atoms existed? For instance, at the primal moments of the Big Bang (see Pecker [1]) p. 309 to 316)..

To sum up, it would seem paradoxical to use our present definition of time, with its current basis, to depict the evolution of the universe. Time should be, on the contrary indissoluble linked to the nature of the matter, which the universe contains. How, for instance, can one compare the time-scale applicable to our era with that of the universe when it had a density approaching nuclear density?

The following two paragraphs resume the essential results of Mathé [2],[3]. Geometric units in which the speed of light c is equal to unity are used.

2 - PHYSICAL DEFINITION OF TIME

Consider a universe U containing a perfect fluid with an equation of state linking that fluid's density ρ and its pressure p. If the fluid is taken as being irrotational, it can be studied in comoving co-ordinate systems as follows:

$$g = e^{2\omega} dt^2 - h_{ij}\, dx_i\, dx_j \qquad (i , j = 1, 2, 3)$$

where h_{ij} is the defined positive metric tensor of space ; p, ρ, ω, h_{ij} functions of (t, x_1, x_2, x_3). The energy-impulse tensor of the fluid is expressed as follows:

$$T_0^0 = \rho ; \quad T_1^1 = T_2^2 = T_3^3 = -p$$

The conservation identities give:

$$\partial_i p + (\rho + p)\partial_i \omega = 0 \qquad 1 \leq i \leq 3 \tag{1}$$

$$\partial_0 \rho + (\rho + p)\partial_0 (\ln\sqrt{h}) = 0 \tag{2}$$

where h stands for the determinant of h_{ij}. When considering F, the index of the fluid:

$$F = \mathrm{Exp}\left(\int dp / (\rho + p) \right) \tag{3}$$

(1) proves that Fe^{ω} is independent of x_1, x_2 et x_3, in other words Fe^{ω} is a function of t only. According to Lichnerowicz [4] p.75 however, the flow lines of the fluid are geodesics of the metric:

$$\gamma = F^2 g = F^2 e^{2\omega} dt^2 - F^2 h_{ij}\, dx_i\, dx_j$$

Consequently, the coefficient of dt^2 within γ is <u>uniquely function of t</u> and a simple change of time-scale gives:

$$d\tau = Fe^{\omega} dt \tag{4}$$

it is possible to write :

$$\gamma = d\tau^2 - F^2 h_{ij}\, dx_i\, dx_j \tag{5}$$

where F et h_{ij} now become functions of (τ, x_1, x_2, x_3).

<u>The metric tensor γ is the frame of the evolution of the fluid. Time τ is thus defined in a univocal manner and should be chosen as an absolute time</u>. This term does not mean a return to Newton's absolute time. The expression "cosmic time" would be suitable but it is usually reserved for time t.

(2) and (3) further give:

$$\partial_0 (\rho + p)/(\rho + p) = \partial_0 F / F - \partial_0 (\sqrt{h})/h \tag{6}$$

which shows:

$$(\rho + p)\sqrt{h} / F = C(x_1, x_2, x_3) \tag{7}$$

3 - APPLICATION TO THE COSMOLOGY

Taking the usual hypotheses with an universe containing a perfect homogenous isotropic fluid, reduced to comoving co-ordinate systems leads to Robertson and Walker's metric tensor and to Friedmann's equations (with a zero cosmological constant).

$$g = dt^2 - R^2 \, dl^2 \qquad (8)$$

where R is a function of t and:

$$dl^2 = (dr^2 + r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2))/ (1 + kr^2/ 4)^2 \qquad (9)$$

where k = -1, 0 or 1according to whether the curvature of the universe is negative, null or positive. We have the Einstein's equations:

$$3 ((dR/dt)^2 + k) / R^2 = 8\pi \rho \qquad (10)$$

$$2 (d^2R/dt^2) / R + (dR/dt)^2 / R^2 + k / R^2 = - 8\pi p \qquad (11)$$

Equation (11) can be replaced by the equation of conservation deduced from (7).

$$(\rho + p) R^3 / F = Cst \qquad (12)$$

According to (3) F is a function of t and (4) gives the absolute time as being:

$$d\tau = F \, dt \qquad (13)$$

(5) gives:

$$\gamma = F^2 g = d\tau^2 - Q^2 \, dl^2 \qquad (14)$$

where the observed scale factor is:

$$Q = FR \qquad (15)$$

If " ′ " is taken as the derivation with respect to τ we get for any derivable function f:

$$df/dt = Ff' \qquad (16)$$

The quantities which characterise the universe and the cosmological fluid should be studied as function of τ and not of t. Using the equation of state and substituting in (3) and (12) we obtain ρ then F as a function of R. Further, (16) gives:

$$dR/dt = FR'$$

The equation (10) then becomes:

$$R'^2 = (- k + 8\pi \rho R^2/3)/ F^2 \qquad (17)$$

Integration of this differential equation gives the variation of R as a function in τ. When equation (3), (12) and (15) permit it ρ, p, F and R can be expressed as a function of Q. Substituting in (17) gives the differential equation that verifies Q. Beyond the fact that is not always possible, the differential equation for Q is often more complicated than that for R. On the other hand, once (17) has been resolved, substituting for R in ρ, p, F and Q gives the variation of these quantities as a function of the time τ. Then the equation (13) gives the expression of t as a function in τ.

$$t = \int (d\tau / F) \qquad (18)$$

The light follows the geodesics of the metric tensor γ and the Hubble's constant is:

$$H = Q'/ Q$$

4 - EQUATION OF STATE FOR COSMOLOGICAL FLUID

Now, we are in the case of simplified hypothesis which follow:

H1) The matter of the universe exists in three forms:

a) dust
b) radiation
c) dense pre-stellar matter

H2) The equations of state are:

a) $p_d = 0 \times \rho_d = 0$ (19a)
b) $p_r = \rho_r / 3$ (19b)
c) $p_m = \rho_m$ (19c)

H3) We can take no care of the interaction of different types of the matter.

The hypothesis (H1c) specially applies to the primordial universe, but some authors as V. Ambartsoumian defend the actual existence of the pre-stellar matter [5].

The relations (H2a and b) are the habitual equations of state.

The relation (H2c) is corroborated by different works on the dense matter, in particular, see S. Tsuruta and A. Cameron [6]. This equation of state corresponds to the Lichnerowicz's case of incompressibility.

The relations (1) and (2) implies, for a perfect fluid, the first law of thermodynamic:

$$(\rho + p)dV + (d\rho)V =0 \qquad (20)$$

vhere V is the volume of any fluid element. Thus with the relations (19):

$$(\rho_d + (4\rho_r /3) + 2 \rho_m) dV + (d\rho_d + d\rho_r + d\rho_m)V = 0 \qquad (21)$$

so:

$$d(\rho_d V) + d(\rho_r V^{4/3}) / V^{1/3} + d(\rho_m V^2) / V = 0 \qquad (22$$

Hence (H3) gives:

$$\rho_d V = C_d \quad \text{(constant)} \tag{23a}$$

$$\rho_r V^{4/3} = C_r \quad \text{(constant)} \tag{23b}$$

$$\rho_m V^2 = C_m \quad \text{(constant)} \tag{23c}$$

As, V is proportional to R^3 we obtain:

$$\rho_d R^3 = \rho_{d0} R_0^3 \tag{24a}$$

$$\rho_r R^4 = \rho_{r0} R_0^4 \tag{24b}$$

$$\rho_m R^6 = \rho_{m0} R_0^6 \tag{24c}$$

The quantities with an indices zero denote the value taken in the actual period.

5 - RESOLUTION AND DISCUSSION OF THE EQUATIONS

After a simple integration, the equations (3) and (24) give:

$$F = 1 + (4 / 3) (\rho_{r0} R_0) / (\rho_{d0} R) + 2 (\rho_{m0} R_0^3) / (\rho_{d0} R^3) \tag{25}$$

To definite the constant of integration, we bear in mind that F tends towards one when R tends towards the infinity (case of universe of dust).
By carrying over (24) and (25) into (17) we obtain:

$$R'^2 = (- k + (8\pi / 3) (\rho_{d0} R_0 / R + \rho_{r0} R_0^4 / R^2 + \rho_{m0} R_0^6 / R^4) / F^2 \tag{26}$$

making a change of variables and parameters, this gives:

$a = 4 \pi\rho_{d0} R_0^3 / 3$
$\lambda = 3 \rho_{r0} / (2 \pi\rho_{d0}^2 R_0^2)$
$\mu = 27 \rho_{m0} / (32 \pi^3\rho_{d0}^4 R_0^6)$
$R = au$
$\tau = a\eta$

(26) becomes:

$$(du / d\eta)^2 = (-k + 2 / u + \lambda / u^2 + \mu / u^4) / (1 + 2\lambda / (3u) + \mu / u^3)^2 \tag{27}$$

With the middle observed values of:

$R_0 = 10^{28}$ cm.
$\rho_{d0} = 3\ 10^{-31}$ g / cm^3.

$\rho_{r0} = 10^{-34}$ g / cm^3.
$\rho_{m0} = 10^{-36}$ g / cm^3.

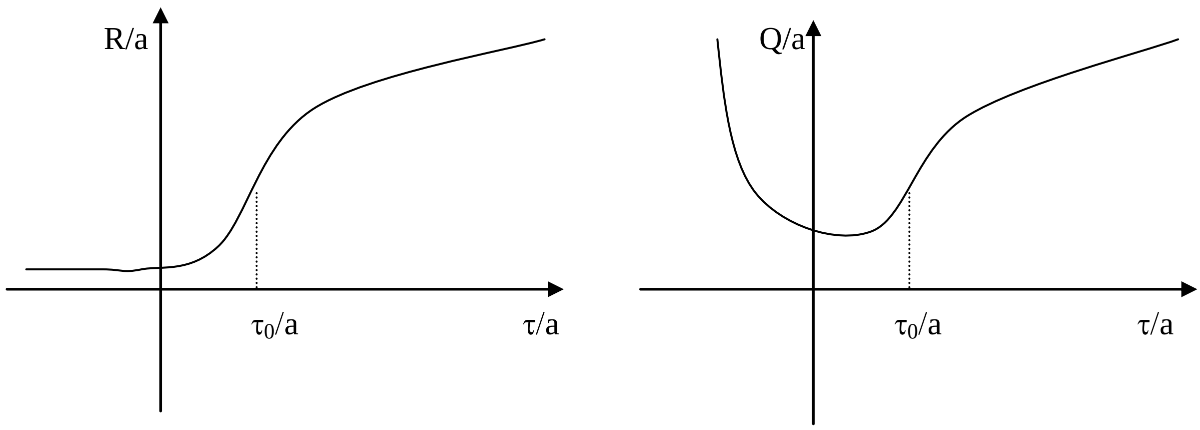
we obtain:

$a = 10^{26}$ cm.
$\lambda = 0.35$
$\mu = 8$

It should be observed that the quantities λ and μ are constants without dimension. These values are near and vary of some units.

The equation (27) resolve with numerical methods and with (15) we obtain the graph of variation of R and Q in function de τ .

These graphs are given in the case where k=0, i.e. for a spatial euclidean metric tensor (some observations, very recent, should prove that is the case).



6 - <u>CONCLUSION</u>

The preceding study shows that the observable universe must be associated with the metric tensor γ this entails the definition of the absolute time τ and a new scale parameter Q. If we also assume that the cosmological fluid is a perfect fluid as defined in state equation (19) then Q is defined for τ varying from ($- \propto$) to ($+ \propto$) and the metric γ has no singularity. In others words, there is no beginning and no end to the history of the universe. Further, since Q never tends towards 0, this means that the universe never tends towards becoming a point although its dimension do vary from time to time. Some obscurities do however remain, particularly when the intermediate quantity R tends towards 0, an event which occurs at least once for each value of k. on these occasions, although Q tends towards infinity. This implies that the observable universe is becoming greater and greater and denser and denser. When the density exceeds the nuclear density, the universe goes into a phase of contraction. Any wave phenomenon emitted during such a state in the past would show up as a shift towards the blue end of the spectrum.

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