NEW RESULTS OF FLAT SPACE-TIME THEORY OF GRAVITATION

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Abstract: An explanation of the anomalous acceleration of spacecrafts in the solar system is given. A new explanation of the observed superluminal velocities at extragalactic objects is stated. The extension of quasars can be larger than generally assumed, i.e. quasars must not be very compact objects. The high energy loss per unit time of quasars can be explained. The virial theorem implies for large galaxies in addition to the law of Newton a further expression. A relation between the measured redshifts of stars rotating about the center of the galaxy and their velocities is derived. An upper bound for the radius of rotating galaxies is given. The upper bound for the structures in the cosmic microwave background stated by the satellite WMAP can be explained. All these results are received from cosmological models studied by flat space-time theory of gravitation and by use of post-Newtonian approximations of perfect fluid in these cosmological models where clocks at earlier times go faster than at present implying that the light velocity at distant objects is greater that the vacuum light velocity measured locally by the observer.

1. Introduction

The starting point of the subsequent considerations is a covariant theory of gravitation in flat space-time [1]. It gives the same results as the general theory of relativity to the accuracy needed by the experiment such as: gravitational redshift, light deflection, perihelion precession, radar time delay, post-Newtonian approximation, gravitational radiation, and the precession of the spin axis of a gyroscope in the orbit of a rotating body. Birkhoff's theorem is not valid by this gravitational theory. A summary of flat space-time theory of gravitation with these results can be found in paper [2] where references to detailed studies are given. The theory is also applied to homogeneous, isotropic cosmological models in several papers (see e.g. [3] where further references can be found). There exist non-singular cosmological models in contrast to Einstein's general theory of relativity. Entropy is produced and the space must be flat as recently observed. In these cosmological models a perfect fluid is considered and post-Newtonian approximations are derived [4].

In this paper the post-Newtonian approximation of a perfect fluid in the universe is used to derive several results:

(1) An explanation of the anomalous acceleration of spacecrafts in the solar system is given. This acceleration is opposite to the direction of the velocity of the spacecraft. In particular, for objects moving radially away from the Sun it is in the direction to the Sun. The study of the radio Doppler data of spacecrafts, in particular of Pioneer 10 and 11 by Anderson et al. [5] yields an anomalous acceleration of spacecrafts of about $8 \cdot 10^{-8} cm/s^2$ in the direction to the Sun. The effect was also confirmed by Markwardt [6]. Several explanations of this anomalous acceleration have been given (see e.g. [5] and the cited literature therein) but Anderson et al. [5] conclude that no mechanism or theory can explain this anomalous acceleration. In the meantime new explanations of this effect appeared. Scheffer [7] explains the effect as the sum of several small accelerations which already have been studied by other authors and Marmet [8] illustrates the anomalous acceleration by the variation of the momentum of Pioneer 10 due to the collision with dust in the Kuiper belt.

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(2) Flat space-time theory of gravitation implies that in the universe clocks at earlier times go faster than clocks at present in contrast to those of Einstein's theory. Therefore, an observer states that the light velocity at distant objects is greater than the measured local vacuum light velocity. This gives a new explanation of the observed superluminal velocities in extragalactic objects. This result was already stated in the paper [3]. It also yields that the estimated upper bounds of quasars are too small and the extension of the quasars can be larger, i.e. quasars must not be as compact as generally assumed.
(3) For spherically symmetric objects the virial theorem is derived from the post-Newtonian approximation in the universe. The connection between the measured shift of spectral lines of stars moving around the center of the galaxy and their velocities is given. For small galaxies the virial theorem in the universe implies Newton's law but for large objects in addition to the Newton law a repulsive force acts. Under certain assumptions on the density of the matter distribution in the galaxy flat rotation curves can arise.
(4) An upper bound of the radius of rotating galaxies is received by considering the virial theorem on large scales using the density distribution which implies flat rotation curves.
(5) Considerations on the attractive Newton force and the repulsive force acting on matter from the beginning of the universe till the time of decoupling of matter and radiation (recombination epoch) give an estimate of the upper bound of the structures in the cosmic microwave background which is stated by the satellite WMAP.
(6) The electromagnetic radiation of moving charges in the gravitational field of a distant object is calculated implying that the high energy loss per unit time of quasars can be explained without the assumption of a black hole.

2. Auxiliary Results

In this section previously studied results of flat space time theory of gravitation on non singular, homogeneous, isotropic cosmological models [3] and on post-Newtonian approximations of matter in these cosmological models [4] are summarized.

2.1 Cosmological Model

The metric of the cosmological model is the pseudo-Euclidean geometry

$$(ds)^2 = -\eta_{\nu\mu} dx^\nu dx^\mu \tag{2.1}$$

with

$$\eta_{ij} = diag(1,1,1,-1) \tag{2.2}$$

where $x^i \, (i = 1,2,3)$ are the Cartesian coordinates and $x^4 = ct$ (t: absolute time). The proper time τ is measured by atomic clocks defined by

$$c^2 (d\tau)^2 = -g_{\nu\mu} dx^\nu dx^\mu \tag{2.3}$$

where

$$g_{ij} = \text{diag}\,(a^2(t),\ a^2(t),\ a^2(t),\ -1/h(t)) \qquad (2,4)$$

The four-velocity is given by

$$u^i = 0 \ (i = 1, 2, 3), \qquad u^4 = c\frac{dt}{d\tau} \qquad (2,5a)$$

with

$$d\tau = dt / \sqrt{h(t)}. \tag{2.5b}$$

The energy-momentum tensors of matter (dust), radiation and additional matter have the form

$$\overset{M}{T}{}^i{}_j = (p_m + p_r + p_a)c^2,\quad i = j = 1,2,3$$
$$= -(\rho_m + \rho_r + \rho_a)c^2,\quad i = j = 4$$
$$= 0,\qquad i \neq j \qquad (2.6)$$

with the equations of state

$$p_m = 0,\quad p_r = \frac{1}{3}\rho_r,\quad p_a = \rho_a \qquad (2.7a)$$

implying

$$\rho_m = \rho_{m0}/h^{1/2},\quad \rho_r = \rho_{r0}/(ah^{1/2}),\quad \rho_a = \rho_{a0}/(a^3 h^{1/2}) \qquad (2.7b)$$

where ρ_{m0}, ρ_{r0} and ρ_{a0} denote the present densities of matter, radiation and additional matter. Put

$$\kappa = 4\pi k / c^4 \qquad (2.8)$$

where k is the gravitational constant then the energy-momentum tensor of the vacuum with cosmological constant Λ is given by

$$\overset{\Lambda}{T}{}^i{}_j = -\frac{\Lambda}{2\kappa} a^3/h^{1/2},\quad i = j = 1,2,3,4$$
$$= 0,\quad i \neq j \qquad (2.9)$$

and that of the gravitational field

$$\overset{G}{T}{}^i{}_j = \frac{1}{16\kappa} L_G,\quad i = j = 1,2,3$$
$$= -\frac{1}{16\kappa} L_G,\quad i = j = 4$$
$$= 0,\quad i \neq j \qquad (2.10)$$

where

$$L_G = \frac{1}{c^2} a^3 h^{1/2}\left(-6\left(\frac{\dot a}{a}\right)^2 + 6\frac{\dot a}{a}\frac{\dot h}{h} + \frac{1}{2}\left(\frac{\dot h}{h}\right)^2\right). \qquad (2.11)$$

Here, the dot denotes the $t-$derivative. Let

$$a(0) = h(0) = 1,\quad \dot a(0) = H_0,\quad \dot h(0) = \dot h_0 \qquad (2.12)$$

be the conditions at present time $t_0 = 0$ where H_0 is the well-known Hubble constant and $\dot h_0$ is an additional constant which is zero for Einstein's theory. Introducing the density parameters

$$\Omega_m = \frac{8\pi k \rho_{m0}}{3H_0{}^2},\quad \Omega_r = \frac{8\pi k \rho_{r0}}{3H_0{}^2},\quad \Omega_a = \frac{8\pi k \rho_{a0}}{3H_0{}^2},\quad \Omega_\Lambda = \frac{\Lambda c^2}{3H_0{}^2} \qquad (2.13)$$

and the constants

$$\frac{1}{2}\frac{\varphi_0}{H_0} = \frac{3}{2}\left(1 + \frac{1}{6}\frac{\dot h_0}{H_0}\right) \qquad (2.14)$$

$$K_0 = (\Omega_m + \Omega_r + \Omega_a + \Omega_\Lambda - 1)/\Omega_m \qquad (2.15)$$

$$K_1 = (1 - \Omega_m - \Omega_r - \Omega_\Lambda)/\Omega_m \qquad (2.16)$$

then it follows

$$\frac{1}{3}\left(\frac{2\kappa c^4 \lambda}{H_0{}^2} - \left(\frac{1}{2}\frac{\varphi_0}{H_0}\right)^2\right) = \Omega_m + \Omega_r + \Omega_a + \Omega_\Lambda - 1 = \Omega_m K_0 \qquad (2.17a)$$

and

$$\Omega_m K_0 + \Omega_m K_1 = \Omega_a . \qquad (2.17b)$$

It must hold

$$K_0 > 0 \qquad (2.18)$$

to get non-singular cosmological models. The function $a(t)$ follows from the differential equation

$$\frac{\dot{a}}{a} = \frac{H_0}{2\kappa c^4 \lambda t^2 + \varphi_0 t + 1}\left[\Omega_m K_1 + \Omega_r a^2 + \Omega_m a^3 + \Omega_\Lambda a^6\right]^{1/2} \qquad (2.19)$$

with the initial condition (2.12) for $a(t)$ at present. Then, the function $h^{1/2}(t)$ follows by

$$a^3 h^{1/2} = 2\kappa c^4 \lambda t^2 + \varphi_0 t + 1 \qquad (2.20)$$

The second law of Thermodynamics is fulfilled for

$$K_1 > 0. \qquad (2.21)$$

Then, the universe is always expanding with entropy production. Under the assumption

$$\Omega_r a^2 << \Omega_m K_1 + \Omega_m a^3 + \Omega_\Lambda a^6$$

for all $a \geq 0$ which is valid for our universe the solution of (2.19) can be given analytically. Put

$$B = \Omega_m K_1 + \frac{1}{2}\Omega_m + (\Omega_m K_1)^{1/2} \qquad (2.22a)$$

$$F(t) = \exp\left(\sqrt{\frac{3K_1}{K_0}}\, arctg\left(\frac{\sqrt{3\Omega_m K_0}\, H_0 t}{1 + \frac{1}{2}\frac{\varphi_0}{H_0} H_0 t}\right)\right) \qquad (2.22b)$$

then, the solution has the form

$$a^3(t) = 2\Omega_m K_1 B F(t) / \left[(B - \frac{1}{2}\Omega_m F(t))^2 - \Omega_m K_1 \Omega_\Lambda F^2(t)\right] \qquad (2.23)$$

Under the assumption

$$0 < K_o << K_1 << 1 \qquad (2.24)$$

the function $a(t)$ takes small values as $t \to -\infty$.

The temperature of the black body radiation at time t is given by

$$T(t) = T_0 / (a h^{1/8}) \qquad (2.25)$$

where T_0 denotes the present temperature. The redshift z of an object is received by

$$a(t) = 1/(1+z). \qquad (2.26)$$

Subsequently, assume

$$\frac{1}{2}\frac{\varphi_0}{H_0} >> \frac{3}{2}(1 + \Omega_\Lambda{}^{1/2}) \qquad (2.27)$$

implying that the always increasing scaling factor $a(t)$ goes to a finite value as t goes to infinity. For small redshifts z it holds

$$h^{1/2} = (1+z)^3 \times$$

$$\left[(\frac{3}{4}\frac{\varphi_0}{H_0} + \frac{3}{4}\frac{\varphi_0}{H_0}(1 + \Omega_m((1+z)^3 - 1))^{1/2} - \frac{9}{4}\Omega_m)/((\frac{1}{2}\frac{\varphi_0}{H_0})^2((1+z)^3 - 1) + \frac{3}{2}\frac{\varphi_0}{H_0} - \frac{9}{4}\Omega_m)\right]^2$$

(2.28)

The presently assumed best cosmological parameters imply

$$\Omega_a << \frac{1}{4}\Omega_m{}^2 / \Omega_\Lambda . \qquad (2.29)$$

Then, the age of the universe measured with proper time is given by

$$\tau(0) \approx \frac{1}{3\Omega_\Lambda^{1/2} H_0} \log\left[1 + 2\frac{\Omega_\Lambda}{\Omega_m} + 2\frac{\Omega_\Lambda^{1/2}}{\Omega_m}\right]. \tag{2.30}$$

The galaxy at redshift z has the distance

$$r = \frac{c}{H_0}\int_0^z \frac{dx}{(\Omega_\Lambda + \Omega_m (1+x)^3)^{1/2}} \tag{2.31}$$

for small values z. The formula (2.31) is identical with the corresponding one of Einstein's theory.

The presently assumed best cosmological parameters are

$$\Omega_m \approx 0.3, \quad \Omega_\Lambda \approx 0.7, \quad H_0 \approx 70\frac{km}{\sec Mpc} \tag{2.32}$$

implying by the relations (2.30), (2.29) and (2.27)

$$\tau(0) \approx 13.5 \cdot 10^9 y, \quad \Omega_a << 0.03, \quad \frac{1}{2}\frac{\varphi_0}{H_o} >> 2.8. \tag{2.33}$$

2.2 Cosmological Post-Newtonian Approximation

In this section the post-Newtonian approximation of a perfect fluid in the universe is given. Let (v^1, v^2, v^3) be the three-velocity, $\rho(x,t)$ the density of perfect fluid in the universe then the gravitational potential U to the lowest order is given by

$$U = k\frac{\sqrt{h}}{a}\int \frac{\rho'}{|x - x'|} d^3x' \tag{2.34}$$

with $\rho' = \rho(x', t)$ where $|\cdot|$ denotes the Euclidean norm in R^3. The proper time $\tilde{\tau}$ of perfect fluid satisfies

$$\frac{dt}{d\tilde{\tau}} = \sqrt{h}(1 + \frac{1}{c^2}U + \frac{1}{2c^2}a^2 h|v|^2). \tag{2.35}$$

The conserved mass M of perfect fluid has the form

$$M = \int \rho^{*'} d^3x' \tag{2.36}$$

with

$$\rho^* = \rho\frac{dt}{d\tilde{\tau}}. \tag{2.37}$$

The potentials of perfect fluid in the universe are:

$$\begin{aligned}
g_{ij} &= a^2(1 + \frac{2}{c^2}U)\delta_{ij}, & i, j = 1,2,3 \\
&= -\frac{4}{c^3}\frac{a}{\sqrt{h}}V_i, & i = 1,2,3, \quad j = 4 \\
&= -\frac{4}{c^3}\frac{a}{\sqrt{h}}V_j, & i = 4, \quad j = 1,2,3 \\
&= -\frac{1}{h}(1 - \frac{2}{c^2}U + \frac{1}{c^4}S), & i = j = 4
\end{aligned} \tag{2.38}$$

where the expressions for V_i and S can be found in paper [4] and they are of the order

$$V_i \sim O(1), \quad S \sim O(1).$$

It is worth mentioning that in paper [4] the cosmological models assume the density parameter $\Omega_a = 0$ but $\Omega_a \neq 0$ can easily be included.

The equations of motion of the perfect fluid in the universe with $\Omega_a = 0$ are found under (3.7) in paper [4]. From these equations the equations of motion of test particles in the universe can be immediately received to post-Newtonian approximation.

Put for the universe with $\Omega_a \neq 0$

$$A(t) = \frac{a}{\sqrt{h}}(h(-3(\frac{\dot{a}}{a})^2 + \frac{\dot{a}}{a}\frac{\dot{h}}{h} + \frac{3}{4}(\frac{\dot{h}}{h})^2) + H_0{}^2(\frac{21}{4}\frac{\Omega_m}{a^3} + \frac{23}{2}\frac{\Omega_r}{a^4} + 24\frac{\Omega_a}{a^6} - \frac{21}{2}\Omega_\Lambda)) . \qquad (2.39)$$

and consider in the cosmological post-Newtonian approximation in addition to the Newtonian approximation only the force with slow decrease for increasing distance then the equations of motion have the form:

$$\frac{\partial a^2\sqrt{h}v^i}{\partial t} + \sum_{\alpha=1}^{3} v^\alpha \frac{\partial a^2\sqrt{h}v^i}{\partial x^\alpha} = -\frac{1}{a\sqrt{h}}k\int \rho^{*'} \frac{x^i - x^{i'}}{\left|x-x^{'}\right|^3} d^3x^{'} + \frac{1}{c^2}A(t)k\int \rho^{*'} \frac{x^i - x^{i'}}{\left|x-x^{'}\right|} d^3x^{'}$$

$$(i = 1,2,3). \qquad (2.40)$$

3. Anomalous Acceleration

The anomalous acceleration of spacecrafts in the solar system is explained by the Newtonian approximation in the universe, i.e. the equations

$$\frac{\partial v^i}{\partial t} + \sum_{\alpha=1}^{3} v^\alpha \frac{\partial v^i}{\partial x^\alpha} + (2\frac{\dot{a}}{a} + \frac{1}{2}\frac{\dot{h}}{h})v^i = -\frac{1}{a^3 h}k\int \rho^{*'} \frac{x^i - x^{i'}}{\left|x-x^{'}\right|^3} d^3x^{'} .$$

These equations are applied to a finite number of point masses l, i.e. the Sun, the planets and the spacecraft with the velocities $v_l = (v_l{}^1, v_l{}^2, v_l{}^3)$ and the conserved masses

$$M_l = \int \rho_l{}^{*'} d^3x^{'} .$$

It follows with the conditions (2.12) in the solar system for the particle l under the assumption that the temporal variation of the velocities of the particles is large compared to the change of the cosmological functions $a(t), h(t), \dot{a}(t)$ and $\dot{h}(t)$

$$\frac{dv_l{}^i}{dt} = -(2H_0 + \frac{1}{2}\dot{h}_0)v_l{}^i - k\sum_{j\neq l}\frac{M_j(x_l{}^i - x_j{}^i)}{\left|x_l - x_j\right|^3} . \qquad (3.1)$$

Assuming

$$0 < H_0 << \dot{h}_0 \qquad (3.2)$$

then equation (3.1) implies

$$\frac{dv_l}{dt} = -\frac{1}{2}\dot{h}_0\, v_l - k\sum_{j\neq l}\frac{M_j(x_l - x_j)}{\left|x_l - x_j\right|^3} . \qquad (3.3)$$

The left hand side of (3.3) is the acceleration of the particle l which is given by the well-known Newton law of all particles acting on particle l and the anomalous acceleration

$$\Delta\frac{dv_l}{dt} = -\frac{1}{2}\dot{h}_0\, v_l . \qquad (3.4)$$

Hence, the anomalous acceleration of any object is opposite to the velocity of the object.

Anderson et al. [5] state for Pioneer 10 and $67AU$ from the Sun a nearly constant velocity relative to the Sun of 12.2 km/s. Assuming the measured acceleration of $8.74 \cdot 10^{-8}\ cm/s^2$ it follows from the relation (3.4)

$$\dot{h}_0 \approx 1.4 \cdot 10^{-13}\ 1/s. \tag{3.5}$$

Using the Hubble constant (2.32) relation (3.2) is confirmed with

$$\dot{h}_0 / H_o \approx 60000. \tag{3.6}$$

Formula (3.4) with (3.5) gives the anomalous acceleration at present, i.e. $t = 0$, for any object in the solar system. In particular, the radial anomalous acceleration relative to the Sun is given by

$$(\Delta \frac{dv_l}{dt})_r = -\frac{1}{2} \dot{h}_0 |v_l| \cos\alpha \tag{3.7}$$

where α is the angle between the velocity of the object and the radius vector from the Sun to this object. Formula (3.7) states for nearly spherical orbits, such as Earth and Mars, a nearly vanishing radial anomalous acceleration, i.e. the orbits of these planets are not changed by this anomalous acceleration. This is in agreement with the orbits of these planets which are known to high accuracy (see Anderson et al. [5]).

The anomalous acceleration (3.4) is the result of the two different times, namely the proper time τ measured by atomic clocks in the universe and the absolute time or system time t of the observer given by relation (2.5b) which implies that at present

$$dt = d\tau$$

i.e. the observer uses the present atomic clocks as it should be. Einstein's general theory of relativity does not distinguish between these two times yielding that the anomalous acceleration (3.4) cannot be received.

4. Superluminal Velocities

The light velocity for the observer at any point in the universe defined by use of time t is given by

$$\left|\frac{dx_L}{dt}\right| = c/(a\sqrt{h}). \tag{4.1a}$$

Let us introduce instead of the comoving coordinates (x^1, x^2, x^3) the real coordinates at time t by

$$\tilde{x}^i = a(t)x^i \qquad (i = 1,2,3)$$

then the local light velocity for the observer at time t is given by

$$\left|\frac{d\tilde{x}_L}{dt}\right| = c/\sqrt{h}. \tag{4.1b}$$

Furthermore, relation (2.14) gives by the use of the value (3.6)

$$\frac{1}{2}\frac{\varphi_0}{H_0} \approx 15000 \tag{4.2}$$

implying inequality (2.27). Hence, relation (2.28) holds for galaxies with small values z.

Let us consider the quasar 1928 + 738 with redshift $z = 0.36$ where several subcomponents have a motion relative to the core with velocities of about 13.85c (Eckart et al. [9]). For the observer the light velocity at this quasar follows by use of (4.1b), (2.28), (4.2) and (2.32):

$$\left|\frac{d\tilde{x}_L}{dt}\right| \approx c \cdot 2.5 \cdot 10^6. \tag{4.3}$$

Hence, the observed superluminal velocities are in reality smaller than the light velocity at this quasar.
Let us introduce the proper time τ defined by (2.5b) and the real coordinates, then it holds

$$\left|\frac{d\tilde{x}_L}{d\tau}\right| = c \tag{4.4}$$

i.e. locally an observer will always measure the vacuum light velocity in the universe at any time. It follows by use of (2.35) instead of (2.5b) that matter in the universe influences the light velocity measured by any observer in the universe. This is well known for the present observers.
Quasars are changing their brightness on small time scales (see e.g. Begelman et al. [11]) implying an upper bound for the extension of these objects. For some objects the variability has a time scale Δt of only some hours implying an upper bound of extension

$$d < \left|\frac{d\tilde{x}_L}{dt}\right| \Delta t = c\Delta t / \sqrt{h(t)} \tag{4.5}$$

where (4.1b) is used. Hence, by virtue of (4.3) the upper bound of extension can be several million times the stated upper bound $c\Delta t$, i.e. quasars must not be as compact as generally assumed.

5. Rotation Curves

Let us assume that the perfect fluid of matter is spherically symmetric with coordinates $(r, \vartheta, \varphi, ct)$ and all physical quantities depend only on r and t. Put

$$B(r) = 4\pi\left[\int_0^r \rho^{*'} r'^2 (1 - \frac{1}{3}(\frac{r'}{r})^2) dr' + \frac{2}{3} r \int_r^\infty \rho^{*'} r' dr'\right] \tag{5.1}$$

then elementary transformations of (2.38) and neglecting small expressions give the potentials

$$\begin{aligned}
g_{ij} &= a^2(1 + \frac{2}{c^2}U), && i = j = 1 \\
&= r^2 a^2 (1 + \frac{2}{c^2}U), && i = j = 2 \\
&= r^2 a^2 (1 + \frac{2}{c^2}U)\sin^2\vartheta, && i = j = 3 \\
&= -\frac{1}{h}(1 - \frac{2}{c^2}U + \frac{1}{c^4}S), && i = j = 4 \\
&= -\frac{1}{c^3} a \frac{\dot{h}}{h} k B(r), && i = 1, j = 4,\ i = 4, j = 1 \\
&= 0, && else
\end{aligned} \tag{5.2}$$

where by virtue of (2.34)

$$U = k\frac{\sqrt{h}}{a} 4\pi\left[\frac{1}{r}\int_0^r \rho' r'^2 dr' - \int_0^r \rho' r' dr'\right]. \tag{5.3}$$

The equations of motion of a test particle in this spherically symmetric fluid have the form (see [10]):

$$\frac{d^2 x^i}{d\tilde{\tau}^2} = -\overset{G}{\Gamma}{}^i_{\nu\mu}\frac{dx^\nu}{d\tilde{\tau}}\frac{dx^\mu}{d\tilde{\tau}} \quad (i = 1,2,3,4) \tag{5.4}$$

where $\overset{G}{\Gamma}{}^i_{jk}$ denote the Christoffel symbols of (5.2). The virial theorem follows by putting

$$\frac{dr}{dt} = 0, \quad \vartheta = \pi / 2 \tag{5.5}$$

implying by (5.4) with $i = 1$:

$$\overset{G}{\Gamma}{}^1_{33}(\frac{d\varphi}{dt})^2 + 2\overset{G}{\Gamma}{}^1_{34} c \frac{d\varphi}{dt} + \overset{G}{\Gamma}{}^1_{44} c^2 = 0 .$$

Elementary calculations give

$$\overset{G}{\Gamma}{}^1_{34} = 0 .$$

Hence, we get the relation

$$\overset{G}{\Gamma}{}^1_{33}(\frac{d\varphi}{dt})^2 + \overset{G}{\Gamma}{}^1_{44} c^2 = 0. \tag{5.6}$$

It follows by the use of (5.2), longer elementary calculations and neglecting small quantities

$$\overset{G}{\Gamma}{}^1_{33} \approx -r \tag{5.7a}$$

$$\overset{G}{\Gamma}{}^1_{44} \approx \frac{1}{c^2} \frac{1}{a^3 h} \frac{4\pi k}{r^2} \int_0^r \rho^{*'} r'^2 dr' - \frac{1}{c^4} \frac{1}{a^2 \sqrt{h}} A(t) k B(r). \tag{5.7b}$$

Therefore, the virial theorem has the form:

$$(r \frac{d\varphi}{dt})^2 = \frac{1}{a^3 h} \frac{4\pi k}{r} \int_0^r \rho^{*'} r'^2 dr' - \frac{1}{c^2} \frac{1}{a^2 \sqrt{h}} A(t) k r B(r). \tag{5.8}$$

The velocity of stars or gas in galaxies is not directly measured but the redshift of these objects. The velocity is then calculated from the redshift by a standard formula. The considerations of chapter 4 have shown that for an observer the light velocity (4.1) at distant objects is different from the vacuum light velocity. Therefore new considerations on the observed redshift of an object and the calculated velocity are needed.

The universe is described in chapter 2 where the potentials are given by (2.4). Let us assume that at time t_e an atom at a distant object emits a light ray with energy E and momentum p_1 to the observer. Then it follows by use of (2.4)

$$p_1 = -a(t_e)\sqrt{h(t_e)} \frac{E}{c} . \tag{5.9}$$

In the non stationary universe the energy $E(t)$ is changing with time and it holds (Petry [12],(5.2))

$$E(t) = E / (a(t)\sqrt{h(t)}) . \tag{5.10}$$

Let us now assume that the distant object is moving with velocity $(v,0,0)$ then it follows (see Petry [13],(5.4) and (5.3)) for the emitted energy $\tilde{E}$ of the moving atom and the energy E of the non-moving atom

$$\tilde{E} / c = E / c \frac{\partial x^4}{\partial \tilde{x}^4} + p_1 \frac{\partial x^1}{\partial \tilde{x}^4} = \gamma (1 + a(t_e)\sqrt{h(t_e)} \frac{v}{c}) E / c \tag{5.11}$$

where

$$\gamma = (1 - a^2(t_e) h(t_e) \left| \frac{v}{c} \right|^2)^{-1/2} . \tag{5.12}$$

Here, relation (5.9) is used. Hence, relation (5.10) has the form

$$E(t) = \gamma^{-1} (1 + a(t_e)\sqrt{h(t_e)} \frac{v}{c})^{-1} \tilde{E} / (a(t)\sqrt{h(t)}). \tag{5.13}$$

It is shown (see Petry [12],(5.4)) that for an atom at rest, i.e. $v = 0$:

$$\widetilde{E} = E(0) = E_0 a(t_e)$$

where E_0 is the energy emitted from the same atom at present. Hence, we get

$$E(t) = \gamma^{-1}(1 + a(t_e)\sqrt{h(t_e)}\frac{v}{c})^{-1} E_0 a(t_e)/(a(t)\sqrt{h(t)}).$$

Therefore, the observer receives from the moving atom the emitted light ray with the energy

$$E(0) = \gamma^{-1}(1 + a(t_e)\sqrt{h(t_e)}\frac{v}{c})^{-1} E_0 a(t_e). \qquad (5.14)$$

Hence, the total redshift z_t is given by

$$z_t = \frac{E_0}{E(0)} - 1 = \frac{1}{a(t_e)}\sqrt{\frac{1 + a(t_e)\sqrt{h(t_e)}v/c}{1 - a(t_e)\sqrt{h(t_e)}v/c}} - 1 \approx \frac{1}{a(t_e)} - 1 + \sqrt{h(t_e)}\frac{v}{c} = z_1 + z_2 \qquad (5.15)$$

where Taylor expansion is used. The redshift

$$z_1 = \frac{1}{a(t_e)} - 1$$

states the expansion of the universe and

$$z_2 \approx \pm\sqrt{h(t_e)}\frac{|v|}{c} = \pm|v|/(c/\sqrt{h(t_e)}) \qquad (5.16)$$

is the redshift (blueshift) of the moving object in the expanding universe. Formula (5.16) shows that the redshift is received in analogy to the standard formula by dividing the absolute value of the velocity of the object through the light velocity.
Let us introduce the mass of the galaxy in a ball with the center of the galaxy and radius r, i.e.

$$M(r) = 4\pi\int_0^r \rho^{*'} r'^2 dr' \qquad (5.17)$$

then relation (5.8) is rewritten by the use of (5.16)

$$(cz_2)^2 = \frac{1}{a^3}(\frac{k}{r}M(r) - \frac{1}{c^2}a\sqrt{h}A(t)krB(r)). \qquad (5.18)$$

The first expression corresponds to the well known Newton law whereas the second expression gives a correction to the Newton law. The distribution of the density of matter in the exterior of the ball must be known to calculate the redshift (5.18) as function of the distance r.
Let us consider two density distributions:
(a) The density is of the form:

$$\rho^* = C/r^2, \quad 0 < r \le R$$
$$= 0, \qquad R < r, \qquad (5.19)$$

i.e. R is the radius of the galaxy. Then, elementary calculations give by the use of (5.1)

$$(cz_2)^2 = \frac{1}{a^3}4\pi kC\left[1 - \frac{2}{3}\frac{1}{c^2}a\sqrt{h}A(t)r^2(\frac{4}{3} + \log(R/r))\right]. \qquad (5.20)$$

Hence, for small galaxies, i.e. small R, the last expression is negligible and the rotation curves are flat. For large R when the last expression is not negligible the rotation curves decrease with increasing distance r.

(b) The density is of the form:

$$\rho^* = C/r, \quad 0 < r \le R$$
$$= 0, \qquad R < r. \qquad (5.21)$$

Then, elementary calculations give by the use of (5.1)

$$(cz_2)^2 = \frac{1}{a^3} 2\pi kCr\left[1-\frac{4}{3}\frac{1}{c^2} a\sqrt{h}A(t)r(R-\frac{3}{8}r)\right]. \tag{5.22}$$

At present it follows by the use of (2.39) and (2.12)

$$(cz_2)^2 = 2\pi kCr\left[1-(\frac{\dot{h}_0}{c})^2 r(R-\frac{3}{8}r)\right]. \tag{5.23}$$

The right hand side of equation (5.23) must be positive for all $0< r \le R$ implying for $r = R$

$$R < \sqrt{8/5}c/\dot{h}_0 \approx 88kpc \tag{5.24}$$

where the value (3.5) is used. Hence, at present all rotating galaxies with density distribution (5.21) have a radius smaller than about $88kpc$.

Let

$$R = \sqrt{9/8}c/\dot{h}_0 \approx 74kpc < 88kpc \tag{5.25}$$

then the rotation curve given by (5.23) is nearly flat from $45kpc$ to $74kpc$ as e.g. the rotation curves of the galaxies UGC 2885 and NGC 6674 (see Sanders[14]). Hence, for large galaxies flat rotation curves can arise by assuming a density distribution of the form (5.21), i.e. the density decreases slower with increasing distance r than the density of small galaxies which imply flat rotation curves.

As application let us consider the galaxy M87 with a distance of $5\cdot 10^7 ly$. The nucleus of M87 is surrounded by luminous gas at a distance of $60ly$ and a rotation velocity of $750km/s$. The estimated central mass is $2.4\cdot 10^9 M_\Theta$ (see Begelman et al.[11]). The Hubble law

$$z_1 \approx \frac{H_0}{c} r_d \tag{5.26}$$

where r_d is the distance from the observer to the galaxy implies

$$z_1 \approx 0.0036. \tag{5.27}$$

It follows from (5.23) by the use of the upper bound (5.25) for the radius of galaxies that the second expression in the virial theorem is negligible at $r = 60ly$. Hence, we get from (5.18)

$$z_2{}^2 = \frac{1}{a^3(t)} \frac{kM(r)}{c^2 r} \tag{5.28}$$

implying

$$z_2 \approx 0.00249. \tag{5.29}$$

It follows from (2.28) by the use of (4.2) at the redshift z_1

$$h^{1/2} = 3.49\cdot 10^{-4} \tag{5.30}$$

implying for the observer the light velocity (4.1b)

$$\left|\frac{d\tilde{x}_L}{dt}\right| = 8.6\cdot 10^8 km/s \tag{5.31}$$

and the rotation velocity (5.16)

$$|v| = \left|r\frac{d\varphi}{dt}\right| = 2.1\cdot 10^6 km/s. \tag{5.32}$$

This is in agreement with the standard methods calculating from the measured redshift z_2 a rotation velocity of about $750km/s$ by the use of the vacuum light velocity c.

6. Structures in the Universe

Let us consider matter points in the universe then equation (2.40) gives for the matter point l:

$$\frac{d}{dt}(a^2\sqrt{h}v_l(t)) = -\frac{1}{a\sqrt{h}}k\sum_{j\neq l}\frac{M_j(x_l - x_j)}{|x_l - x_j|^3} + \frac{1}{c^2}A(t)k\sum_{j\neq l}\frac{M_j(x_l - x_j)}{|x_l - x_j|}. \qquad (6.1)$$

The left hand side is the force acting on the mass point l. It is the sum of the attractive Newton force (first expression) and a repulsive force (second expression). These equations must be solved by the use of (2.23), (2.20) and (2.39) to know the distribution of matter in the universe an any time t but the initial conditions in the beginning of the universe are unknown. Hence, only an upper bound of the extension of the structures in the microwave background will be derived. Equation (6.1) is rewritten:

$$\frac{d}{dt}(a^2\sqrt{h}v_l(t)) = -\frac{1}{a\sqrt{h}}k\sum_{j\neq l}\frac{M_j(x_l - x_j)}{|x_l - x_j|^3}(1 - \frac{1}{c^2}a\sqrt{h}A(t)|x_l - x_j|^2). \qquad (6.2)$$

Assume

$$1 - \frac{1}{c^2}a\sqrt{h}A(t)|x_l - x_j|^2 < 0 \quad (> 0)$$

for the point mass j then the acting force of it on the point mass l is repulsive (attractive).Therefore, all the matter with a distance

$$r(t) > c/(a\sqrt{h}A(t))^{1/2} \qquad (6.3)$$

is pushed off from the center. Hence, structures with a diameter

$$2r(t) > 2c/(a\sqrt{h}A(t))^{1/2} =: d(t) \qquad (6.4)$$

cannot arise as long as $d(t)$ is increasing with increasing time.

This inequality is applied to estimate the maximal structures in the microwave background, i.e. till the recombination epoch. To do this the formulas (2.23), (2.20), (2.22) and (2.17b) are used. Hence, the constant K_1 must be known where (2.24) and (2.29) have to be fulfilled. Assume

$$\Omega_m K_1 = 2\cdot 10^{-4} \qquad (6.5)$$

then relation (2.24) is satisfied and relation (2.17b) gives

$$\Omega_a \approx \Omega_m K_1 \approx 2\cdot 10^{-4} << 0.03 \qquad (6.6)$$

as demanded by (2.33). It is useful to introduce the time t_γ by

$$H_0 t_\gamma = -(\frac{1}{2}\frac{\varphi_0}{H_0} + \gamma\sqrt{\Omega_m K_1})/(\frac{2\kappa c^4\lambda}{H_0^{\ 2}}). \qquad (6.7)$$

Equation (2.23) is approximated by

$$a^3(t_\gamma) \approx 4\Omega_m K_1 \exp(\frac{3}{\gamma})/((\Omega_m + 2\sqrt{\Omega_m K_1})(1 - \frac{\Omega_m}{\Omega_m + 2\sqrt{\Omega_m K_1}}\exp(\frac{3}{\gamma}))^2) \qquad (6.8)$$

and relation (2.10) gives

$$a^3(t_\gamma)\sqrt{h(t_\gamma)} \approx \gamma^2\Omega_m K_1/(\frac{1}{2}\frac{\varphi_0}{H_0})^2. \qquad (6.9)$$

To approximate $A(t_\gamma)$ at first eliminate $\dot{h}$ by differentiation of equation (2.20) and then eliminate $\dot{a}$ by the use of (2.19) and (2.20). Then longer calculations yield the result:

$$(a\sqrt{h}A)(t_\gamma) = H_0{}^2((42+12\gamma^2)\frac{\Omega_m K_1}{a^4} + \frac{59}{2}\frac{\Omega_r}{a^2} + \frac{93}{4}\frac{\Omega_m}{a} + \frac{15}{2}\Omega_\Lambda a^2 +$$

$$+32\gamma\frac{\sqrt{\Omega_m K_1}}{a^2}(\frac{\Omega_m K_1}{a^4} + \frac{\Omega_r}{a^2} + \frac{\Omega_m}{a} + \Omega_\Lambda a^2)^{1/2}) \qquad (6.10)$$

where $a = a(t_\gamma)$ is calculated by the use of (6.8).

(1) Put

$$\gamma = -2.6 \qquad (6.11)$$

then it follows

$$a(t_\gamma) \approx 0.115, \quad \sqrt{h(t_\gamma)} \approx 3.96 \cdot 10^{-7}, \quad T(t_\gamma) \approx 3000^0 K, \quad d(t_\gamma) \approx 5.2 \cdot 10^9 ly.$$

(2) Put

$$\gamma = -2.7 \qquad (6.12)$$

then it follows

$$a(t_\gamma) \approx 0.118, \quad \sqrt{h(t_\gamma)} \approx 3.95 \cdot 10^{-7}, \quad T(t_\gamma) \approx 2920^0 K, \quad d(t_\gamma) \approx 5.4 \cdot 10^9 ly.$$

(3) Put

$$\gamma = -2.8 \qquad (6.13)$$

then it follows

$$a(t_\gamma) \approx 0.121, \quad \sqrt{h(t_\gamma)} \approx 3.94 \cdot 10^{-7}, \quad T(t_\gamma) \approx 2850^0 K, \quad d(t_\gamma) \approx 5.6 \cdot 10^9 ly.$$

The decoupling takes place at a temperature of about $3000^0 K$ being in agreement with the above examples.

The diameter $d(t)$ defined by equation (6.4) is increasing from zero in the beginning of the universe to a maximal value at t_γ with $\gamma = -8$ and $d(t_\gamma) = 10^{10} ly.$ Then $d(t)$ decreases, i.e. it is increasing till the recombination epoch and reaches its maximal value thereafter. Hence, the largest structures in the universe till the decoupling of matter and radiation are about $5.6 \cdot 10^9 ly.$ In the WMAP structures larger than $6 \cdot 10^9 ly$ could not be found (see e.g. [15]) which is in agreement with the above result. An explanation of the result of WMAP could not be given till now.

It follows for the universe at present

$$d(0)/2 = 260000 ly. \qquad (6.14)$$

Hence, the Magellan clouds with a distance of about $220000 ly$ are in the region of attraction of the Milky Way. It seems that the whole "Local Group" is in a region of attraction.

7. Electromagnetic Radiation

In this section the electromagnetic radiation of a charge moving in the gravitational field of a distant object in the universe is calculated. The Poynting vector of the radiation field is given.

The Lagrangian of the electromagnetic field has the form

$$L_E = \frac{1}{4}(\frac{-G}{-\eta})^{1/2} g^{\alpha\beta} g^{\nu\mu} F_{\alpha\nu} F_{\beta\mu} + A_\alpha J^\alpha \qquad (7.1)$$

where J^i $(i = 1,2,3,4)$ is the current four-vector, A_i the electromagnetic potential and

$$F_{ij} = A_{j|i} - A_{i|j} \qquad (7.2)$$

the electromagnetic field strength. Here, the bar "|" denotes the covariant derivative relative to the flat space-time metric (2.1). The Lagrangian implies by Euler's differential equations the field equations

$$((\frac{-G}{-\eta})^{1/2} g^{i\mu} g^{\alpha\nu} F_{\nu\mu})_{|\alpha} = J^i \quad (i = 1,2,3,4). \tag{7.3}$$

Furthermore, it follows by the use of (7.2)

$$F_{ij|k} + F_{jk|i} + F_{ki|j} = 0. \tag{7.4}$$

Relation (7.3) gives by the use of (7.2) the conservation law

$$J^{\alpha}{}_{|\alpha} = 0. \tag{7.5}$$

The equations (7.3) and (7.4) are the electromagnetic field equations in covariant form. They can be rewritten by the use of classical derivatives in the more conventional form

$$\frac{1}{\sqrt{-\eta}} \frac{\partial}{\partial x^{\alpha}} (\sqrt{-G} g^{i\mu} g^{\alpha\nu} F_{\nu\mu}) = J^i \quad (i = 1,2,3,4). \tag{7.6}$$

$$\frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^j} = 0 \tag{7.7}$$

$$F_{ij} = \frac{\partial A_j}{\partial x^i} - \frac{\partial A_i}{\partial x^j} \tag{7.8}$$

with

$$\frac{1}{\sqrt{-\eta}} \frac{\partial}{\partial x^{\alpha}} (\sqrt{-\eta} J^{\alpha}) = 0. \tag{7.9}$$

The energy-momentum tensor of the electromagnetic field also follows from the Lagrangian (7.1) and it holds

$$\overset{E}{T}{}^i{}_j = \sqrt{\frac{-G}{-\eta}} (g^{i\beta} g^{\nu\mu} F_{j\nu} F_{\beta\mu} - \frac{1}{4} \delta^i{}_j g^{\alpha\beta} g^{\nu\mu} F_{\alpha\nu} F_{\beta\mu}).. \tag{7.10}$$

This theory is considered in the universe described by (2.1) to (2.4). The equations (7.6) give the relations

$$\sum_{\alpha=1}^{3} \frac{\partial}{\partial x^{\alpha}} (\frac{1}{a\sqrt{h}} F_{\alpha i}) - \frac{\partial}{\partial ct} (a\sqrt{h} F_{4i}) = J^i \quad (i = 1,2,3) \tag{7.11a}$$

$$\sum_{\alpha=1}^{3} \frac{\partial}{\partial x^{\alpha}} (a\sqrt{h} F_{4\alpha}) = J^4. \tag{7.11b}$$

Let us introduce by the use of (7.8) the potentials A_i instead of the field strength and assume the gauge condition

$$\frac{1}{a\sqrt{h}} \sum_{\alpha=1}^{3} \frac{\partial A_{\alpha}}{\partial x^{\alpha}} - \frac{\partial}{\partial ct} (a\sqrt{h} A_4) = 0 \tag{7.12}$$

then, the equations (7.11) can be rewritten

$$\sum_{\alpha=1}^{3} \frac{\partial}{\partial x^{\alpha}} (\frac{1}{a\sqrt{h}} \frac{\partial A_i}{\partial x^{\alpha}}) - \frac{\partial}{\partial ct} (a\sqrt{h} \frac{\partial A_i}{\partial ct}) = J^i \quad (i = 1,2,3) \tag{7.13a}$$

$$\sum_{\alpha=1}^{3} \frac{\partial}{\partial x^{\alpha}} (a\sqrt{h} \frac{\partial A_4}{\partial x^{\alpha}}) - a\sqrt{h} \frac{\partial}{\partial ct} (a\sqrt{h} \frac{\partial}{\partial ct} (a\sqrt{h} A_4)) = -J^4 \tag{7.13b}$$

whereas the equations (7.7) are fulfilled.

A wave solution of (7.13) is studied. Assume that at time t_e a wave is emitted from an object at distance x' and put

$$u(x, x', t) = t_e + \int_{t_e}^{t} \frac{dt}{a\sqrt{h}} - |x - x'| / c \tag{7.14}$$

then the retarded solutions of (7.13) have the form

$$A_i(x,t) = -\frac{1}{4\pi}\int J^i(u(x,x',t))/|x-x'|d^3x' \quad (i=1,2,3)$$

$$A_4(x,t) = \frac{1}{a\sqrt{h}}\frac{1}{4\pi}\int J^4(u(x,x',t))/|x-x'|d^3x'. \tag{7.15}$$

Elementary calculations imply by the use of the conservation law (7.9) that the gauge condition (7.12) is fulfilled. The electric field $E=(E_1,E_2,E_3)$ and the magnetic field $B=(B_1,B_2,B_3)$ are defined by the field strength

$$E_i = F_{4i} \ (i=1,2,3), \quad B_1=-F_{23}, \quad B_2=-F_{31}, \quad B_3=-F_{12}. \tag{7.16}$$

Then, the equations (7.11) and (7.7) can be rewritten

$$\frac{1}{a\sqrt{h}}rotB - \frac{\partial}{\partial ct}(a\sqrt{h}E) = J = (J^1,J^2,J^3)$$

$$div(a\sqrt{h}E) = J^4 \tag{7.17}$$

and

$$rotE + \frac{\partial}{\partial ct}B = 0$$

$$divB = 0. \tag{7.18}$$

At present, these equations agree with the equations of Maxwell. The conservation law of the energy-momentum in the exterior of the current four-vector has for the metric (2.1) with (2.2) the form

$$\frac{\partial}{\partial x^\alpha}\overset{E}{T}{}^\alpha{}_i = 0 \qquad (i=1,2,3,4). \tag{7.19}$$

The equations (7.10) give

$$\overset{E}{T}{}^i{}_4 = \frac{1}{a\sqrt{h}}\sum_{\alpha=1}^{3}F_{i\alpha}F_{4\alpha} \quad (i=1,2,3)$$

$$\overset{E}{T}{}^4{}_4 = -\frac{1}{2}(a\sqrt{h}\sum_{\alpha=1}^{3}(F_{4\alpha})^2 + \frac{1}{2}\frac{1}{a\sqrt{h}}\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}(F_{\alpha\beta})^2) \tag{7.20}$$

implying by the use of (7.16)

$$(\overset{E}{T}{}^1{}_4,\overset{E}{T}{}^2{}_4,\overset{E}{T}{}^3{}_4) = -\frac{1}{a\sqrt{h}}E\times B$$

$$\overset{E}{T}{}^4{}_4 = -\frac{1}{2}\frac{1}{a\sqrt{h}}(|E|^2+|B|^2). \tag{7.21}$$

Hence, relation (7.19) gives with $i=4$ for the energy flow

$$\frac{\partial}{\partial ct}(-\overset{E}{T}{}^4{}_4) = -\frac{1}{a\sqrt{h}}div(E\times B). \tag{7.22}$$

These results are applied to a point charge with the density of charge $\rho_c(x,t)$ and the trajectory $x(t)=(x^1(t),x^2(t),x^3(t))$, i.e.

$$J^i = \frac{1}{c}\rho_c\frac{dt}{d\tau}\delta(x-x(t))\frac{dx^i}{dt} \quad (i=1,2,3,4). \tag{7.23}$$

The law (7.9) gives the conservation of the charge

$$Q = \int \rho_c{}'\frac{dt}{d\tau}d^3x'. \tag{7.24}$$

Then the relations (7.15) yield

$$A_i(x,t) = -\frac{Q}{4\pi}\frac{1}{c}\frac{dx^i(u(x,x(t),t))}{du}/|x - x(t)| \quad (i = 1,2,3)$$

$$A_4(x,t) = \frac{1}{a(t)\sqrt{h(t)}}\frac{Q}{4\pi}/|x - x(t)|. \tag{7.25}$$

It holds at the time t_e of emission and at the position of the charge, i.e. $x = x(t_e)$

$$u(x(t_e), x(t_e), t_e) = t_e \tag{7.26a}$$

and at present $t = 0$ when the observer at the position $x = 0$ receives the wave

$$u(0, x(0), 0) = t_e + \int_{t_e}^{o}\frac{dt}{a\sqrt{h}} - \frac{|x(0)|}{c} \approx t_e. \tag{7.26b}$$

Here relation (4.1a) for the light velocity in the universe is used and the distance to the emitting object is nearly unchanged, i.e.

$$|x(0)| \approx |x(t_e)|.$$

Therefore, it follows for the observer at present with $i, j = 1,2,3$:

$$\frac{\partial A_i(0,0)}{\partial x^j} = -\frac{Q}{4\pi}\frac{1}{c^2}\frac{d^2x^i(t_e)}{dt_e^{\,2}}\frac{x^j(0)}{|x(0)|^2} + O(\frac{1}{|x(0)|^2})$$

$$\frac{\partial A_i(0,0)}{\partial x^4} = -\frac{Q}{4\pi}\frac{1}{c^2}\frac{d^2x^i(t_e)}{dt_e^{\,2}}\frac{1}{|x(0)|} + O(\frac{1}{|x(0)|^2})$$

$$\frac{\partial A_4(0,0)}{\partial x^i} = O(\frac{1}{|x(0)|^2}).$$

Let the distant object be in the direction x^1 with the distance d, i.e.

$$d = x^1(0) = |x(0)|, \quad x^2(0) = x^3(0) = 0$$

then the relations imply for $i = 1,2,3$:

$$\frac{\partial A_i(0,0)}{\partial x^1} = \frac{\partial A_i(0,0)}{\partial x^4} = -\frac{Q}{4\pi}\frac{1}{c^2}\frac{d^2x^i(t_e)}{dt_e^{\,2}}\frac{1}{d} + O(\frac{1}{d^2})$$

whereas all other derivatives are of order $O(\frac{1}{d^2})$. Hence we get

$$F_{i4}(0,0) = \frac{Q}{4\pi}\frac{1}{c^2}\frac{d^2x^i(t_e)}{dt_e^{\,2}}\frac{1}{d} + O(\frac{1}{d^2}) \ (i = 1,2,3)$$

$$F_{i1}(0,0) = \frac{Q}{4\pi}\frac{1}{c^2}\frac{d^2x^i(t_e)}{dt_e^{\,2}}\frac{1}{d} + O(\frac{1}{d^2}) \ (i = 2,3).$$

The relations (7.20) give at time $t = 0$ at the position of the observer

$$\overset{E}{T}{}^1{}_4 = (\frac{Q}{4\pi c^2})^2\sum_{\alpha=2}^{3}(\frac{d^2x^\alpha(t_e)}{dt_e^{\,2}})^2\frac{1}{d^2} + O(\frac{1}{d^3}) \tag{7.27a}$$

$$\overset{E}{T}{}^i{}_4 = -(\frac{Q}{4\pi c^2})^2\frac{d^2x^i(t_e)}{dt_e^{\,2}}\frac{d^2x^1(t_e)}{dt_e^{\,2}}\frac{1}{d^2} + O(\frac{1}{d^3}) \quad (i = 2,3). \tag{7.27b}$$

Hence, the accelerated point charge radiates energy by virtue of (7.22). In the direction of the observer it is given by equation (7.27a). The energy flow is by (7.27) perpendicular to the vector of the acceleration of the charge.

Let us assume that the point mass is moving in the gravitational field of a mass M then it follows from (2.40) to the lowest order approximation

$$\frac{d^2 x^i(t_e)}{dt_e^{\,2}} \approx -\frac{1}{(a^3 h)(t_e)} kM \frac{x^i(t_e) - x_o^{\,i}}{\left|x(t_e) - x_o\right|^3} \qquad (i = 1,2,3)$$

where $x_0 = (x_o^{\,1}, x_0^{\,2}, x_o^{\,3})$ is the center of the mass M. Hence, the acceleration of the charged point mass at time t_e is the standard acceleration multiplied by the factor

$$1/(a^3(t_e)h(t_e)). \tag{7.28}$$

Therefore, the energy loss per second of the accelerated point particle in the gravitational field of a distant object is by virtue of (7.27) and (7.28) the standard value multiplied with the factor

$$1/(a^6(t_e)h^2(t_e)). \tag{7.29}$$

Let us consider an object with the redshift $z = 0.3.$ Then it follows by the use of (7.29),(2.26)and (2.28) that the energy loss per second is by the factor $1.8 \cdot 10^{29}$ higher than the standard value.

Hence, the high energy loss of quasars by electromagnetic radiation can be explained without the assumption of a massive black hole.

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