COMPLETTING EINSTEIN'S OPERATIONAL DERIVATION OF LORENTZ TRANSFORMATION

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Abstract: The validity of the almost forgotten operational method that Einstein used in his original paper on relativity [1] to deduce Lorentz transformation (LT) is confirmed here by revealing the physics that should warrant LT. There becomes evident that the lack of that physics was mainly responsible for the engendered ill-speculations and misleading interpretations discrediting the special theory of relativity (STR) since its foundation, as well as for harsh unceasing controversies.

l.Introduction

Einstein's 1905 derivation on physical grounds of LT [1] was never concluded. The axial diagram he chose in [1] did not put forward him the problem of adding travel times of crossing light signals as scalar quantities. Becoming doubtful of his operational method, Einstein abandoned it soon [2] (and for ever) for challenging pure mathematical methods. Showing the identical graphical and mathematical descriptions of the uniform rectilinear motions of an object relative to both a "stationary" coordinate system (CS)• and a CS at absolute rest in Sect.2.1, approaching the addition of light travel times as scalar quantities in Sect.2.2, and identifying the CS Ξ at absolute rest associated to k in Sect.4 (all these pre-existing in [1] and being correctly solved by Einstein by ingenious mathematical "tricks" -i.e., true but physically unjustified mathematical operations-, I complete in Sects.5, 6 Einstein's operational derivation of LT just sketched in Sect.3 with physical content. The operational derivation of the vector LT in Sect.7 and the operational proof in Sects.8, 9 that co-linear and non-co-linear LTs form groups confirm the validity of Einstein's operational method, revealing that the principle of the physical determination of the equations works in STR, too. Conclusions are drawn in Sect.9.

2.Preliminary considerations

2.1.Coordinate systems at absolute rest associated to "stationary" coordinate systems

A. Consider the diagrams in Fig.1. k and K are parallel 3-dimensional CSs of origins O', O in uniform relative motion along the common x',x axis. O, O' coincide at t=0. P is a fixed point in k. The velocity v is imparted to O' relative to a K at absolute rest in the 1st diagram and to a K at rest in a "stationary" space in the 2nd diagram. Both the upper and the resulting bottom diagrams predict

x'=x-vt (1)

Thus *the motion of a body (k) relative to a CS K also in uniform co-linear motion is actually graphically refered to a CS at absolute rest rigorously associated to that K and mathematically described by Eq.(1).*

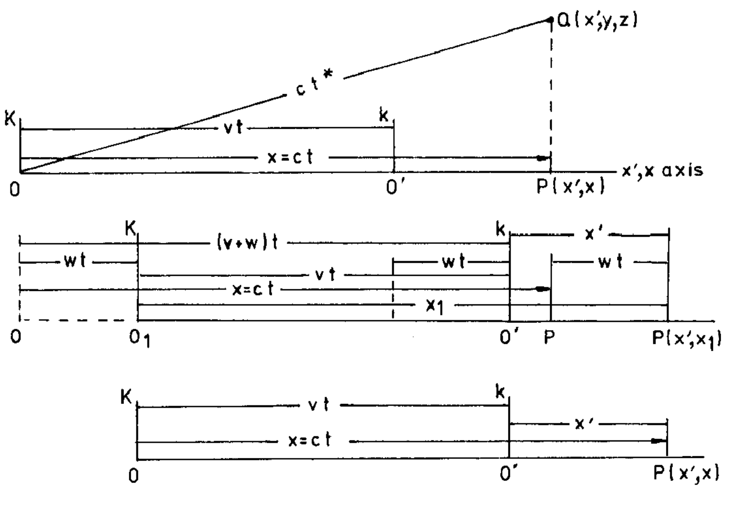

Fig.1

As the radius vector of P is the same in the upper and bottom diagrams, the paths of the light signals tracing it in the two diagrams are identical, too. These signals leave O simultaneously with the origin of k, reaching P when the last arrives at O'.

B. Consider the diagrams in Fig.2. K is a CS at absolute rest associated to the "stationary" CS by the bottom diagram in Fig.1. k_A , k_B and K, of origins O'_A, O'_B and O, coincide at t=0. The motion of k_B is refered to that of k_A which in its turn moves uniformly. The last three diagrams are obtained as the bottom diagram in Fig.l was. There results that O'_B moves relative to the CS at absolute rest K'_A, associated to k_A, with a velocity u obtained by simplifying the equation

$$u(t-wvt/c^2)=(w-v)t, \qquad (2)$$

which is just that we know as the relativistic law for the composition of parallel velocities -with the difference that u, v and w are now absolute velocities. A light signal leaving O'_A simultaneously with the origin of k_B will reach P in the last diagram at time

• Einstein conceived "stationary" CSs (later named inertial CSs [3], [4]) associated with "stationary" spaces [1] to be exclusively in uniform rectilinear motion and a stationary CS [1] to be at rest in a "stationary" space. This because he considered in [1] that "no properties of phenomena attach to the idea of absolute rest".

$t'=t-wvt/c^2$

when the last reaches the point O'_B moving with velocity u. As

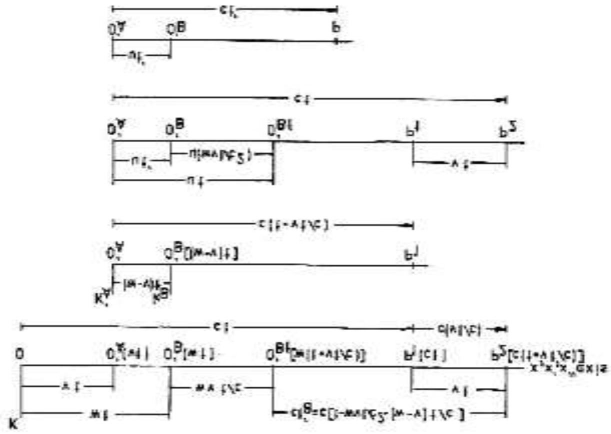

Fig.2

$ut'=(w-v)t$,

from the last diagram in Fig.2 we have the equations

$x'=x-vt,\ y'=y,\ z'=z,\ t'=t-vx/c^2$, (3)

with $x=wt$, relating the translatory motion of constant velocity u of an object (k_B) relative to a CS K'_A at absolute rest, associated to a "stationary" CS k_A, to the translatory motion of constant velocity w of that object relative to another CS K at absolute rest. The last of Eqs.(3) is not the time-equivalent of the first. Eqs.(1) and (3) did not assume the constancy of the speed of light relative to "stationary" CSs, but relative to empty space••.

2.2. Adding light travel times as scalar quantities

A. For parallel light signals

It is well known that whenever a light signal travels a sequence of co-linear segments OA_1, $A_1A_2,\dots, A_{n-1}A_n$ in empty space, the time in which it travels the line segment commencing at O, viz.

$OA_n = OA_1+A_1A_2+\dots+A_{n-1}A_n$ (4)

is obtained dividing Eq.(4) by c. As the time in which a light signal travels any line segment is the difference between the times indicated by synchronous clocks located at its endpoints at the arrival of that light signal, we have

$t(OA_n)=t(OA_1)+t(A_1A_2)+\dots+t(A_{n-1}A_n)$, (5)

with $t(OA_n)=OA_n/c=t(A_n)-t_O$, $t(OA_1)=OA_1/c=t(A_1)-t(O)$, $t(A_1A_2)=A_1A_2/c=t(A_2)-t(A_1),\dots,t(A_{n-1}A_n)=A_{n-1}A_n/c= t(A_n)-t(A_{n-1})$. Eq.(5) is the time-equivalent of (4).

B. For crossing light signals

Consider the origin of k in Fig.3 to leave the origin O of K -moving uniformly along the x', x axis- simultaneously with the light signal tracing the radius vector of Q. Whenever co-linear line segments -like OO' and O'P- depend on travel times of non-co-linear light signals -like those tracing OQ and O'Q-, we need to convert those travel times to travel times of light signals travelling along one and the same direction we name t*ime-axis*. This because line segments like OO' and O'P are covered by the projections onto the x', x axis (more generally onto the direction of motion of k) of the tips of the light signals tracing the radii vectors of a geometrical point Q relative to K (OQ) and k(O'Q) at the differing velocities $c\times\cos\alpha$ and $c\times\cos\theta$, respectively, making impossible a passage from

$OO'+O'P=OP$

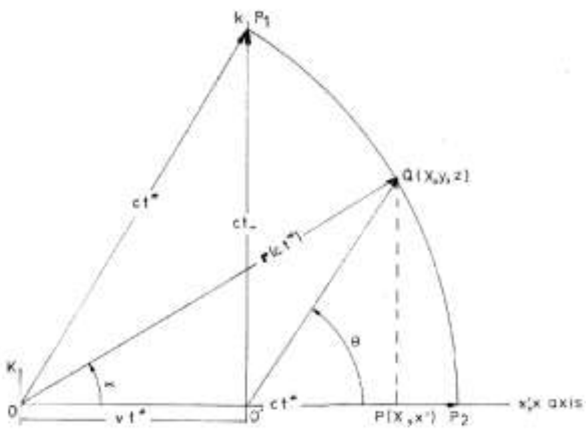

Fig.3

to the time-equivalent

•• Actually Einstein's formulation of the principle of the constancy of the speed of light as [1] "Every light ray moves in the 'stationary' CS with the fixed velocity c, independently of whether this ray is emitted by a stationary or a moving body" also refers to the speed of light in empty space. The velocities $c\pm v$ are seen by observers in CSs "at rest" which combine mathematically the independent motions of a light signal and k.

$OO'/c+O'P/c=OP/c$
analogously to that from (4) to (5) in Sect.2.2.A. Or only this way light travel times like OO'/c and $O'P/c$ can be graphically added as scalar quantities in a theory joining CSs in uniform rectilinear motion by light signals.
The geometry of Fig.3 shows that a time axis is always orthogonal to **v**. From the right triangle OP_1O' we have
$t^*=\beta t\_$ (6)
where $\beta=(1-v^2/c^2)^{-1/2}$. This procedure is consistent with the time-equation of the vector LT which time t is defined by $(\mathbf{r}.\mathbf{v})/cv$. Laying O'O on the time-axis is straightforward. That of O'P will be given in Sect.5 below.

3. Einstein's 1905 operational derivation of LT
In Sect.I.2 of [1] Einstein took into account the uniform rectilinear motion of k relative to a stationary CS K in a "stationary" space and, in view of the diagram in Fig.4, calculated what he called to be the time of k ,τ, in terms of the time of K, t. Resuming, Einstein associated in this aim the time

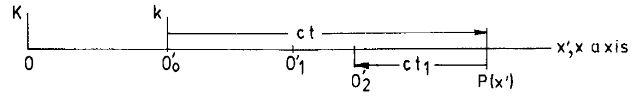

Fig.4

$\tau_O=\tau[0,0,0,t]$
to the departure of a light signal from the origin of k (O'_o in Fig.4) along the common x', x axis towards the P(x') at rest in k, the time
$\tau_P=\tau[x',0,0,t+x'/(c-v)]$
to its reflection at P(x') back to the origin of k, and the time
$\tau'_O=[0,0,0,t+x'/(c-v)+x'/(c+v)]$
to its arrival at the position O'_2 of the origin of k, then wrote the equation
$\tau_O+\tau'_O=2\tau_P$,
defining clocks running in synchrony. By choosing x' infinitely small, he obtained the equation
$\partial\tau/\partial x'+[v/(c^2-v^2)]\partial\tau/\partial t=0$,
and on its integration the time equation
$\tau=a[t-vx'/(c^2-v^2)]$, (7)
with a defined as a function $\phi(v)$, later shown to have unity value. Since Einstein obtained (7) provided that $\tau=t=0$ [implying $x'=x=0$ by Eq.(1)], his true working diagram was, by the change of CS in Fig.1, the upper diagram in Fig.5 with K a CS at absolute rest. At time t=0, the origin O' of k and the light signal tracing the radius vector of the fixed point P(x') in k leave the origin O of K, reaching O'_1 and P at time t. Reflected at P, that light signal reaches the origin of k at O'_2 in time t_1. Inserting (1) in (7) and putting

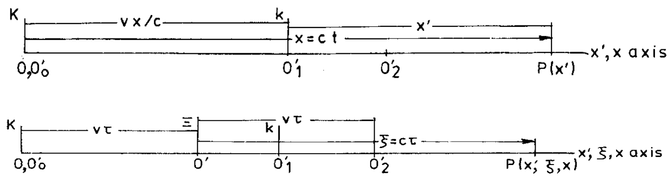

Fig.5

$\xi=c\tau$, (8)
Einstein finally should obtain the set of equations
$\xi=\beta^2(x-vt)$, $\eta=\beta y$, $\zeta=\beta z$, $\tau=\beta^2(t-vx/c^2)$, (9)
and not the linear equations in β he wrote either by merely dropping a factor β or by including that β in x, y, z and t in (9) with no physical explanation [Einstein's 1st "trick"]•••. Specific to Eqs.(9), as well as to their counterpart linear in β, is that the last of them is the time-equivalent of the first. In order to change the set of linear equations in β to LT, Einstein broke this equivalence merely by putting x=wt in them [Einstein's 2nd "trick"].

4. The coordinate system Ξ at absolute rest
Einstein's extrapolation of the validity of the equation defining synchronous clocks at rest in a CS at absolute rest to clocks at rest at O', P and O'_2 in the "stationary" CS k in the upper diagram in Fig.5 [Einstein's 3rd "trick"] –with the a priori ignorance of an eventual effect that motion could have upon their machineries, ignorance in accordance with the principle of relativity but dropped as concerned the time dilation- leads, in addition to (7), to:
i) $O'_oP=x=ct=\xi+v\tau$ (10)
by inserting (1) in (7) then (7) in (8),
ii) $O'_oO'_2=2v\tau$
by inserting $O'_oO'_1=vx'/(c-v)$, $O'_1O'_2=vx'/(c+v)$ in $O'_o O'_2=O'_o O'_1+ O'_1O'_2$ and, by laying $O'_oO'= O'O'_2= O'_oO'_2/2$ and O'P upon the x', x axis, to the bottom diagram in Fig.5, associating the CS Ξ (of origin O') at absolute rest to the "stationary" CS k. Thus, by $O'PO'=2c\tau$, Einstein's 3rd "trick" was actually useful to define the

••• Against Prokhovnik, who claimed in [5] that Einstein included that β in a function $\psi(v)=\beta\phi(v)$, there is no $\psi(v)$ in [1]. Moreover, as $\phi(v)$ in Einstein's linear equations in β is just that he associated with (7), as well as that he later determined, it is evident that he did not (actually he could not do it in any way) embody a β factor in $\phi(v)$.

work in synchrony of the clocks located at O' and P in Ξ. As concerns Eqs.(9), they relate coordinates defined relative to the CSs Ξ and K at absolute rest.

5. Physical reasons validating Einstein's 1st "trick"

Consider the diagram in Fig.6. P(X) and $P_1(\beta X)$ are fixed points in k. At time t=0, the origin of k and the light signal travelling OP(X) leave the origin O of K. At time T, they reach, respectively, O'_o and P(X). We lay the bottom diagram in Fig.5 at O'_o on the time-axis. For the reason leading to (6) in Sect.2.2.B we have

$T=\beta t,\ X=cT=\beta ct=\beta x,\ OO'_o=vT=v\beta t.$ (11)

We need further to remove the dependence of $O'_oP(X)$ (O'P in Fig.3) on the time in which light travels O'_oQ (yet an ignored issue). To this end, by Eqs.(10) and (8), we determine ξ and τ in terms of X and T as

$\xi=\beta(X-vT),\ \eta=y,\ \zeta=z,\ \tau=\beta(T-vX/c^2).$ (12)

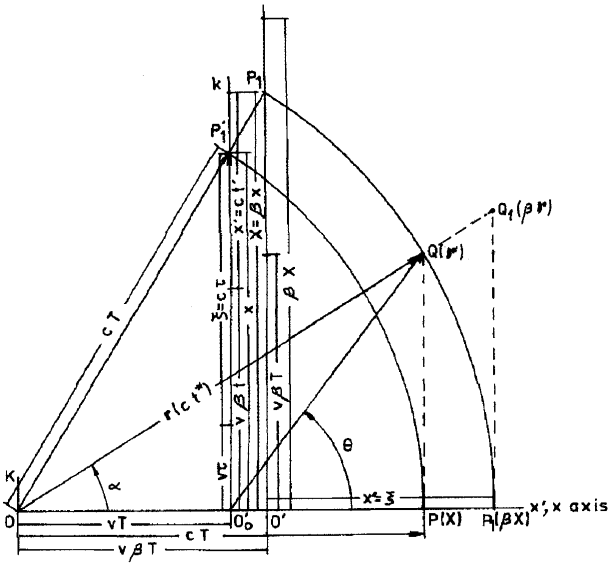

Fig.6

For X, T given by (11) and y, z changed to βy, βz, Eqs.(12) identify with (9). So we identified a point $Q_1(\beta\mathbf{r})$ which coordinate ξ (the same with respect to both k and Ξ) is a line segment traveled by a light signal in a time independent of the time $O'_o\,Q_1/c$ in which a light signal travels $O'_o\,Q_1$. We can pass now from Eqs.(9) to equations linear in β, viz

$\xi=\beta(x-vt),\ \eta=y,\ \zeta=z,\ \tau=\beta(t-vx/c^2),$ (13)

by looking at x in (13) not as to the coordinate of P(X), the projection of $Q(\mathbf{r})$ onto the x', x axis, but as to the coordinate of a point $P_1(\beta X)$, the projection of $Q_1(\beta\mathbf{r})$ onto the same axis, sited β times far from O, and at βy, βz as to the other coordinates of $Q_1(\beta\mathbf{r})$. Yet the first and the fourth of Eqs.(13) are, respectively, of type (4) and (5).

6. Physical reasons validating Einstein's 2nd "trick": The Lorentz transformation

Let us consider the Q's (implicitly their projections P) in Fig.6 to move relative to the CS k also in translatory motion relative to K. We are in the case considered in Sect.2.1.B above. So that we must pass from a description of the motion of Q relative to the "stationary" CS k to one with respect to the CS K'_A at absolute rest associated to k_A in Sect.2.1.B. We identify the projection P of Q with the origin of k_B. By the last diagram in Fig.2 we have, analogously to Eqs.(3), Eqs.(13) with x=wt. Thus, like the 1st and the 3rd "tricks", Einstein's 2nd "trick", changing the set of equations (13) to LT by the additional equation, proves to be physically well founded. Unlike the 4th of Eqs.(13), the 4th of the LT equations is not the time equivalent of the equation in ξ and x. LT just relates mathematically "stationary" observers in a theory connecting them by light signals as STR is.

7. Vector Lorentz transformation

Consider the diagram in Fig.7. k moves rectilinearly with constant velocity v relative to K along the direction $\mathbf{v}_o=\mathbf{v}/v$. A light signal traveling OP in time T is used, just like in Sect.5 above (O'P' playing the role of time-axis), to remove the dependence of OP and O'P on t* and O'Q/c, respectively, by passing from Q and O' to Q_1 and O'_1 with $OP_1=\beta OP$ and $OO'_1=\beta OO'$. From the right triangles $O'Q_1P_1$ and OQP we have

$\mathbf{r}'=\mathbf{Q_1P_1}+\mathbf{O'_1P_1}$

with

$\mathbf{Q_1P_1}=\mathbf{r}-(\mathbf{r}.\mathbf{v}_o)\mathbf{v}_o$

and

$\mathbf{O'_1P_1}=\mathbf{OP_1}-\mathbf{OO'_1}=\beta[(\mathbf{r}.\mathbf{v}_o)\mathbf{v}_o-\mathbf{v}T],$

that by noting $t'=(r'.\mathbf{v}_o)/c$ and $T=(r.\mathbf{v}_o)/c$, provides the vector LT as

$\mathbf{r}'=\mathbf{r}-(\mathbf{r}.\mathbf{v}_o)\mathbf{v}_o+\beta[(\mathbf{r}.\mathbf{v}_o)\mathbf{v}_o-\mathbf{v}T],\ t'=\beta[T-(\mathbf{r}.\mathbf{v})/c^2].$ (14)

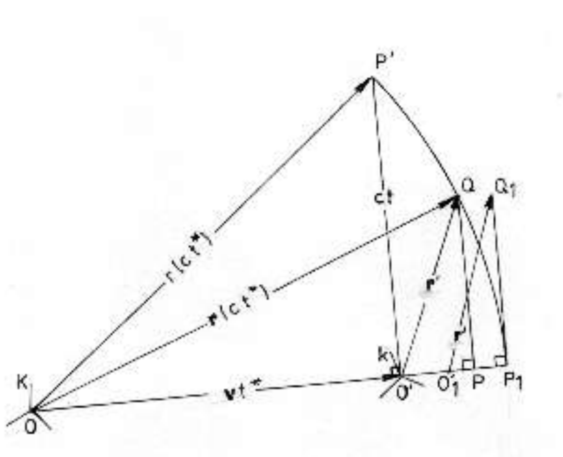

Fig.7

8. Group properties

The main requirement for a set of transformations to form a group is that they to accomplish the transitivity property which stipulates that, successively performed, any two of them engender an equivalent one. To prove operationally that co-linear and non-co-linear LTs form a group mainly amounts to identify one or more light signal which -just for the reason for which those tracing OP in Figs.6, 7 led, respectively, to the standard and the vector LT, and in an analogous manner that tracing OP in Fig.8 (OP_B in Fig.9) leads to the vector LT

$\mathbf{r}''=\mathbf{r}-(\mathbf{r}.\mathbf{w}_o)\mathbf{w}_o+\gamma[(\mathbf{r}.\mathbf{w}_o)\mathbf{w}_o-\mathbf{w}T]$, $t''=\gamma[T-(\mathbf{r}.\mathbf{w})/c^2]$, (15)

where $\mathbf{w}_o=\mathbf{w}/w$, $\gamma=(1-w^2/c^2)^{-1/2}$ and $T=(\mathbf{r}.\mathbf{w}_o)/c$- to imply a new vector LT, related to (14) and (15) and similar to them, by tracing $O'_{If}P_{IB}$ and $O'_{If}P_C$ (O'_{If} being the origin of the CS K'_A at absolute rest associated to k_A as in Sect.2.1.B) in Figs.8 and 9, respectively. These signals will leave O'_{If} when O'_{If} and the origin of k_B in Fig.8 (k'_B in Fig.9)coincide, and will reach P_{IB} in Fig.8 (P_{If},P_C in Fig.9) simultaneously with the light signal leaving O together with the origins of k_A and k_B, when the origin of k_B reaches O'_{IB} in Fig.8 ($O'_{IB'}$, O'_{IB} in Fig.9).

A. For co-linear LTs

For the co-linear LTs (14), (15) we have from Fig.8

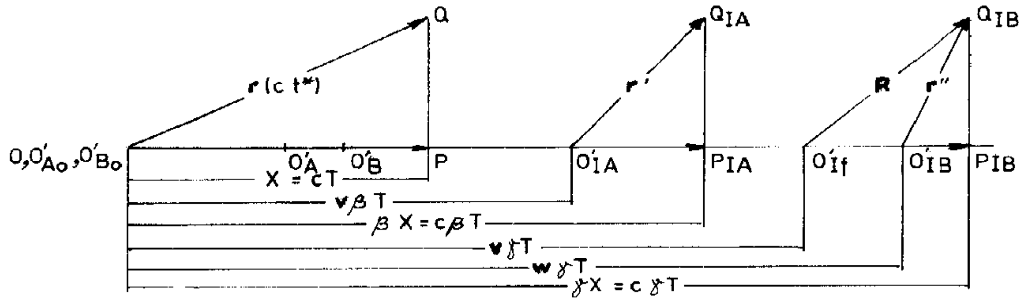

Fig.8

$\mathbf{r}''=\mathbf{R}-\mathbf{O}'_{If}\mathbf{O}'_{IB}$

with

$O'_{If}O'_{IB}=w\gamma T-v\gamma T$,

$(\mathbf{R}.\mathbf{w}_o)=\gamma X-v\gamma T=\gamma(X-vT)=\gamma\beta^{-1}x'$,

where x' is just ξ in (12), and

$x''=(\mathbf{r}''.\mathbf{w}_o)=\gamma\beta^{-1}x'-(w-v)\gamma T=\gamma\beta^{-1}x'-\gamma(w-v)\beta t'-\gamma(w-v)\beta vx'/c^2=\gamma\beta^{-1}x'[1-(w-v)v/(c^2-v^2)]-\gamma\beta(w-v)t'=$
$\gamma\beta(1-wv/c^2)x'-\gamma\beta(w-v)t'$,

where t' is just τ in (12), u is that given by Eq.(2) and $\delta=(1-u^2/c^2)^{-1/2}$. By $\mathbf{v}_o||\mathbf{w}_o||\mathbf{u}_o$, with $\mathbf{u}_o=\mathbf{u}/u$, the relationships

$\gamma\beta(1-wv/c^2)=\delta$, $\gamma\beta(w-v)=\delta u$ (16)

and $(\mathbf{r}'.\mathbf{u}_o)=ut'$, we get the new vector LT

$r''=\mathbf{r}'-(\mathbf{r}'.\mathbf{u}_o)\mathbf{u}_o+\delta[(\mathbf{r}'.\mathbf{u}_o)\mathbf{u}_o-\mathbf{u}t']$, $t''=\delta[t'-(\mathbf{r}'.\mathbf{u})/c^2]$, (17)

where $t'=(\mathbf{r}'.\mathbf{u}_o)/c$ and $t''=(\mathbf{r}''.\mathbf{u}_o)/c$, which relates position vectors of geometrical points relative to k_B and k_A. Thus the transitivity condition is proved for co-linear LTs.

B. For non-co-linear LTs

Showing operationally that the non-co-linear LTs (14), (15) -also implied by the light signals tracing, respectively, OP_A, OP_B in Fig.9- form a group requires for a CS k'_B parallel to k_B which to cover in the time T a distance equal to $OO'_AO'_B$ along OP_A at a constant velocity *w*, as well as for an identical CS k''_B which to start moving along $O'_AO'_B$ when the origin of k'_B reaches O'_A. So we can pass from the relative velocity $|\mathbf{w}-\mathbf{v}|$ to the relative velocity *w*-v by

$|\mathbf{w}-\mathbf{v}|T=(w-v)T$,

and from the motion of k_B relative to k_A to its motion relative to the CS K'_A at absolute rest, associated to k_A in Sect.2.1.B above, by

$(w-v)T=(T-wvT/c^2)u\mathbf{u}_o$

with $u=(w-v)/(1-wv/c^2)$ and $\mathbf{u}_o=(\mathbf{w}-\mathbf{v})/|\mathbf{w}-\mathbf{v}|$ (Rewritten as

$\mathbf{w}-\mathbf{v}=(w-v)\mathbf{u}_o$, (18)

this equation expresses the true relativistic law for the composition of non-parallel velocities••••.)

At the times βT, γT [with $\gamma=(1-w^2/c^2)]^{-1/2}$ the light signals leaving O simultaneously with k_A, k_B and k'_B reach, respectively, P_{IA} and P_{If}, P_{IB}, while the origins of k_A and k_B arrive, respectively, at O'_{IA} and O'_{If},O'_{IB}. In accordance with Sect.2.1.B, O'_{If} is the origin of the CS K'_A at absolute rest at time γT. By the above definition of k'_B and k''_B, the origin of k'_B finds at time γT at a distance equal to $O'_{If}O'_{IB}$ from O'_{If} along OP_{If}, namely at $O'_{IB'}$ in Fig.9. The light signals leaving O'_{If} simultaneously with the origins of k'_B and k''_B will travel equal distances along the directions of motion of k'_B and k''_B, viz.

$O'_{If}P_{If}=O'_{If}P_C$.

Since $O'_{If}P_{If}$ is the projection of $\mathbf{R}$ onto the direction of $\mathbf{v}$, $O'_{If}P_C$ will be the projection of a vector $\mathbf{R}'$ of magnitude $|\mathbf{R}|$ that makes with $\mathbf{u}_o$ an angle equal to that $\mathbf{R}$ makes with $\mathbf{v}$. From

$O'_{If}P_{If}=(\mathbf{R}.\mathbf{v}_o)=\gamma[(\mathbf{r}.\mathbf{v}_o)-vT]$,

and equation

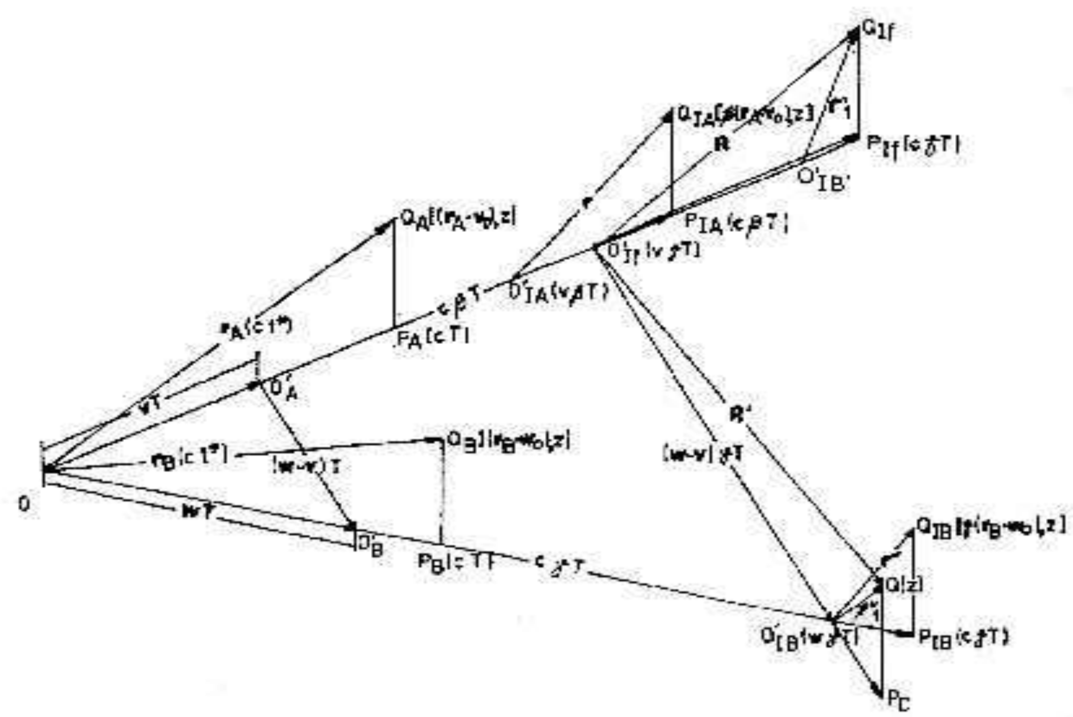

Fig.9

$(\mathbf{r}'.\mathbf{v}_o)=\beta[(\mathbf{r}.\mathbf{v}_o)-vT]$,

resulting from the first of Eqs.(14), we have

$(\mathbf{R}.\mathbf{v}_o)=\gamma\beta^{-1}(\mathbf{r}'.\mathbf{v}_o)$

with

$$(\mathbf{R}.\mathbf{v}_o)\mathbf{u}_o=\gamma\beta^{-1}(\mathbf{r}'.\mathbf{v}_o)\mathbf{u}_o \quad (19)$$

along $\mathbf{u}_o$.

By inserting (19), the inverse of the last of Eqs.(14) and Eq.(18) in $(\mathbf{R}.\mathbf{v}_o)\mathbf{u}_o-\mathbf{u}'\gamma T$, we obtain

$(\mathbf{R}.\mathbf{v}_o)\mathbf{u}_o-(\mathbf{w}-\mathbf{v})\gamma T=\gamma\beta^{-1}(\mathbf{r}'.\mathbf{v}_o)\mathbf{u}_o-\mathbf{u}_o(w-v)\gamma\beta t'-\mathbf{u}_o(w-v)\gamma\beta v(\mathbf{r}'.\mathbf{v}_o)/c^2=\gamma\beta^{-1}(\mathbf{r}'.\mathbf{v}_o)\mathbf{u}_o[1-(w-v)v/(c^2-v^2)]-\mathbf{u}_o(w-v)\gamma\beta t'=$
$\gamma\beta(1-wv/c^2)(\mathbf{r}'.\mathbf{v}_o)\mathbf{u}_o-\mathbf{u}_o(w-v)\gamma\beta t'$.

In view of Eqs.(16), also valid for w, we have

$O'_{IB}P_C=(\mathbf{R}.\mathbf{v}_o)\mathbf{u}_o-(\mathbf{w}-\mathbf{v})\gamma T=\delta[(\mathbf{r}'.\mathbf{v}_o)-ut']\mathbf{u}_o=\delta[(\mathbf{r}'.\mathbf{u}_o)\mathbf{u}_o-\mathbf{u}t']$.

Because $Q_{If}P_{If}=Q_{IB}P_{IB}=QP_C$

by virtue of

$Q_AP_A=Q_BP_B$,

and

$|\mathbf{r}''_1| = |\mathbf{O}'_{IB'}\mathbf{Q}_{If}| = |\mathbf{O}'_{IB}\mathbf{Q}|$

with

••••The standard relativistic formula for the composition of non-parallel velocities [6] -that predicted the famous, experimentally unproved [7], Thomas precession- is physically non-true merely because ignored that the uniform translatory motion of an object relative to a CS also in uniform translatory motion is (as shown in Sect.2.1.A) refered to a CS at absolute rest associated to the last.

$\mathbf{O'}_{IB}\mathbf{Q}=\mathbf{QP}_C+\mathbf{O'}_{IB}\mathbf{P}_C$,
we have, respectively,
$\mathbf{QP}_C=\mathbf{r}'-(\mathbf{r}'.\mathbf{u}_o)\mathbf{u}_o$
and

$$\mathbf{r}''_1=\mathbf{r}'-(\mathbf{r}'.\mathbf{u}_o)\mathbf{u}_o+\delta\,[(\mathbf{r}'.\mathbf{u}_o)\mathbf{u}_o-\mathbf{u}t'],\; t''=\delta[t'-(\mathbf{r}'.\mathbf{u})/c^2] \qquad (20)$$

where $t''=(\mathbf{r}''_1.\mathbf{u}_o)/c=(\mathbf{r}''_1.\mathbf{w}_o)/c$.
Similar to the initial LTs (14) and (15), the resulting vector LT (20) proves that the non-co-linear LTs satisfy the transitivity property, forming a group without requiring rotations of "stationary" (inertial) CSs in this aim.

9. Conclusions

1°. Unlike the ordinary time-dependent coordinate transformations, LT can be written *only when* the radius vector of a geometrical point in a “stationary” space is physically traced by a light signal emitted by an observer "at rest" relative to that space before to be traced with a pencil in a graph.
2°. LT relates coordinates of an object moving uniformly and rectilinearly with absolute velocity u (known us as “relativistic” velocity) relative to the CS K'_A (at absolute rest) –associated to the “stationary” CS k- to those defined relative to the CS at absolute rest –associated to the “stationary” CS of an observer-, identical with those defined with respect to the last and the “stationary” CS k.
3°. A peculiarity of operational origin of the LT is that the spacial coordinates and the times appearing in its inverse refer to a different event. This because the CS Ξ associated to the moving K by $\xi=\beta^2x$ differs from the CS Ξ associated to the moving k by $\xi=\beta^2x'$ [predicted by (8) in view of (7) and (3)].
4°. The mixture of spacial coordinates and times in LT originates just in connecting "stationary" CSs by light signals. That of the "space" and "time" components of the four-quantities in their dependence on spacial coordinates and times mixed by LT. As concerns the newtonian concepts of space and time, they keep -by virtue of our results- their original meaning, in deep agreement with the everyday common sense experience which ,for about a century, underwent unjustified killing attacks.
5°. The operational method strictly makes of the minkowskian metric a relationship by which any observer "at rest" can experimentally read a time measured by any observer in uniform rectilinear motion relative to him after the last converted that time into a light signal travelling along his time-axis.
6°. Succeeding in rigorously applying the operational method to prove that the *non-co-linear LTs form a group without requiring rotations of inertial CSs in this aim*, we strengthen the validity of LT, implicitly that of STR.
7°. Consistent with the corpus of classical physics, STR predicts by LT no true Lorentz-FitzGerald contraction, as well as no true time dilation. Experiments proved the existence of larger lifetimes for faster moving particles, a result in direct relation with one proving that [8] mass is the coupling “constant” between spinning clouds of subquantum particles constituting them.
8°. The operational derivation of LT proves that the principle of the physical determination of the equations actually works in STR and should work in any relativistic theory as well.
9°. Disclosing the CS Ξ at absolute rest in Einstein's 1905 derivation of LT, and relating in consequence physical quantities measured in “stationary” CSs to quantities defined in CSs at absolute rest by LT, we prove that Einstein’s theory of special relativity has actually nothing in common with the almighty misleading relativism governing our century. By refusing the absolute truth and the natural way to reach it under convergent accumulations of information, this relativism also refuses any absolute value, weakening both the scientific and common thinking (in physics this meant to claim that the principle of the physical determination of the equations does not work in STR and relativistic quantum mechanics, strongly altering particularly the experimental investigation of the subquantum structure of matter with major technological applications [9]) and blowing up the foundations of the political, economical and social lives of the mankind. The absolute truth has nothing in common with claims and relativistic truths rendered absolutes by personal wishes/interests, as well as with the absolutisms that the last always raised and will further raise under scientific ignorance.

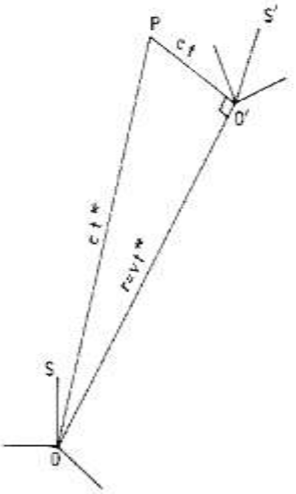

Fig.10

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