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Comparison of the classical and relativistic Doppler formulae to those of Einstein-Minkowski and Poincare-Lorentz.

ABSTRACT

Doppler formulae for the change of wave frequency in case of a moving source of electromagnetic waves are compared to those resulting from the Einstein-Minkowski formula. Doppler formulae for the change of wave frequency in case of a moving observer are compared to those resulting from Poincare-Lorentz relativity formulae. In both cases the comparison is performed for the product of two formulae: one for the change of wave period when the source-observer distance increases, the other for the change of wave period when the distance decreases.

It is shown that, the Einstein-Minkowski formula is not comparable with the product of relativistic Doppler formulae. A condition is specified for the consistency between the Einstein-Minkowski formula for a moving source of electromagnetic waves and the product of corresponding classical Doppler formulae, in conjunction with the additionally required modifications of the fundamentals of the Einstein-Minkowski formulation.

It is shown that, the product of the relativistic version of Doppler formulae for a moving observer is consistent with the corresponding product resulting from Poincare-Lorentz formulae. However, this is not the case when the special relativity corrections are removed. To achieve full consistency, the Poincare-Lorentz formulae for distance and time must be modified in such a way that the special relativity coefficient appears in the nominator and the factor (1-u/c), after changing the sign, appears in the denominator. As a result of such a reformulation, full consistency with Doppler formulae (both classical and relativistic) is achieved, but validity of modified Poincare-Lorentz formulae for distance and time is rather lost (now velocity of moving object behaves in the same manner as the phase velocity of light). Possible formulae of classical relativity for moving objects are discussed. Possibility that the relativistic formulae for waves need not to be the same as these for moving objects is considered.

1 Introduction

Apart from being recently a subject of heated debates (Hannon 2004) the Poincare-Lorentz formulae show the effect, which the observer velocity has on physical qualities such as the object velocity, distance and time as well as the period and the length of electromagnetic waves. The Einstein-Minkowski expression is used to indicate how the object velocity affects the mass of a moving object, the vectors of electric and magnetic fields as well as the period and the length of electromagnetic waves.

As far as waves are concerned the Poincare-Lorentz (P-L) and the Einstein-Minkowski (E-M) formulae can be compared to the Doppler formulae for the change of the wave period in the case of a moving observer and/or the source of electromagnetic waves. According to classical Doppler formulae the period of wave is affected by the velocity of the observer in a different way than by velocity of the wave source. Seeing the difference we can determine the corrections necessary for making the effect of a moving observer identical with that of the moving source of waves. Such a situation, when we know a priori what should give P-L or E-M relativity formulae, happens in the case of waves exclusively.

Let us start with a moving source of waves. There are two classical Doppler formulae for the period of electromagnetic waves **T** when the source of waves is moving with the **v** velocity; one formula for the distance increasing (index +v) and the other one for the distance decreasing (index -v). The product of these two formulae reads:

$$(T_{-v})(T_{+v}) = [T(1 - v/c)][T(1 + v/c)] = T^2(1 - v^2/c^2) \tag{1}$$

The corresponding product of the classical Doppler formulae for the period of electromagnetic waves when the observer is moving with the **u** velocity can be written as follows:

$$(T_{-u})(T_{+u}) = \left(\frac{T}{1 + u/c}\right)\left(\frac{T}{1 - u/c}\right) = \frac{T^2}{1 - u^2/c^2} \tag{2}$$

It is evident that in order to obtain relativity the classical Doppler formulae for the moving source of waves has to be divided by relativity coefficient with the **v** velocity while the corresponding formulae for a moving observer has to be multiplied by relativity coefficient with the **u** velocity. Introducing to equation (1), the coefficients responsible for relativity we obtain equation (3)

$$\left(T'_{-v}\right)\left(T'_{+v}\right)=\left(\frac{T_{-v}}{\sqrt{1-v^2/c^2}}\right)\left(\frac{T_{+v}}{\sqrt{1-v^2/c^2}}\right)=T^2 \tag{3}$$

Correspondingly, introducing to equation (2), the coefficients responsible for relativity we obtain equation (4)

$$\left(T'_{-u}\right)\left(T'_{+u}\right)=\left(T_{-u}\sqrt{1-u^2/c^2}\right)\left(T_{+u}\sqrt{1-u^2/c^2}\right)=T^2 \tag{4}$$

When compared, the (3) and (4) indicate that introduction of coefficients responsible for relativity results in the same value on the right hand side of the equations. In both cases the product of corresponding formulae equals, the period of wave raised to the second power.

The most general form of the classical Doppler formula for a change of electromagnetic wave period is as follows

$$T_{+u,-v}=T\frac{c-v}{c-u} \tag{5}$$

where the index determines that the observer moves in such a way that the distance increases and the source of waves moves in such a way that the distance decreases.

Having considered the coefficients responsible for relativity, the formula (5) should read:

$$T'_{+u-v}=T\frac{c-v}{\sqrt{1-c^2/v^2}}\frac{\sqrt{1-c^2/u^2}}{c-u} \tag{6}$$

It is evident, from the equation (5) and (6), that when the velocity of a source equals the velocity of the observer, the period of wave remains unchanged.

2 Comparing the Poincare-Lorentz and the Doppler formulae

Product of the Doppler formulae, after having the coefficients responsible for relativity introduced, can be compared to the product of the two corresponding Poincare-Lorentz formulae

$$t'_{-u} = \frac{t - ux/c^2}{\sqrt{1-u^2/c^2}} \qquad t'_{+u} = \frac{t + ux/c^2}{\sqrt{1-u^2/c^2}} \tag{7), (8}$$

which, considering the relation between time, distance and the velocity of moving object

$$x = vt \tag{9}$$

can be written as

$$t'_{-u} = t\frac{1 - uv/c^2}{\sqrt{1-u^2/c^2}} \qquad t'_{+u} = t\frac{1 + uv/c^2}{\sqrt{1-u^2/c^2}} \tag{10), (11}$$

Assuming that the object velocity equals the velocity of light **c**, and that in such a case the time **t** can be replaced by the period of wave **T,** we finally obtain

$$(T'_{-u})(T'_{+u}) = \left(T\frac{1-u/c}{\sqrt{1-u^2/c^2}}\right)\left(T\frac{1+u/c}{\sqrt{1-u^2/c^2}}\right) = T^2 \qquad ?\,? \tag{12}$$

Comparing equation (12) to equation (4) it is easy to see that the product of the P-L formulae gives the same as the product of the relativistic Doppler formulae for a moving observer. However the components, from which the products are built up, are not the same. After taking off from equation (12) the coefficients responsible for Special Relativity the remaining equation does not agree with the product of classical Doppler formulae numbered as equation (2). Fortunately, however, the identity

$$\frac{1-u/c}{\sqrt{1-u^2/c^2}} = \frac{\sqrt{1-u^2/c^2}}{1+u/c} \tag{13}$$

provides us with the unique possibility of being able to present the Special Relativity formulae for the period of electromagnetic waves in a form that so far has not been used (marked by asterisk), thus resulting in the following equation for the product of two formulae for the period of waves in the case of a moving observer.

$$\left(T'^{*}_{-u}\right)\left(T'^{*}_{+u}\right)=\left(T\,\frac{\sqrt{1-u^2/c^2}}{1+u/c}\right)\left(T\,\frac{\sqrt{1-u^2/c^2}}{1-u/c}\right)=T^2 \tag{14}$$

Only this new form of the P-L formulae does agree with both versions of the Doppler formulae for moving observer i.e. the relativistic formula presented in equation (4) and the classical one presented in equation (2).

Taking into account that the phase velocity of electromagnetic wave is constant, and that any change of a wave period is accompanied by a corresponding change of the wavelength we can write:

$$\left(\lambda'^{*}_{-u}\right)\left(\lambda'^{*}_{+u}\right)=\left(\lambda\,\frac{\sqrt{1-u^2/c^2}}{1+u/c}\right)\left(\lambda\,\frac{\sqrt{1-u^2/c^2}}{1-u/c}\right)=\lambda^2 \tag{15}$$

Please note that for elastic waves there should be a corresponding formulae for the shear and the pressure waves, in which the velocity of light **c** will be replaced by the corresponding velocities of elastic waves.

What about the formulae in equation (12)? After closer consideration, these formulae are found to be of no application for a moving observer, however, astonishingly enough, applicable for a moving source of waves. Replacing the **u** velocity of the observer by the **v** velocity of the source, we can rewrite equation (12) as follows:

$$\left(T'^{*}_{-v}\right)\left(T'^{*}_{+v}\right)=\left(T\,\frac{1-v/c}{\sqrt{1-v^2/c^2}}\right)\left(T\,\frac{1+v/c}{\sqrt{1-v^2/c^2}}\right)=T^2 \tag{16}$$

thus obtaining the relativistic Doppler formulae for a moving source of waves. The classical Doppler formulae for a moving source of waves shown in equation (1) can be obtained by removing the coefficients responsible for Special Relativity from equation (16).

3 Comparing the Einstein-Minkowski and the Doppler formulae

Let us now consider the Einstein-Minkowski formula for the time-space interval **s**

$$s^2 = t^2 - \frac{x^2}{c^2} \qquad \text{which for} \qquad x^2 = v^2 t^2 \qquad (17), (18)$$

should read $$s^2 = t^2\left(1 - v^2/c^2\right) \qquad (19)$$

As the time-space interval has to be constant, so for the object velocity equal zero there should be

$$s^2 = t_0^2 \qquad (20)$$

while in the case when velocity of an object is different from zero we should have

$$s^2 = t_v^2\left(1 - v^2/c^2\right) \qquad (21)$$

or $$s^2 = \left(t_{-v}\right)\left(t_{+v}\right)\left(1 - v^2/c^2\right) \qquad (22)$$

Equations (20) and (21) result in

$$t_v = \frac{t_0}{\sqrt{1 - v^2/c^2}} \qquad (23)$$

while equations (20) and (22) result in the equation

$$\left(t_{-v}\right)\left(t_{+v}\right) = \frac{t_0}{\left(1 - v/c\right)} \frac{t_0}{\left(1 + v/c\right)} \qquad (24)$$

Assuming that in case of waves the proper time $\mathbf{t_0}$ can be replaced by proper period of wave **T** and that time $\mathbf{t_v}$ dependent upon object velocity can be replaced by period of wave $\mathbf{T_v}$ dependent upon the source velocity, the equation (24) can be written as follows:

$$\left(T_{-v}\right)\left(T_{+v}\right) = \left(\frac{T}{1 - v/c}\right)\left(\frac{T}{1 + v/c}\right) \qquad (25)$$

Comparing equation (25) to the equations (1) and (2) we can see that the equation (25) derived from the E-M expression for the case of moving source of waves does not correspond to the equation (1) for moving source of waves, but unexpectedly corresponds to equation (2), for moving observer. This is somewhat disappointing . Moreover in equation (25) there are not coefficients responsible for relativity. May be that instead of using equation (24) we should use the equation (23) giving only coefficient responsible for relativity. However the situation, when E-M formula is considered as giving only correction for obtained otherwise the Doppler formulae is rather not acceptable.

The other way of handling the situation is to exchange the meaning of **s** and **t** . If time **t** could be considered as constant and the proper time **s** could be considered as dependent upon object velocity than we would obtain the formulae corresponding to those of classical Doppler formulae. However, the condition mentioned above changes physical meaning of the whole Einstein-Minkowski expression.

Given the E-M expression and following to the investigation, we have to state that, it is rather impossible to obtain the formulae that could correspond to the Doppler relativistic formulae. The formulae, which more or less correspond to those of Doppler, can be derived from the P-L formulae exclusively.

4 Classical relative time and velocity

The changes proposed for the P-L formulae for waves when the observer moves open the problem of relativistic formulae for moving objects. As for waves in the case of a moving observer, one of the relativistic formulae reads

$$T'^{*}_{+u} = T\frac{\sqrt{1-u^2/c^2}}{1-u/c} \tag{26}$$

then correspondingly for photons (dropping the lower indexes) there should be

$$t'^{*} = t\,\frac{\sqrt{1-u^2/c^2}}{1-u/c} \tag{27}$$

and after removing the coefficient responsible for Special Relativity

$$t^{*} = \frac{t}{1-u/c} \tag{28}$$

The simplest formula of classical relativity for objects moving with the velocity **v** would be

$$t^{*} = \frac{t}{1-u/v} \tag{29}$$

Such a formula of classical relativity, perfectly fits the notion that for any inertially moving observer the distance (not the time) has to be invariant. Product of the classical relative velocity

$$v^{*} = v - u \tag{30}$$

and the classical relative time proposed here gives the distance unchanged

$$x = v^{*}t^{*} = (v-u)\frac{t}{1-u/v} = vt \tag{31}$$

5 Conclusions

1 Due to identity (13) the relativistic Doppler formulae for a moving observer can be also written in the form usually used for a moving source of waves and vice versa i.e. the relativistic Doppler formulae for a moving source of waves can be written in the form usually used for a moving observer. In order to choose the right form the coefficients responsible for relativity have to be removed from the considered Poincare-Lorentz formulae for the period of wave and then the formulae need to be compared with the corresponding classical Doppler formulae.

2 As a matter of fact, the Poincare–Lorentz formulae - rewritten for waves and considered as appropriate for a moving observer - describe the change of the wave period for a moving source of waves. Consequently, the new formula for the period of wave in the case of moving observer - fully consistent with the classical and the relativistic Doppler formulae – is proposed.

3 An attempt to treat the Einstein-Minkowski expression for moving objects as the searched-for formula for the period of waves in the case of a moving source of waves turned out to have been unsuccessful.

4 The introduction of a new formulae for waves in the case of a moving observer affects both the classical and the special relativity formulae regarding distance, time and velocity of moving objects. We cannot just copy the relations for waves (phase velocity invariant) because for moving objects the velocity is not invariant. We propose the new formulae of classical relativity, in which it is the distance (and not the time as it has been the case so far) that remains invariant.

References

Robert J. Hannon, The Mysteries of the Lorentz Transformation, Galilean Electrodynamics, Vol.15, Number 3, May/June 2004.