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ABERRATION AND LUMINOSITY-DISTANCE

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Abstract

It is currently accepted in cosmology that due to nebular recession luminosity-distance d_L is related to actual distance d_0 by

$$d_L = (1+z)d_0$$

Here z is redshift. This relation originated from Tolman(1930)and was used in early observational work by Hubble on the nature of the red-shift. The assumptions underlying this formula were questioned by Vogt (1937) from whose work it would follow that

$$d_L = d_0$$

However, after a paper by Robertson (1937) the existence of the factor (1+z) has always been accepted as correct and not questioned further. The writer however maintains that Vogt was right, Tolman and Robertson having entirely omitted to consider aberration. The consequences of this being so could be important for cosmology since the Hubble diagram may need reinterpretation.

1. Luminosity-distance and the Tolman reduction factor.

The standard method of estimating distances in astronomy is by the relation between observed and estimated luminosity. A stationary source having absolute luminosity L placed at a distance d will have apparent luminosity l given by the inverse square law

$$l = L/4\pi d^2 \qquad (1.1)$$

L is the total luminous flux (rate of emission of luminous energy) and l the luminous flux per unit perpendicular area received at a distance d from the source. If L can be estimated from knowledge of the type of the source observed, then d can be calculated since l is directly measurable. This gives the *luminosity distance* d_L.

As first pointed out by Tolman (1930), if the source is receding in the line of sight then the luminosity actually observed will not be l but a reduced value owing to:

- *The number effect*: the reduction in the number of photons arriving because of the lengthening of the travel path of a receding nebula.

This reduces incident radiation by a factor (1+z) where z is red-shift.

- *The energy effect*: the energy of photons arriving is reduced because redshift lowers their frequency. The reduction of incident radiation is by a further factor (1+z).

The reduced value of luminosity observed, l_{obs} say, is given by:

$$l_{obs} = l/(1+z)^2 \qquad (1.2)$$

The factor $(1+z)^2$ will be called the *Tolman double reduction factor*. The standard procedure now is to identify d as true distance d_0 at the time of observation and so get *Tolman's formula*:

$$l_{obs} = L/4\pi d_0^2(1+z)^2 \qquad (1.3)$$

from which luminosity distance is identified as

$$d_L = (1+z)d_0 \qquad (1.4)$$

- a formula which has become part of accepted theory.

We shall maintain that, although it is correct to use the Tolman double reduction factor (1.2), the Tolman formula (1.3) and the formula for luminosity distance (1.4) are incorrect.

2. Vogt's criticism

In the early days the Tolman double reduction factor was used to test the hypothesis of the expanding universe (Hubble & Tolman 1935). If redshift were not a Doppler effect, only the frequency effect with a luminosity reduction by a single factor (1+z) would occur. Hubble's examination of the data gave no grounds for believing in the need for the double factor after which he had doubts about the expansion hypothesis and searched for possible explanations (Hubble 1936).

Soon afterwards, Vogt (1937) wrote a short note pointing out the necessity of taking into account the motion of the source from the time of light emission. He found that the appropriate magnitude correction would exactly cancel out the effect of the Tolman's double factor.

This same conclusion follows more simply from the obvious principle that received light is observed to be propagated from the point of emission regardless of any subsequent movement of the source

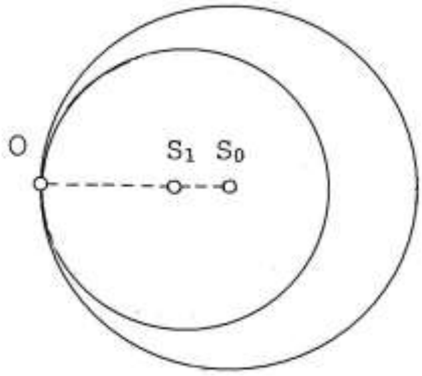

Fig 1

Consider fig.1. Let O denote the position of the observer, S_1 the position of the source (nebula) when the light was emitted and S_0, its present position. On reception the wave front of emitted light will be a sphere centre S_1 at distance d_1 not the sphere centred at S_0 at the true distance d_0. It is d_1 which must be used in the inverse square law.

Now the relation between d_0 and d_1 on the assumption of uniform motion of the source is

$$d_0/d_1 = (c+v)/c = (1+z) \tag{2.1}$$

This will be approximately satisfied if the motion is nonuniform.

On identifying d in the inverse square law as distance d_1 at time of emission, there follows

$$d_L = (1+z)d_1 = d_0 \tag{2.2}$$

Thus for a receding source, even though the Tolman reduction in observed luminosity occurs, the luminosity distance estimated from the inverse square law is still the true distance at time of observation.

3. **Robertson's argument and aberration**

In 1937 Robertson published a paper whose main aim was to disprove arguments brought by Vogt and others and to re-establish Tolman's formula on the basis of relativity theory. Though his arguments used both the General and the Special Theory of Relativity, his principal argument against Vogt was a very simple one phrased in the Special Theory which is what we now consider.

Robertson argued as follows (in our notation). The luminosity relative to the source S is

$$l = \frac{L}{4\pi d^2} \tag{3.1}$$

so that using Einstein (1905),an observer O moving with velocity v will assign to it a value

$$l. \frac{\sqrt{(1 - v/c)}}{\sqrt{(1 + v/c)}} \approx \frac{1}{(1+z)^2} \qquad (3.2)$$

So observed luminosity from O is, in agreement with Tolman's formula

$$l = \frac{L}{4\pi d^2 (1+z)^2} \qquad (3.3)$$

The error here is that aberration has been ignored. No mention was made of aberration by Tolman, Robertson or any of their co-workers. That aberration could play any part in observations of nebulae and other distant objects was apparently never considered, which is understandable since aberration occurs only with motions across the line of sight and for motions truly along the line of sight no aberration of direction is possible.

Nevertheless, aberration does affect the observation since light received is never strictly parallel. The light emitted from a point source in the nebula is always received over a certain area, i.e. the telescope mirror. If a ray is received a distance b off-centre as in fig.2 it is inclined to the line of sight at an angle

$$\theta_0 = b/d_0 \qquad (3.4)$$

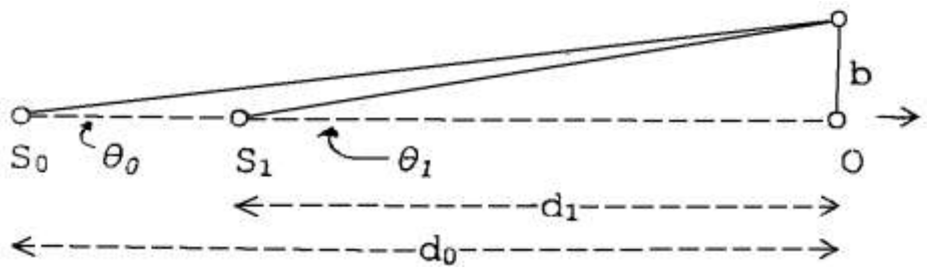

Fig.2

Using Einstein's 1905 paper again we find that in O's frame the ray will be observed travelling at an angle θ_1 given by

$$\cos \theta_1 = \frac{\cos \theta_0 - v/c}{1 - \cos \theta_0 . v/c} \qquad (3.5)$$

This equation is easily transformed to

$$\sin \theta_1 = \frac{\sin \theta_0}{\gamma (1 - \cos \theta_0 . v/c)} \qquad (3.6)$$

where

$$\gamma = 1/\sqrt{(1-v^2/c^2)} \qquad (3.7)$$

from which, for small angles, we get

$$\frac{\theta_1}{\theta_0} = \frac{\sqrt{(1-v^2/c^2)}}{(1-v/c)} = \frac{\sqrt{(1+v/c)}}{\sqrt{(1-v/c)}} \approx 1+z \qquad (3.8)$$

Observer O sees the light inclined to the line of sight at an angle

$$\theta_1 = \theta_0(1+z) = b(1+z)/d_0 = b/d_1 \qquad (3.9)$$

ie. to the observer O, the light is seen as coming from S_1 consistent with the conclusion from Vogt's comment.

The important point to note in Robertson's calculation is the switch in the frames of reference. So, starting relative to the source, the value of d was initially taken as d_0 and it was allowed to remain at that value after switching to the observer's frame. But, because of aberration, the value of d should have been changed to d_1 on switching frames.

4. Light cones

The effect of aberration is seen more clearly by considering the light cone received from a point source on the telescope mirror. The fig.3 below is drawn from the standpoint of the observer O with the source moving towards the right. The telescope, centred at O, is perpendicular to the line of sight.

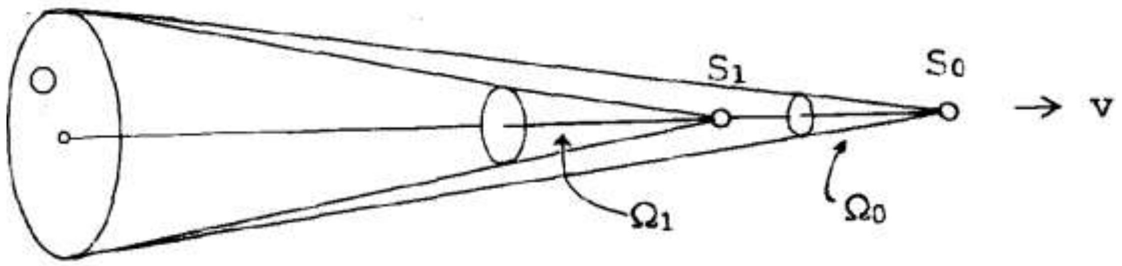

Fig.3

The small solid angle Ω of a light cone is given in terms of the small receptor area A and distance d by

$$\Omega = A/d^2 \qquad (4.1)$$

This takes two values Ω_0, Ω_1 for values d_1, d_2 of d.

1) Relative to observer O

Received light is seen as emitted from vertex S_1 in a cone having solid angle

$$\Omega_{1.} = A/d_1^2 \qquad (4.2)$$

Luminous flux received is, taking into account Tolman reduction,

$$\frac{1}{(1+z)^2}\frac{\Omega_1}{4\pi} L = \frac{1}{(1+z)^2}\frac{A\ L}{4\pi d_1^2} \qquad (4.3)$$

Note that the expression on the right can be interpreted as meaning that the light is spread out over a sphere of radius d_1, part of it falling on area A and reduced by the Tolman factor.

2) Relative to source S

Light is seen emitted in a cone of vertex S_0 having cone solid angle

$$\Omega_{0.} = A/d_0^2 \qquad (4.4)$$

Luminous flux emitted is

$$\frac{\Omega_0}{4\pi} L = \frac{A}{4\pi d_0^2} L \qquad (4.5)$$

Here the expression on the right can be interpreted as meaning that the light is spread out over a sphere of radius d_0, part of it falling on area A without Tolman reduction.

The luminous fluxes calculated relative to observer and relative to source are the same when

$$d_0 = (1+z)\ d_1 \qquad (4.6)$$

5. General Relativity

The relation (4.6) is either true or approximately true in the classical and Special Relativity cases depending on the law of recession. In the General Relativity case though, it is exactly true.

In General Relativity we are more easily led to the neglect of aberration since there exists no theory of aberration similar to that of Special Relativity. It is this fact which apparently led to the original neglect of this factor by Tolman. The situation may be clarified by showing the relation with the classical and the Special Theory cases by means of the light cones of fig.3. For the standard theory in General Relativity, using light cones with the Friedmann model gives values similar to those above (Weinberg 1972).

Let r be coordinate distance and R(t) the radius scale factor. Then (4.6) follows from

$$d_0 = r\,R(t_0),\ d_1 = r\,R(t_1), \qquad (5.1)$$

and the cosmological redshift equation:

$$R(t_0)/R(t_1) = 1 + z \qquad (5.2)$$

Tolman's formula as usually stated is:

$$l_{obs} = \frac{L}{4\pi r^2 R(t_0)^2 (1+z)^2} = \frac{L\ R(t_1)^2}{4\pi r^2 R(t_0)^4} \qquad (5.3)$$

(Robertson 1937, Bondi 1960, Weinberg 1972, Sandage 1988 etc) It is clear from the light cones that this formula should be:

$$l_{obs} = \frac{L}{4\pi r^2 R(t_1)^2 (1+z)^2} = \frac{L}{4\pi r^2 R(t_0)^2} \qquad (5.4)$$

the two expressions on the right being the values calculated relative to observer and relative to source respectively.

Standard derivations of Tolman's formula (e.g. that of Weinberg) make the same mistake as Robertson, i.e. switching frames of reference, making the Tolman correction for luminosity reduction while using the smaller cone angle at S_0.

6. References

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