ABOUT POSSIBILITY OF QUALITATIVELY NEW EXPERIMENTAL TEST OF SPECIAL RELATIVITY

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1. Introduction

At present time all known experimental facts in space-time physics are in a full agreement with the special theory of relativity, which constitutes a basis of modern physics. At the same time, at last decades some ether theories have been developed (see, e.g. [1-6]), which are non-distinguishable from STR on experimental level with respect to all experiments gathered up to now in space-time physics. A common sign of all such "covariant ether theories" is their compatibility with the general relativity principle. A possible existence of such theories follows from non-equivalence of the particular Einstein's relativity principle and general relativity principle in case of inertial motion in an empty space. (The simplest proof of non-equivalence of both relativity principles for inertial motion has been presented in [5]). On the other hand, during many years such "covariant ether theories" were considered as a pure mathematical game, as even "ether formulation of relativity" [3], which does not predict really observable non-relativistic phenomena.

The present paper develops a new approach to analysis of the "covariant ether theories" which is based on a difference of "physical" and "experimentally measured" values (and their transformation rules) in hypothetical curvilinear geometry of an empty space-time. This way finds a kind of phenomena giving different unambiguous predictions for STR and "covariant ether theories" and gives a key for search of the experiments for qualitatively new test of STR.

2. About a special case of transformation from pseudo-Euclidean to curvilinear geometry

It is well-known, that STR establishes a pseudo-Euclidean geometry in an empty physical space-time. At the same time, a pseudo-Euclidean geometry is an exclusive, wherein the experimentally measured $x_{\rm ex}$ and physical $x_{\rm ph}$ space-time four-vectors directly coincide with each other under proper measurements [1, 5-7]: $x_{\rm ph} = x_{\rm ex} = x_{\rm L}$, where $x_{\rm L}$ is the Minkowskian four-vector, subjected to the Lorentzian transformation **L** between two arbitrary inertial frames K and K':

$$x_{{\rm L}i} = L_{ij} x'^{j}_{\rm L}, \tag{1}$$

i,j=0...3.

Now let us formally go from pseudo-Euclidean geometry to curvilinear geometry according to some admissible transformation **B**:

$$\left(dx^{\rm N}_{\rm ph}\right)_i = B_{ij}\left(dx^{\rm E}_{\rm ph}\right)^j, \tag{2}$$

where the upper subscripts E and N signify the pseudo-Euclidean and curvilinear geometry, respectively. In non-Euclidean geometry the physical $d\,x_{\rm ph}^{\rm N}$ and experimentally measured $d\,x_{\rm ex}^{\rm N}$ space-time four-vectors are not equal, in general, to each other. Hence, a corresponding admissible transformation from pseudo-Euclidean to curvilinear geometry for $dx_{\rm ex}$ should be written as

$$\left(dx_{\rm ex}^{\rm N}\right)_i = C_{ij}\left(dx_{\rm ex}^{\rm E}\right)^j, \tag{3}$$

where, in general, the matrix **C** differs from the matrix **B** in (2), but depends on **B**. In this connection one may ask the following question: what should be a form of the matrix **B** in order to provide the equality **C**=**E** in each four-dimensional point, where **E** is the unit matrix 4×4? In this particular case we will have

$$\left(dx_{\rm ex}^{\rm N}\right)_i = \left(dx_{\rm ex}^{\rm E}\right)_i. \tag{4}$$

In order to give answer on this question let us introduce the conventional measuring methods in each infinitely small four-dimensional volume of space-time: a use of infinitely small "unit scales" for a measurement of length; a use of standard clocks for a measurement of time, and the Einstein's method for synchronization of clocks separated by infinitely small spatial interval.

Then the magnitude of experimentally measured length $\left(dx_{\rm ex}^{\rm N}\right)^\alpha$ in physical space-time $x_{\rm ph}^{\rm N}$ is equal to the ratio $\left((d\,x_{\rm ph}^{\rm N})^\alpha / (dx_{\rm phu}^{\rm N})^\alpha\right)$, where $\left(dx_{\rm phu}^{\rm N}\right)^\alpha$ is the infinitely small "unit scale" in curvilinear physical space-time (hereinafter the Greek subscripts correspond to three-dimensional space, α=1...3)[1]. Now let us write a relationship between the space components of $dx_{\rm ph}^{\rm N}$ and four-vector $dx_{\rm ph}^{\rm E}$:

$$\left(dx_{\rm ph}^{\rm N}\right)_\alpha = B_{\alpha j}\left(dx_{\rm ph}^{\rm E}\right)^j = B_{\alpha 0}\left(dx_{\rm ph}^{\rm E}\right)^0 + B_{\alpha\beta}\left(dx_{\rm ph}^{\rm E}\right)^\beta. \tag{5}$$

For "unit scale" $\left(dx_{\rm phu}^{\rm N}\right)_\alpha$ in physical space-time we also can write the similar relation:

$$\left(dx_{\rm phu}^{\rm N}\right)_\alpha = B_{\alpha 0}\left(dx_{\rm ph}^{\rm E}\right)^0 + B_{\alpha\beta}\left(dx_{\rm phu}^{\rm E}\right)^\beta, \tag{6}$$

where $\left(dx_{\rm phu}^{\rm E}\right)^\alpha$ is the corresponding "unit scale" in Euclidean space. Dividing (5) by (6), one obtains:

$$\frac{\left(dx_{\rm ph}^{\rm N}\right)_\alpha}{\left(dx_{\rm phu}^{\rm N}\right)_\alpha} = \frac{B_{\alpha 0}\left(dx_{\rm ph}^{\rm E}\right)^0 + B_{\alpha\gamma}\left(dx_{\rm ph}^{\rm E}\right)^\gamma}{B_{\alpha 0}\left(dx_{\rm ph}^{\rm E}\right)^0 + B_{\alpha\beta}\left(dx_{\rm phu}^{\rm E}\right)^\beta}. \tag{7}$$

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[1] In order to keep the experimentally measured length $\left(dx_{\rm ex}^{\rm N}\right)^\alpha$ as infinitely small value, we can multiply the ratio $\left((d\,x_{\rm ph}^{\rm N})^\alpha / (dx_{\rm phu}^{\rm N})^\alpha\right)$ by $\left((dx_{\rm phu}^{\rm N})^\alpha / (x_{\rm phu}^{\rm N})^\alpha\right)$, where $(x_{\rm phu}^{\rm N})^\alpha$ is the finite unit scale in the reference frame under consideration. Such the re-definition has no importance for the proving theorem.

This expression allows further transformation in the particular case $B_{\alpha 0}$=0 with account of the obvious equality for Euclidean space

$$\left(x_{\mathrm{ph}}^{\mathrm{E}}\right)^{\beta} \Big/ \left(x_{\mathrm{ph}}^{\mathrm{E}}\right)^{\alpha} = \left(x_{\mathrm{phu}}^{\mathrm{E}}\right)^{\beta} \Big/ \left(x_{\mathrm{phu}}^{\mathrm{E}}\right)^{\alpha} .$$

Hence,

$$\frac{\left(dx_{\mathrm{ph}}^{\mathrm{N}}\right)_{\alpha}}{\left(dx_{\mathrm{phu}}^{\mathrm{N}}\right)_{\alpha}} = \frac{B_{\alpha\gamma}\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)^{\gamma}}{\left(dx_{\mathrm{phu}}^{\mathrm{E}}\right)^{\alpha}\left[B_{\alpha\alpha} + \left(1/\left(dx_{\mathrm{phu}}^{\mathrm{E}}\right)^{\alpha}\right)\sum_{\beta\neq\alpha} B_{\alpha\beta}\left(dx_{\mathrm{phu}}^{\mathrm{E}}\right)^{\beta}\right]} =$$

$$= \frac{B_{\alpha\gamma}\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)^{\gamma}}{\left(dx_{\mathrm{phu}}^{\mathrm{E}}\right)^{\alpha}\left[B_{\alpha\alpha} + \left(1/\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)^{\alpha}\right)\sum_{\beta\neq\alpha} B_{\alpha\beta}\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)^{\beta}\right]} = \tag{8}$$

$$= \frac{\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)_{\alpha}\left(B_{\alpha\gamma}\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)^{\gamma}\right)}{\left(dx_{\mathrm{phu}}^{\mathrm{E}}\right)_{\alpha}\left(B_{\alpha\beta}\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)^{\beta}\right)} = \frac{\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)_{\alpha}}{\left(dx_{\mathrm{phu}}^{\mathrm{E}}\right)_{\alpha}} .$$

The equality (8) means that the measured magnitudes $\left(dx_{\mathrm{ex}}^{\mathrm{N}}\right)_{\alpha}$ in $x_{\mathrm{ph}}^{\mathrm{N}}$ coordinates coincide with the measured magnitudes $\left(dx_{\mathrm{ex}}^{\mathrm{E}}\right)_{\alpha}$ in $x_{\mathrm{ph}}^{\mathrm{E}} \equiv x_{\mathrm{L}}$ coordinates, *i.e.*,

$$\left(dx_{\mathrm{ex}}^{\mathrm{N}}\right)_{\alpha} = \left(dx_{\mathrm{ex}}^{\mathrm{E}}\right)_{\alpha} = \left(dx_{\mathrm{L}}\right)_{\alpha} \tag{9}$$

under the adopted condition $B_{\alpha 0}$=0 for the transformation (2). Physically this result means that the distortion of length dx^{α} being induced by the transformation **B** from pseudo-Euclidean to curvilinear geometry is not detectable experimentally under $B_{\alpha 0}$=0 because of proportional distortion of the "unit scale" dx_{u}^{α} .

Further, let us write a relationship between time components of the four-vectors $dx_{\mathrm{ph}}^{\mathrm{N}}$ and $dx_{\mathrm{ph}}^{\mathrm{E}}$:

$$\left(dx_{\mathrm{ph}}^{\mathrm{N}}\right)_{0} = B_{00}\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)^{0} + B_{0\alpha}\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)^{\alpha} . \tag{10}$$

For two events in a fixed spatial point ($\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)^{\alpha}$ =0),

$$\left(dx_{\mathrm{ph}}^{\mathrm{N}}\right)_{0} = B_{00}\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)^{0} . \tag{11}$$

Hence, the coefficient B_{00} describes a change of rate of clock in a fixed spatial point under transformation **B** from pseudo-Euclidean to curvilinear geometry. Such the change takes place for both standard clock and physical time interval. Therefore, the experimentally measured time interval in a fixed spatial point is equal to

$$\left(dx^{\mathrm{N}}_{\mathrm{ex}}\right)_0 = \frac{1}{B_{00}}\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_0 . \qquad (12)$$

For the time interval in two different spatial points separated by the distance $\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_\alpha$ one should write

$$\left(dx^{\mathrm{N}}_{\mathrm{ex}}\right)_0 = \frac{1}{B_{00}}\left[\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_0 + \Delta\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_0\right], \qquad (13)$$

where $\Delta\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_0$ is the error of synchronization of clocks separated by the distance $\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_\alpha$ in curvilinear physical space-time. The value of $\Delta\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_0$ can be found from the equality

$$\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{02} = \left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{01}/2 , \qquad (14)$$

(Einstein's method of clocks synchronization), where $\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{01}$ is the propagation time of light from the first clock Cl_1 (in the point $\left(x^{\mathrm{N}}_{\mathrm{ph}}\right)_\alpha$) to the second clock Cl_2 (in the point $\left[\left(x^{\mathrm{N}}_{\mathrm{ph}}\right)_\alpha + \left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_\alpha\right]$) and backward according to Cl_1, while $\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{02}$ is the indication of Cl_2 at the moment of arrival of light pulse (here we take $\left(x^{\mathrm{N}}_{\mathrm{ph}}\right)_0 = 0$ for Cl_1 at the moment of start of synchronization). In curvilinear space-time the propagation time of light from Cl_1 to Cl_2 $\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{0+}$ is not equal, in general, to the propagation time in the reverse direction $\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{0-}$. Hence, an implementation of the equality (14) is possible only in the case where the readings of both clocks at initial moment of time differ by the value $\Delta\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_0$, and

$$\begin{aligned}\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{01} &= \frac{1}{B_{00}}\left[\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{0+} + \left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{0-}\right],\\ \left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{02} &= \frac{1}{B_{00}}\left[\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{0+} + \Delta\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_0\right]\end{aligned} \qquad (15)$$

(Here we do not take into account a dependence of the coefficient B_{00} on the $\left(x^{\mathrm{N}}_{\mathrm{ph}}\right)^i$. One can easy show that under smooth function $B_{00}\left((x^{\mathrm{N}}_{\mathrm{ph}})^i\right)$ this dependence is negligible for infinitely small distance between Cl_1 and Cl_2). Hence, we obtain with account of (14):

$$\Delta\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_0 = \frac{1}{2}\left[\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{0-} - \left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{0+}\right]. \qquad (16)$$

The expressions for $(dx^{\mathrm{N}}_{\mathrm{ph}})_{0+}$ and $(dx^{\mathrm{N}}_{\mathrm{ph}})_{0-}$ can be found from (2):

$$\begin{aligned}\left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{0+} &= B_{00}\left(dx^{\mathrm{E}}_{\mathrm{ph}}\right)^0 + B_{0\alpha}\left(dx^{\mathrm{E}}_{\mathrm{ph}}\right)^\alpha ,\\ \left(dx^{\mathrm{N}}_{\mathrm{ph}}\right)_{0-} &= B_{00}\left(dx^{\mathrm{E}}_{\mathrm{ph}}\right)^0 - B_{0\alpha}\left(dx^{\mathrm{E}}_{\mathrm{ph}}\right)^\alpha .\end{aligned} \qquad (17)$$

Substituting (17) into (16), one gets:

$$\Delta\left(dx_{\mathrm{ph}}^{\mathrm{N}}\right)_0 = -B_{0\alpha}\left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)^{\alpha}. \qquad (18)$$

Further substitution of (18) and (10) into (13) gives:

$$\left(dx_{\mathrm{ex}}^{\mathrm{N}}\right)_0 = \left(dx_{\mathrm{ph}}^{\mathrm{E}}\right)_0 = \left(dx_{\mathrm{ex}}^{\mathrm{E}}\right)_0 = \left(dx_{\mathrm{L}}\right)_0. \qquad (19)$$

Therefore, we conclude that for any admissible transformation **B**, an experimentator will not detect a deflection of his local geometry from pseudo-Euclidean under measurement of the differentials of time intervals. Besides, under the condition $B_{\alpha 0}$=0 the same conclusion is additionally valid for the infinitely small length, and if the equality $B_{\alpha 0}$=0 takes place in whole four-dimensional space, a deflection of a geometry of physical space-time from pseudo-Euclidean in each infinitely small four-dimension volume is not detectable experimentally. On the other hand, the transformation **B** under $B_{\alpha 0}$=0 in whole space-time belongs to a kind of transformations $x'^0 = x'^0(x^i), x'^{\alpha} = x'^{\alpha}(x^{\beta})$, acting within a fixed reference frame. Thus, we have proved the following general theorem:

- under any admissible transformation B from pseudo-Euclidean to curvilinear geometry within a fixed frame of references ($B_{\alpha 0}$=0) a deflection of local geometry from pseudo-Euclidean is not experimentally observable.

3. Special theory of relativity and "covariant ether theories"

The formal theorem proven above can be useful under application to some particular problems of both special and general relativity. In the present paper we will restrict ourselves by a case of inertial motion in an empty space only, where equality (4) is valid not only for differentials of space-time four-vectors, but for their any finite values in a properly constructed inertial reference frame, *i.e.*

$$\left(x_{\mathrm{ex}}^{\mathrm{N}}\right)^i = \left(x_{\mathrm{ex}}^{\mathrm{E}}\right)^i = \left(x_{\mathrm{L}}\right)^i. \qquad (20)$$

Thus, we conclude that for any hypothetical curvilinear geometry of an empty physical space-time, related with pseudo-Euclidean one by means of transformation **B** under $B_{\alpha 0}$=0, the experimentally measured space-time four-vectors obey to the Lorentzian transformations. In order to derive physical inferences from this result, it is necessary to find possible physical meaning for such **B**-transformations. It can be made by the following way.

Let us consider a class of space-time theories of inertial motion in an empty space satisfying to the following general principles:

- space-time homogeneity;
- space isotropy;
- causality principle;
- general relativity principle (GRP).

In these theories a general transformation between two arbitrary inertial reference frames K and K' in physical space-time can be written as

$$x_{\mathrm{ph}i} = A_{ij} x'^{j}_{\mathrm{ph}}, \qquad (21)$$

where the transformations **A** in (21) are linear due to the space-time homogeneity and constitute a ten-parametrical Lee group due to adoption of the GRP [8]. For simplicity we further omit the trivial translations and rotations of space, considering three-parametrical transformations **A**($\vec{v}$), satisfying to the reciprocity principle [9]

$$\mathbf{A}(\vec{v}) = \mathrm{A}^{-1}(-\vec{v}). \qquad (22)$$

Here $\vec{v}$ is a relative velocity of two arbitrary inertial frames. Further, in general, we do not suppose that a geometry of physical space-time has to be pseudo-Euclidean in any inertial reference frame. At the same time, due to the isotropy of physical space, such an inertial reference frame K_0 exists wherein the speed of light is isotropic and equal to *c*, *i.e.*, its geometry should be pseudo-Euclidean for any particular choice of transformation **A**. In order to specify this requirement: an existence of at least one K_0 frame with pseudo-Euclidean geometry for any admissible transformation **A**, we can formally introduce into consideration the Minkowskian co-ordinates x_{L} and demand the equality (23) for the frame K_0:

$$x'_{\mathrm{ph}i} \doteq x'_{\mathrm{L}i} \qquad (23)$$

(hereinafter the primer four-vectors belong to K_0), and in general case **A**≠**L** equality (23) is valid only for this ("absolute") frame. One can additionally show that (23) simultaneously ensures that the x_{ph} coordinates are "admissible" wherein the known relationships between components of the metric tensor ***g*** take place [1], and a velocity of light is finite [5].

Using (1), (21), (23), one can find a relation between x_{ph} and x_{L} in an inertial frame K moving at a constant velocity $\vec{v}$ in K_0:

$$x_{\mathrm{ph}i} = B_{ij}(\vec{v}) x^{j}_{\mathrm{L}}, \qquad (24)$$

where the introduced matrix **B** is determined by the relationship

$$B_{ij}(\vec{v}) = A_i^k(\vec{v}) L^{-1}_{kj}(\vec{v}). \qquad (25)$$

Further, using a known form of the matrix **L** (see, for instance [7]) one obtains from (25) and (22), that

$$B_{\alpha 0} = 0$$

for any admissible transformation **A**, that is the necessary and sufficient condition for implementation of the equality (20). Hence, we conclude that **for any space-time theory of inertial motion, satisfying to general symmetries of space-time and general relativity principle, the experimentally measured space-time four-vectors always obey to the Lorentzian transformation L regardless of the concrete form of physical space-time transformations.** One follows from there that the fact of Lorentzian transformation for experimentally measured space and time intervals formally gives no information about physical space-time and, generally, does not reject the theories with not pseudo-Euclidean geometry of space-time. One can show that a number of such admissible “covariant ether theories” is equal to infinity [10]. That is why all the experiments for verification of

Lorentzian transformations find an infinite number of alternative explanations of their results.

An exclusive place of STR among all such covariant ether theories is defined by the fact, that it directly takes an equality of experimentally measured and physical space-time four-vectors, *i.e.*, the equality of the matrices **A** and **L**. This circumstance allows to propose the relativistic postulate (instead of two well-known Einstein’s postulates) in the following form: in an empty space one can always choose such a proper measuring method, which provides a coincidence of the experimentally measured and physical space-time four-vectors with a vanishing measuring error in macrophysics scale. (In a brief form it can be called as a postulate about equality of physical and experimentally measured four-vectors).

On the other hand, this postulate follows from nothing: in general, a question about coincidence of "measured" and "physical" values even under optimal measurements is one of initial questions to Nature. That is why it is especially interesting to analyse the negative answer on the question for deeper understanding of physical essence of relativity. One can easy see that such negative answer means that **A**≠**L**.

Under **A**≠**L** a geometry of physical space-time is not pseudo-Euclidean, and we must distinguish space-time transformations for physical and experimentally measured four-vectors:

$$x_{\mathrm{ph}i} = A_{ij} x'^{j}_{\mathrm{ph}}, \tag{26}$$

$$x_{\mathrm{ex}i} = L_{ij} x'^{j}_{\mathrm{ex}}, \tag{27}$$

where the primer four-vectors, as before, belong to the “absolute” frame K_0. It means, in particular, that the transformation (27) does not yet solve a main kinematical problem for experimentally measured space-time four-vectors (determination of space-time transformations between two arbitrary inertial frames): it acts only in special case, defined by the equality (23). In order to find a transformation between two such arbitrary inertial frames K and K”, we should write

$$x_{\mathrm{ex}i} = L_{ij}(\vec{v}_1) x'^{j}_{\mathrm{ex}}, \tag{28}$$

$$x''_{\mathrm{ex}i} = L_{ij}(\vec{v}_2) x'^{j}_{\mathrm{ex}}, \tag{29}$$

where $\vec{v}_1, \vec{v}_2$ are the “absolute” velocities of the frames K and K”, respectively. Excluding four-vector x'^{j}_{ex} from (28), (29), we obtain a general transformation between experimentally measured space-time four-vectors in two arbitrary inertial frames as

$$x_{\mathrm{ex}i} = L_{ij}(\vec{v}_1)[L^{-1}(\vec{v}_2)]^{jk} x''_{\mathrm{ex}k}. \tag{30}$$

The same way can be applied for determination of space-time transformations in physical space-time:

$$x_{\mathrm{ph}i} = A_{ij}(\vec{v}_1)[A^{-1}(\vec{v}_2)]^{jk} x''_{\mathrm{ph}k}, \tag{31}$$

where the matrix **A** can be taken in arbitrary admissible form. (In case of inertial motion in an empty space we can nothing say about the concrete form of the matrix **A**, we only can take these or that assumptions).

Thus, the contrary to STR, in the covariant ether theories (adopting **A≠L)**, Nature does not "know" a direct relative velocity of two arbitrary inertial frames K and K": it is always composed as a sum $\vec{v}_1 \oplus \vec{v}_2$, where $\vec{v}_1$ and $\vec{v}_2$ are the corresponding velocities of K and K" in the "absolute" frame K_0. It means, that direct rotation-free Lorentzian transformation between experimentally measured space-time four-vectors in K and K" is impossible: according to general group properties of these transformations, an additional rotation of the coordinate axes of the frames K and K" appears at some angle Ω, depending on $\vec{v}_1$ and $\vec{v}_2$. It is quite important that such a rotation occurs in experimentally measured coordinates, *i.e.*, it can be really detected. It defines a principal possibility to experimentally distinguish the hypotheses **A**=**L** and **A**≠**L**.

Before consideration of such experiments and their physical meaning, we would like to stress, that our analysis reveals an exclusive place of STR as the simplest space-time theory among others, satisfying to GRP, because it directly takes an equality of physical and experimentally measured space-time four-vectors, and **A**=**L**. Among admissible space-time theories, taking **A**≠**L**, the simplest case corresponds to a choice **A**=**G**, where **G** is the matrix of Galilean transformation: G_{ii}=1, $G_{\alpha 0}$ =-v_α, and all others G_{ij} =0. Substituting the matrices **G** and **L** into (25), one gets the following coefficients of matrix **B**:

$$B_{00} = \gamma, B_{\alpha 0} = 0, B_{0\alpha} = \frac{v_\alpha}{c^2}\gamma, B_{\alpha\beta} = \delta_{\alpha\beta}\frac{v_\alpha v_\beta}{v^2}\left(1 - \frac{1}{\gamma}\right), \tag{32}$$

where $\delta_{\alpha\beta}$ is the Cronekker symbol. Further substitution of (32) into (24) with account of the obvious equality $x_{\mathrm{L}} = x_{\mathrm{ph}}(v=0)$ allows to determine a dependence of physical space-time four-vectors on the "absolute" velocity $\vec{v}$ of some arbitrary inertial reference frame K [5]:

$$\vec{r}_{\mathrm{ph}}(\vec{v}) = \vec{r}_{\mathrm{ph}}(v=0) + \frac{\vec{v}\left(\vec{r}_{\mathrm{ph}}(v=0), \vec{v}\right)}{v^2}\left[\sqrt{1-(v^2/c^2)} - 1\right], \tag{33}$$

$$t_{\mathrm{ph}}(\vec{v}) = \frac{t_{\mathrm{ph}}(v=0)}{\sqrt{1-(v^2/c^2)}} + \frac{\vec{r}_{\mathrm{ph}}(v=0)\vec{v}}{c^2\sqrt{1-(v^2/c^2)}}\ . \tag{34}$$

For the time intervals in a fixed spatial point of the frame K (r_{ph}=0), we obtain the dependence of t_{ph} on $\vec{v}$ (see, (34)):

$$t_{\mathrm{ph}}(\vec{v}) = t_{\mathrm{ph}}(v=0)/\sqrt{1-(v^2/c^2)}, \tag{35}$$

that means an "absolute" dilation of time by $\sqrt{1-(v^2/c^2)}$ times. Furthermore, one obtains from (33):

$$\left(\vec{r}_{\rm ph}(\vec{v}),\vec{v}\right)=\left(\vec{r}_{\rm ph}(v=0),\vec{v}\right)\sqrt{1-v^2/c^2}\,,$$
$$\left[\vec{r}_{\rm ph}(\vec{v})\times\vec{v}\right]=\left[\vec{r}_{\rm ph}(v=0)\times\vec{v}\right], \tag{36}$$

that means an "absolute" contraction of moving scale along a vector of "absolute" velocity (Fitzgerald-Lorentz hypothesis). Finally, transformation (31) (under **A**=**G**)

$$x_{{\rm ph}i}=[G_{ij}(\vec{v}_1-\vec{v}_2)]x''^{j}_{\rm ph} \tag{37}$$

leads to the Galilean law of speed addition for the physical light velocity $c_{\rm ph}$.

Thus, we have got a full set of the Lorentz ether postulates in case **A**=**G**[2]. However, the physical space-time $x_{\rm ph}$ in the Lorentz ether theory is not observable in arbitrary inertial reference frame, while the experimentally measured four-vectors $x_{\rm ex}$ obey to the Lorentzian transformations in form of (30). (This important circumstance about a difference of physical and experimentally measured four-vectors in non-Euclidean geometry was dropped by Lorentz and his successors). Therefore, we may consider the Lorentz ether theory as one of the covariant ether theories, defined above, and the simplest among them. Just due to this fact an application of the Lorentz ether postulates for explanation of "null" results of all experiments for search of "ether wind speed" was always successful.

4. About a new experimental test of special relativity

The obtained in section 3 results allow to formulate the experimental criteria for a crucial choice: **A**=**L** (STR) or **A**≠**L** (all theories of "post-relativity ether"). In particular, one can easy see that in all relativistic interference experiments (Michelson-Morley and others) where measuring instruments (interferometers) do not contain moving inertial parts, we have to apply for x_{ex} a direct Lorentzian transformation from the absolute frame K_0 to a laboratory frame K. Then (30) transforms into

$$x_{{\rm ex}i}=L_{ij}(\vec{v})x'^{\rm j}_{\rm ex}\,, \tag{38}$$

(where $\vec{v}$ is the velocity of K in K_0), that is non-distinguishable from STR. One follows from there that for experimental choice between the hypotheses either **A**=**L** or **A**≠**L**, an instrument for measurement of hypothetical "absolute" velocity must contain moving inertial parts, so that to deal with successive space-time transformations. Then a general idea of such experiment can be described with help of a diagram in Fig. 1. It shows the absolute frame K_0, laboratory frame K (moving at the constant "absolute" velocity $\vec{v}$) and frame K_i, attached

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[2] Let us recall the postulates of Lorentz ether theory in its modern form:

1. There is an "absolute" reference frame K_0, wherein a light velocity is isotropic and equal to c.
2. In arbitrary reference frame K, moving at constant velocity $\vec{v}$ in K_0, the velocity of light is equal to $\vec{c}\,'=\vec{c}-\vec{v}$.
3. In this reference frame K time is dilated by $\sqrt{1-v^2/c^2}$ times.
4. In this reference frame K a linear scale is contracted by $\sqrt{1-v^2/c^2}$ times along the vector $\vec{v}$.

with some moving inertial part of measuring instrument in K. In our laboratory we specify a velocity $\vec{u}$ of K_i in K. In such a case for the hypothesis **A**=**L** we apply direct rotation-free Lorentzian transformation $K \to K_i$ under calculation of indication of measuring device. Hence, according to STR we get a null value of "absolute" velocity. Under the hypothesis **A**≠**L** Nature does not "know" a direct rotation-free Lorentzian transformation between K and K_i, and "operates" with the "absolute" velocities of these frames $\vec{v}$ and $v \oplus \vec{u}$. Hence, in order to calculate an indication of measuring device, we must apply the successive Lorentzian transformations $K \to K_0 \to K_i$ according to (30). (A direct Lorentzian transformation from K to K_i is also possible, but it will be not a rotation-free). In this case the axes of the frames K and K_i are turned out at some angle Ω, that, in principle, changes the state of a measuring instrument. Since Ω depends on the "absolute" velocity $\vec{v}$ of the laboratory frame K, that the state of measuring instrument will depend on $\vec{v}$, too. There is only one particular case ($\vec{v}$ is collinear to $\vec{u}$) where Ω=0, and the state of measuring instrument has to be unchanged for any magnitude of "absolute" velocity of the laboratory frame. This is an unambiguous inference of the GRP, and all experiments for search of "ether wind" velocity with moving inertial parts in experimental instruments, aiming to measure non-relativistic effects under collinear $\vec{v}$ and $\vec{u}$ (see, e.g., [11-13]) checked, in fact, the GRP, but not the particular relativity principle.

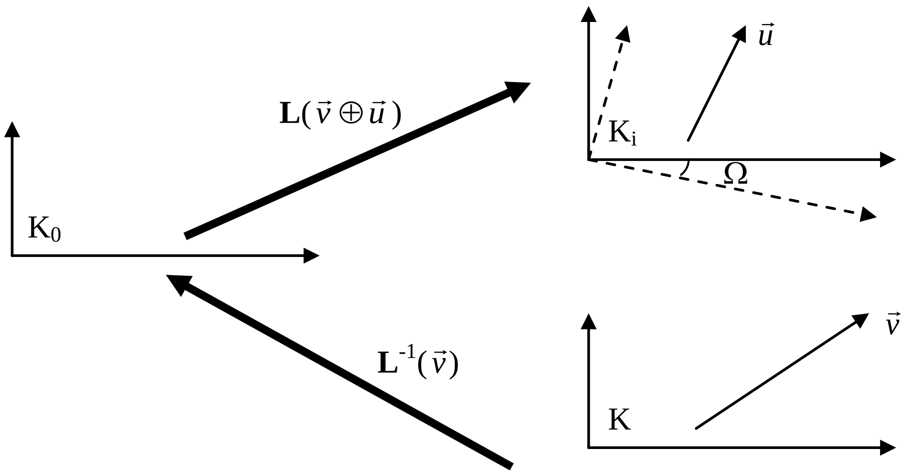

Fig. 1. General idea of the experiment for a new test of STR.

There is a sole experimental fact - precession of Thomas [14] – dealing with addition of Lorentzian boosts with non-collinear relative velocities. One can shows that this fact can be explained not only in STR, but for any admissible transformation **A**≠**L**, acting in physical space-time (see Appendix A).

Hence, we conclude, that a measurement of hypothetical dependence of $\Omega(\vec{v})$ under non-collinear $\vec{v}$ and $\vec{u}$ would be a qualitatively new test of special relativity. This is the answer on the second main question of the present paper (to find a kind of phenomena, giving different unambiguous predictions for STR and all alternative space-time theories).

It is necessary to emphasize that a dependence of Ω on $\vec{v}$ for x_{ex} is predicted in the theories, where physical and experimentally measured space-time four-vectors differ from each other, and we may omit the problem to seek its physical meaning in the x_{ex} coordinates.

One can show that this dependence is explained in terms of physical space-time, even under impossibility to directly measure x_{ph} [10].

Thus, an experiment for qualitatively new test of STR must contain moving inertial part (parts) with non-collinear velocities $\vec{v}$ and $\vec{u}$, and be aimed to measure a dependence of the angle Ω on the "absolute" velocity of a laboratory frame. In the order of magnitude c^{-2}, and for orthogonal vectors of $\vec{v}$ and $\vec{u}$, this dependence has the form [7]

$$\Omega \approx \frac{uv}{2c^2}. \qquad (39)$$

A direct measurement of this dependence in a laboratory scale experiment is practically impossible. Indeed, the "absolute" velocity v could be taken as about $10^{-3}c$ (typical velocities of Galax objects). The maximum value of u could be about 10^3 m/s. Hence, the angle Ω takes on the value 3×10^{-9}, *i.e.*, below any limit of practicability in the experiments with moving inertial parts.

An analysis of possible experimental schemes for indirect measurement of the angle Ω can be greatly simplified under numerical estimation of eventual non-relativistic effects, proceeding from their dimension. Indeed, the experiments, looking for the change of length being associated with $\Omega(\vec{v})$ dependence, give the effect in the order of magnitude $L\Omega$. Here L is some length, which is equal to about 1 m in the laboratory scale. In such a case we get $L\Omega\approx3\times10^{-9}$ m, that is impossible to measure in practice. A corresponding change of time has a dimension $\frac{L\Omega}{u} \approx \frac{Lv}{c^2} \approx 3$ ps, a time interval within the range of present technology, but not for mechanical parts necessarily involved. Finally, one can rearrange the experiment into a "speed experiment" looking at the term Ωu, and the latter arrangement could be further transformed into frequency measurement via the Doppler effect ($\Omega u/c$). For the last case one is looking at the term u^2v/c^3, the latter being about 10^{-14} (for $u=10^3$ m/s) - a value accessible in the most convenient way by the Mössbauer effect and practically only by this technique at least as far as the laboratory scale experiment is considered.

Thus, under modern development of experimental technique a search of $\Omega(\vec{v})$ dependence could be made only on the basis of the Mössbauer effect, where for some proper experimental schemes eq. (39) induces a relative energy shift between emission and absorption lines at the level u^2v/c^3, where u is a typical velocity of an emitter (absorber) in the laboratory frame. For $u\approx10^3$ m/s, the term $u^2v/c^3\approx10^{-14}$, that lies in the range of the Mössbauer effect sensitivity.

One of possible schemes of such an experiment (high-sensitivity modified experiment by Champeney et al.) has been considered in [5, 15]. Ref. [16] proposes an experiment based on synchrotron Mössbauer radiation of the isotope ^{67}Zn. Its approximate scheme is depicted in Fig. 2.

The first Zn target is placed in the synchrotron radiation beam on the axis of a rotor, while the second Zn target is distributed on the outer radius of the rotor. Forward scattered Mössbauer radiation from both targets is registered by a detector, whose axis constitutes some angle α with the axis of the synchrotron beam. The value of $\alpha\pm\Delta\alpha\approx\omega\tau/2\pm\omega\tau/2$, where ω is the rotation frequency, while τ is the half-life of ^{67}Zn (≈9 μs). In such a case the calculated relative energy shift between emission and absorption lines is equal to

$$\frac{\Delta E}{E} = \frac{u^2}{2c^2} + \frac{u^2 v_{xy} \cos(\varphi - \alpha)}{4c^3}, \tag{40}$$

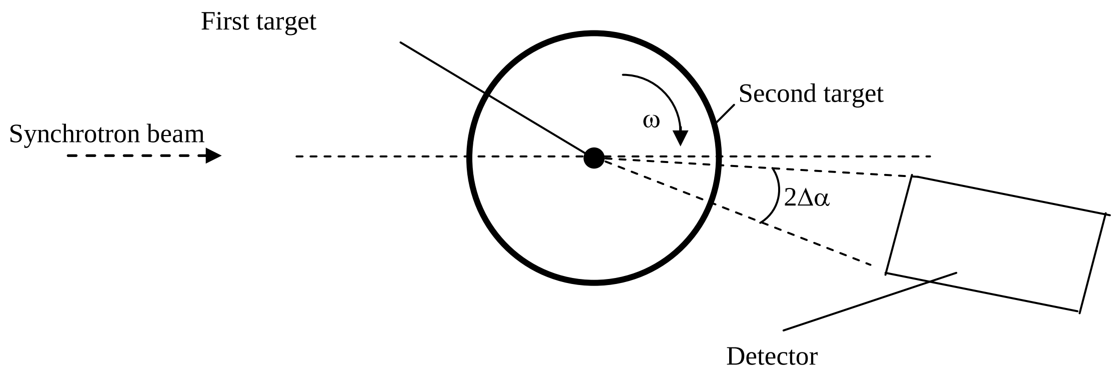

Fig. 2. An approximate scheme of the proposed experiment on the Mössbauer synchrotron radiation.

where φ is the angle between the vector $\vec{v}_{xy}$ and the axis x ($\vec{v}_{xy}$ is the projection of the absolute velocity vector onto xy plane). Here the first term in the right part of (40) corresponds to conventional second order Doppler effect, while the third order term being proportional to the "absolute" velocity, is induced by the dependence (39).

The detector registers the beats of intensity in time scale, and their frequency is proportional to the energy shift between emission and absorption lines. The "reference" quantum beat frequency is determined by the second order Doppler shift in (40), while the third order term in (40) causes its variation during 24 h. Under the value of u about 300 m/s, the term $u^2/2c^2 \approx 5\times10^{-13}$, and for identical chemical compounds of both targets, the "reference" frequency is $\nu \approx 1.1\times10^{7}$ s^{-1}. It means, that during the decay time of ^{67}Zn we are able to observe approximately 100 maximums of quantum beats. One can show that the third order term $u^2 v/4c^3 \approx 2.5\times10^{-16}$ would produce a maximum variation of time interval between the first and last maximums of quantum beats at the level ≈9 ns, that should be quite detectable.

The experiment can be realized on the storage ring PETRA (DESY, Hamburg). On the contrary to all known experiments in space-time physics, it will allow to choose either the special relativity, or "covariant ether theories", and hence, this will be a qualitatively new test of the particular Einstein relativity principle.

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Conclusions

1. Consideration of all hypothetical theories of an empty space-time with curvilinear geometry should be based on distinguishing of physical and experimentally measured space-time four-vectors. General analysis of the properties of admissible space-time transformations shows that in any theory, adopting the general relativity principle and symmetries of space-time, the experimentally measured space and time intervals always obey to the Lorentzian transformation regardless of concrete form of physical space-time transformation. The latter circumstance defines a possibility to explain all known experimental results in space-time physics within infinite number of space-time theories.

2. The special theory of relativity is an exclusive among all other admissible theories of an empty space-time, because it directly takes an equality of experimentally measured and physical four-vectors under proper measurements. An adoption of such the equality defines a possibility of direct rotation-free Lorentzian transformation between two arbitrary inertial frames. This is impossible in all other admissible space-time theories called by us as "covariant ether theories", that leads to the dependence of Ω on an "absolute" velocity $\vec{v}$ in the experimentally measured space-time co-ordinates. Hence, this hypothetical dependence is a sole observable physical phenomenon, admitting an unambiguous direct test of the special relativity.

3. Practically single method for the measurement of $\Omega(\vec{v})$ dependence under modern development of laboratory experimental technique is the Mössbauer spectroscopy. One of possible experiments of such a kind can be realized on Mössbauer synchrotron radiation of the isotope ^{67}Zn at the storage ring PETRA (DESY, Hamburg).

Appendix A
Thomas precession in the "covariant ether theories"

It is well known that Thomas precession explained on classical level an appearance of the multiplier ½ in the expression for spin-orbit interaction in atoms. Proceeding from classical model of a motion of electron in atom, Thomas showed that the appearance of this multiplier occurs due to additional space turn Ω of the co-ordinate axes of the reference frames under successive Lorentzian transforms. This fact is considered until the moment as a strong confirmation of the STR. We assert that the Thomas precession can be explained for any admissible transformation **A**≠**L** acting in physical space-time, and demonstrate this assertion for the simplest case where **A**=**G**. Such the choice allows us to use the derived above expressions (35), (36) (Lorentz ether postulates) in further consideration.

Let an electron with its own reference frame K_e rotates about a nucleus, which, for simplicity, rests in the "absolute" frame K_0. Let the rotational frequency of the electron is equal to $\vec{\omega}_0$. Then, according to the "absolute time dilation effect" (eq. (35)), the rotational frequency of nucleus about electron in the frame K_e is $\vec{\omega}'=\vec{\omega}_0\sqrt{1-v^2/c^2}$, where $\vec{v}$ is the orbital velocity of the electron. Thus, the rotational frequency of the electron about nucleus is not equal to the rotational frequency of nucleus about electron. It is possible only in the case where the coordinate axes of K_e rotates about coordinate axes of K_0 at the frequency

$$\vec{\omega}=\vec{\omega}_0-\vec{\omega}'=\vec{\omega}_0\left(1-\sqrt{1-v^2/c^2}\right) \tag{41}$$

Taking into account, that for rotation motion of the electron

$$\dot{\vec{v}} = [\vec{\omega}_0 \times \vec{v}],$$

and using the vector identity $[\vec{a} \times [\vec{b} \times \vec{c}]] = \vec{b}(\vec{a}\vec{c}) - \vec{c}(\vec{a}\vec{b})$, one can easy prove that

$$\vec{\omega}_0 = \frac{[\vec{v} \times \dot{\vec{v}}]}{v^2}. \qquad (42)$$

Substituting (42) into (41), we obtain with the accuracy c^{-2}:

$$\vec{\omega} = \frac{1}{2}\frac{[\vec{v} \times \dot{\vec{v}}]}{c^2}, \qquad (43)$$

that coincides with the well-known expression for Thomas precession in the STR.

One can show that the same expression (43) can be obtained for any other admissible choice of the matrix **A**, and the requirement for a rest of nucleus in the “absolute” frame K_0 is not essential.

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