The author's collection of relativistic paradoxes in classical electrodynamics

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1. Introduction

It is well-known that Maxwell-Lorentz classical electrodynamics represents a powerful tool for investigation of the properties of space time (see, *e.g.*, [1]), and it played a very important role in formation of special relativity. Few last decades gave a number of modern covariant ether theories, which successfully explain all experimental observations with alternative to special relativity physical interpretation. Therefore, the question concerning the possible existence of an absolute inertial frame has remained a central issue in physics. In the author's opinion, an essential progress in resolution of this problem can be achieved within classical electrodynamics, if we again look for this theory, first of all, as a tool for investigation of properties of empty space-time. Within such an approach I have collected a number of physical problems of classical electrodynamics in form of relativistic paradoxes, considered in this paper.

A presentation of the paradoxes is beginning with a physical problem, which was found under research of the Lorentz non-invariance of the Faraday induction law [2-5]. Then a number of problems are described, which deal with relativistic transformation of force and/or four-current density [6, 7]. Finally, we will consider the paradoxes, where an application of classical electrodynamics predicts a perpetual motion of the closed systems.

For the sake of simplicity we further adopt the accuracy of calculations to the order c^{-2}.

2. A rectangular conducting closed circuit and charged particle in a relative motion

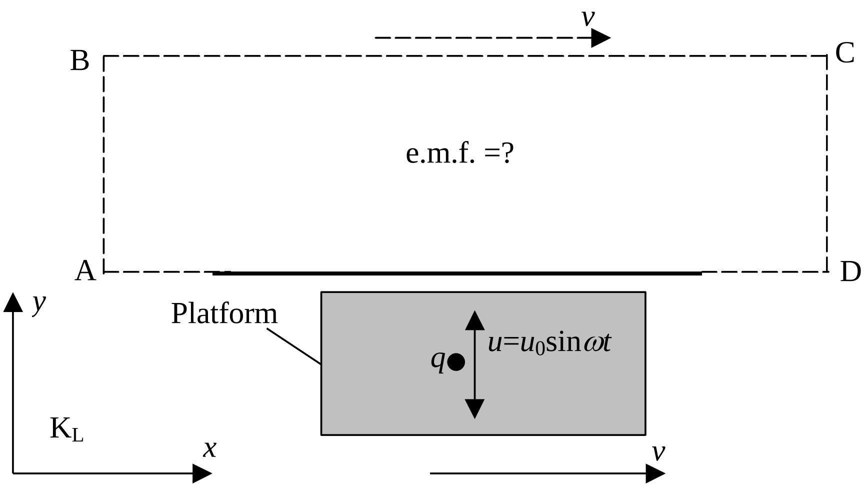

Fig. 1. The rectangular closed circuit A-B-C-D and charged particle *q* in a relative motion, as shown.

Let us consider the problem, which is depicted in Fig. 1. A rectangular conducting closed circuit A-B-C-D initially rests in a laboratory frame K_L. A charged particle *q* oscillates near the circuit along the axis *y*. This oscillator is attached to the platform P, which initially rests in the laboratory. One wants to compute an e.m.f. in the circuit A-B-C-D, assuming that the sides of the circuit are very large. In such a case the particle interacts only with the straight line AD, which is taken infinitely long. Then the e.m.f. can be written as

$$\varepsilon = \int_A^D \vec{E} d\vec{l} = \int_{-\infty}^{+\infty} E_x dx , \tag{1}$$

where $\vec{E}$ is the electric field of charged particle. One sees that under a relative rest of the platform P and the circuit A-B-C-D, the e.m.f. in Eq. (1) is equal to zero. Indeed,

$\vec{E} = -\nabla\varphi - \partial\vec{A}/\partial t$,

where $\varphi, \vec{A}$ are the scalar and vector potential, respectively. Noting that

$$\int_{-\infty}^{+\infty} \nabla\varphi dx = 0 \text{, and } A_x = 0 \tag{2}$$

for the oscillating motion of particle along the *y* axis, we find that $\varepsilon = 0$.

Now let the platform P with oscillating particle acquires a constant velocity *v* along the axis *x* of the laboratory frame K_L. For such a case classical electrodynamics unambiguously predicts the appearance of non-vanishing e.m.f. in the circuit A-B-C-D. Indeed, according to transformation of four-potential $\{\varphi, \vec{A}\}$ [1] from the rest frame of platform K_P to the laboratory frame K_L,

$\varphi_P = \gamma\varphi_L$, $A_{Px} = \gamma v \varphi_L / c^2$,

where $\gamma = 1/\sqrt{1 - v^2/c^2}$. Then the induced electric field in Eq. (1) is defined as

$$E_x = -\frac{\partial A_{Px}}{\partial t} = -\frac{\gamma v}{c^2}\frac{\partial \varphi_L}{\partial t} .$$

In turn, to the adopted accuracy of calculations $\frac{\partial \varphi_L}{\partial t} = \frac{\partial}{\partial t}\left(\frac{q}{4\pi\varepsilon_0 r}\right)$, where *r* is the distance from the particle to a designated point on the line AD with the *x* coordinate. If the momentary distance between the particle and the line is equal to *h*, then $r = \sqrt{h^2 + x^2}$, and

$$\frac{\partial \varphi_L}{\partial t} = \frac{\partial}{\partial t}\left(\frac{q}{4\pi\varepsilon_0\sqrt{h^2 + x^2}}\right) = -\frac{qh}{4\pi\varepsilon_0\left(h^2 + x^2\right)^{3/2}}\frac{dh}{dt} = -\frac{qhu_0 \sin\omega t}{4\pi\varepsilon_0\left(h^2 + x^2\right)^{3/2}} . \tag{3}$$

Hence, $E_x = -\frac{\gamma v}{c^2}\frac{\partial \varphi_L}{\partial t} = \frac{\gamma q h v u_0 \sin\omega t}{4\pi\varepsilon_0 c^2\left(h^2 + x^2\right)^{3/2}}$. Substituting this value into Eq. (1), we obtain

$$\varepsilon = \int_{-\infty}^{+\infty} E_x dx \approx \frac{qhvu_0 \sin\omega t}{4\pi\varepsilon_0 c^2}\int_{-\infty}^{+\infty}\frac{dx}{\left(h^2 + x^2\right)^{3/2}} = \frac{qvu_0 \sin\omega t}{2\pi\varepsilon_0 c^2 h} . \tag{4}$$

Thus, under a motion of platform along the axis *x*, the harmonically changed e.m.f. appears in the circuit A-B-C-D. Such a result is not mysterious, it just reflects the fact of relative motion of the circuit and the platform P.

A paradox appears, when we further assume that the circuit A-B-C-D also acquires the constant velocity *v* along the axis *x* of the frame K_L. In such a case for an observer, attached to the circuit, the platform again occurs to be at rest, and no e.m.f. is induced in the circuit under oscillating motion of the charge along the *y* axis (see, Eqs. (2)). On the other hand, for a laboratory observer a motion of infinitely long homogeneous conducting line AD along its own axis does not influence the integral (4), which gives the non-vanishing e.m.f.

Thus, we see that in the laboratory frame the e.m.f. in the moving circuit A-B-C-D is not vanishing, while for the observer, attached to the circuit, the e.m.f. is equal to zero. It obviously contracts the causality principle.

3. Two point-like oppositely charged particles bounded mechanically inside a parallel plate condenser

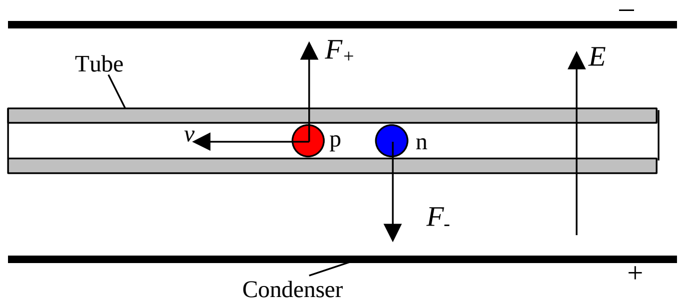

Fig. 2. The oppositely charged point-like particles "p" and "n" are placed into a hollow tube. All system is in the inner volume of the parallel plate condenser, which creates the electric field E along the axis y. The particle "n" is fixed in the tube and rests with respect to condenser. The particle "p" moves at the constant velocity v along the axis x. Here we assume a presence of an external mechanical force along the axis x, which compensates an electrical attraction of the particles.

Consider a problem as follows. Let there be two charges point-like particles with the charge $+q$ and $-q$, respectively, being inside a hollow neutral tube. The tube is placed into a parallel plate condenser, creating a homogeneous electric field E along the axis y. The tube has a single degree of freedom to move along the axis y. A gravitation field is absent, so that the masses of particles are not relevant. Initially both particles are fixed inside the tube and they rest with respect to each other and the condenser. The resultant force acting on the tube along the axis y is composed as the sum of qE and $-qE$, and equal to zero. Now we imagine that the positively charged particle (p) can move inside the tube, and it acquires a constant velocity v in the negative x-direction (Fig. 2). One requires to compute the force acting on the tube along the axis y.

It seems that the problem is trivial: the magnetic field is absent in the rest frame of condenser (K_C), and the total force, acting on the moving particle "p" is directed along the axis y and equal to $F^+=qE$, like in the case v=0. Since the force, acting on the particle "n" is $F^-=-qE$, that the resultant force, exerted on the tube by two charged particles, is equal to zero.

A paradox appears, when we compute the same force in the rest frame K_p of particle "p". It is known that the force $\vec{F}$, acting on a particle, is a relative quantity, and its transformation between two inertial reference frames K and K' in special relativity has the form [8]

$$\vec{F}'=\frac{\sqrt{1-v^2/c^2}\,\vec{F}+\dfrac{\vec{v}\cdot\left(\vec{v}\cdot\vec{F}\right)\left(1-\sqrt{1-v^2/c^2}\right)}{v^2}+\dfrac{\vec{v}\cdot\left(\vec{u}\cdot\vec{F}\right)}{c^2}}{1+\left(\vec{v}\cdot\vec{u}\right)/c^2}, \qquad (5)$$

where $\vec{v}$ is the velocity of the frame K in K', and $\vec{u}$ is the velocity of particle in K. In particular case, where $\vec{u}=0$ (K is the rest frame of particle),

$$\vec{F}'=\sqrt{1-v^2/c^2}\,\vec{F}+\frac{\vec{v}\cdot\left(\vec{v}\cdot\vec{F}\right)\left(1-\sqrt{1-v^2/c^2}\right)}{v^2}. \qquad (6)$$

In such a case we see that for orthogonal $\vec{F}$ and $\vec{v}$,

$$\vec{F}_\perp=\frac{\vec{F}'_\perp}{\sqrt{1-v^2/c^2}}, \qquad (7)$$

while for collinear vectors $\vec{F}$ and $\vec{v}$,

$$\vec{F}_{//}=\vec{F}'_{//}. \qquad (8)$$

In the problem under consideration we have to apply transformation (7) for determination of the force, acting on the "p" particle in its rest frame K_P:

$$F'^{+}{}_{y} = F^{+}{}_{y} \Big/ \sqrt{1-v^2/c^2} = qE \Big/ \sqrt{1-v^2/c^2} \,. \qquad (9)$$

Computing the force, acting on the particle "n" in this frame K_p, we have to accomplish a reverse force transformation from K_p to K_C, taking into account that in the frame K_C, $F^{-}=-qE$. Then we obtain from Eq. (7):

$$F'^{-}{}_{y} = F^{-}{}_{y} \sqrt{1-v^2/c^2} = -qE\sqrt{1-v^2/c^2} \,. \qquad (10)$$

Let us show that Eqs. (9) and (10) are in agreement with direct calculation of the Lorentz forces, acting on both particles in the frame K_p. Indeed, according to the field transformation, the moving in the K_p frame condenser produces the electric field $E'_y = E\Big/\sqrt{1-v^2/c^2}$ and the magnetic field $B'_z = vE\Big/c^2\sqrt{1-v^2/c^2}$. Hence, the force acting on the particle "p" is $F'^{+}{}_{y} = qE'_y = qE\Big/\sqrt{1-v^2/c^2}$, which coincides with Eq. (9). The force acting on the moving particle "n" is written in accordance with the Lorentz force law as

$$F'^{-}{}_{y} = -qE'_y - qvB'_z = -\frac{qE}{\sqrt{1-v^2/c^2}} - \frac{qv^2E}{c^2\sqrt{1-v^2/c^2}} = -qE\sqrt{1-v^2/c^2} \,,$$

which coincides with Eq. (10). Therefore, the resultant force, acting on the tube along the axis y in the frame K_p is determined as the sum of the forces (9) and (10):

$$F'_{\Sigma y} = F'^{+}{}_{y} + F'^{-}{}_{y} = \frac{qE}{\sqrt{1-v^2/c^2}} - qE\sqrt{1-v^2/c^2} \neq 0 \,. \qquad (11)$$

Thus, we derive the paradoxical result: in the K_p frame the tube acquires an acceleration along the axis y due to the force (11), while in the rest frame of condenser it should remain in rest.

4. A charged particle and L-shaped closed circuit with steady current under progressive relative motion

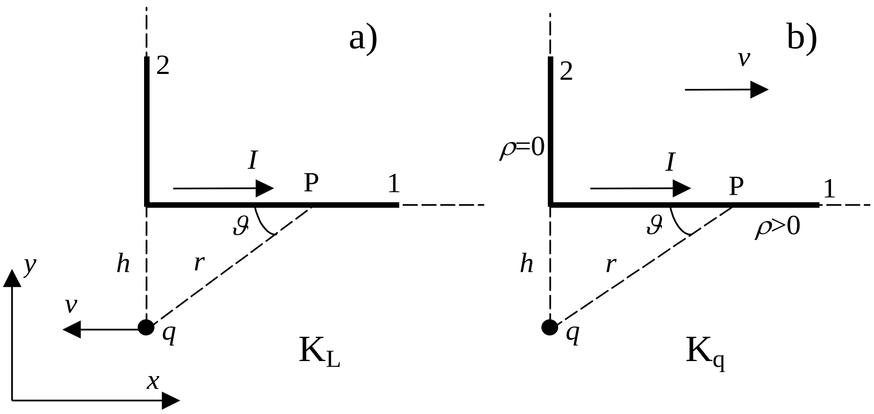

Fig. 3, a – The closed circuit with very long sides 1 (z=0, y=h) and 2 (x, z=0) rests in the laboratory frame K_L. It carries a steady current I, flowing in the counter-clockwise direction. The charged particle q with the coordinates x, y, z=0 at t=0 moves at the velocity v along the axis x of a laboratory frame. One seeks to determine the force, acting on the particle; b – the same problem from the rest frame of particle K_q.

In this section we consider a circuit, composed from two orthogonal lines semi-infinite in length carrying a steady current I (Fig. 3). We assume that these lines form a closed circuit somewhere at infinity. Simultaneously we assume that there is a charged particle q, and its velocity lies along the axis x of a laboratory frame K_L (rest frame of the circuit) at t=0.

We want to compute the force, exerted on the circuit by moving particle, in the frame K_L and in the rest frame of particle K_q as well.

First calculate the force acting on the segment 1 in the laboratory frame K_L. To the adopted accuracy of calculations the magnetic field of moving particle can be written as

$$\vec{B} = \frac{q(\vec{v} \times \vec{r})}{4\pi\varepsilon_0 c^2 r^3}, \tag{12}$$

where r is the distance from the particle to the point of observation. Then the force, acting on a line element dx at the point P of segment 1 is

$$d\vec{F}' = (\vec{I} \times \vec{B})dx. \tag{13}$$

One sees that

$$F'_{x1} = 0, \tag{14}$$

while the y-component of force is $F'_{y1} = \int_0^\infty \frac{qvI\sin\vartheta}{4\pi\varepsilon_0 c^2 r^2} dx$. Using the equalities $\sin\vartheta = h/r$, $r = \sqrt{x^2 + h^2}$, we obtain

$$F'_{y1} = \int_0^\infty \frac{qvIh}{4\pi\varepsilon_0 c^2 (x^2 + h^2)^{3/2}} dx = \frac{qvIh}{4\pi\varepsilon_0 c^2} \left. \frac{x}{h^2\sqrt{x^2 + h^2}} \right|_0^\infty = \frac{qvI}{4\pi\varepsilon_0 c^2 h}. \tag{15}$$

Further, we compute the force acting on the segment 2 in the frame K_L. This force can be found as

$$\vec{F}'_2 = \int_0^\infty (\vec{I} \times \vec{B})dy, \tag{16}$$

where the magnetic field $\vec{B}$ is defined by Eq. (12). Combining Eqs. (12) and (16), we get:

$$F'_{x2} = \int_h^\infty \frac{qvI}{4\pi\varepsilon_0 c^2 y^2} dy = \frac{qvI}{4\pi\varepsilon_0 c^2 h}, \tag{17}$$

$$F'_{y2} = 0. \tag{18}$$

From there we obtain that the resultant force, acting on the L-shaped circuit, has the components in the laboratory frame K_L:

$$F'_x = F'_{x1} + F'_{x2} = \frac{qvI}{4\pi\varepsilon_0 c^2 h} \text{ (see, Eqs. (14), (17))}, \tag{19}$$

$$F'_y = F'_{y1} + F'_{y2} = \frac{qvI}{4\pi\varepsilon_0 c^2 h} \text{ (see, Eqs. (15), (18))}. \tag{20}$$

Now let us compute the same forces in the frame K_q (Fig. 3, b). In this frame the resting particle creates the electric field

$$\vec{E} = \frac{q\vec{r}}{4\pi\varepsilon_0 r^3}, \tag{21}$$

which acts on the charges of segments 1 and 2. First determine the force exerted on the segment 1. The force acting on the elemental volume at the point P is

$$d\vec{F}_1 = \frac{q\vec{r}(\rho_+ + \rho_-)Sdx}{4\pi\varepsilon_0 r^3}. \tag{22}$$

From there

$$F_{x1} = \int_0^\infty \frac{q(\rho_+ + \rho_-)\cos\vartheta S dx}{4\pi\varepsilon_0 r^2} = \frac{q(\rho_+ + \rho_-)S}{4\pi\varepsilon_0} \int_0^\infty \frac{xdx}{(h^2 + x^2)^{3/2}} = \frac{q(\rho_+ + \rho_-)S}{4\pi\varepsilon_0 h}.$$

Taking into account that for the segment 1 $\rho_+ + \rho_- \approx jv/c^2$ due to the law of transformation of charge density [1], we get

$$F_{x1} \approx \frac{qjSv}{4\pi\varepsilon_0 c^2 h} = \frac{qvI}{4\pi\varepsilon_0 c^2 h}. \tag{23}$$

Further,

$$F_{y1} = \int_0^\infty \frac{q(\rho_+ + \rho_-)\sin\vartheta S dx}{4\pi\varepsilon_0 r^2} = \frac{q(\rho_+ + \rho_-)S}{4\pi\varepsilon_0}\int_0^\infty \frac{hdx}{\left(h^2+x^2\right)^{3/2}} = \frac{q(\rho_+ + \rho_-)S}{4\pi\varepsilon_0 h} \approx \frac{qvI}{4\pi\varepsilon_0 c^2 h}. \tag{24}$$

Now look for the segment 2. It continues to be electrically neutral in the frame K_q, and we immediately obtain

$$F_{x2} = 0,\ F_{y2} = 0. \tag{25}$$

The obtained Eqs. (23)-(25) allow us to derive the total force, acting on the circuit:

$$F_x = F_{x1} + F_{x2} = \frac{qvI}{4\pi\varepsilon_0 c^2 h} \tag{26}$$

(see, Eqs. (23), (25)),

$$F_y = F_{y1} + F_{y2} = \frac{qvI}{4\pi\varepsilon_0 c^2 h} \tag{27}$$

(see, Eqs. (24), (25)). Comparison of Eqs. (26), (27) with Eqs. (19), (20) gives an expected result for relativistic physics: to the adopted accuracy of calculations, the total force acting on the circuit due to the charged particle is the same for both reference frames K_L and K_q. However, in the K_L frame the force is applied to both segments 1 and 2, while in the frame K_q it is applied to the segment 1 solely. This circumstance indeed becomes crucial, if we notice that an electrical connection of segments of a closed circuit does not yet mean their corresponding mechanical connection. For example, the segments 1 and 2 can be electrically connected via a mercury cup. Such a connection makes them mechanically independent on each other, and hence, the equality of the forces in the frames K_L and K_q must be implemented for each segment. However, it is not the case for our problem. In particular, for the segment 1 $F'_{y1} = F_{y1}$ (see Eqs. (15) and (24)), but $F'_{x1} \neq F_{x1}$ (compare Eqs. (14) and (23). For the segment 2 we have $F'_{y2} = F_{y2} = 0$ (see Eqs. (18) and (25)), but $F'_{x2} \neq F_{x2}$ (compare Eqs. (17) and (25). The obtained inequalities represent an essence of the paradox.

5. A square circuit, carrying a steady current, and charged particle in a relative motion.

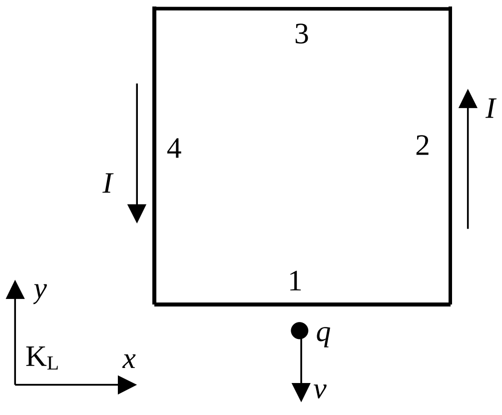

Fig. 4. The electrically neutral rigid square circuit rests in the *xy*-plane of a laboratory frame K_L. At the initial time moment the charged particle q with the coordinates x, z=0, y=-h, moves in the negative y-direction at the velocity v. The x-coordinates of left (4) and right (2) sides of the square are –L/2 and +L/2, respectively. The y-coordinates of bottom (1) and upper (3) sides of the square are 0 and L, respectively.

Next consider the following paradox. Let there be a rigid square closed circuit with the side L, resting on the plane xy of a laboratory frame K_L, and a charged particle q moving along the axis of symmetry of the circuit at the velocity v in the negative y-direction at t=0 (Fig. 4). One wants to compute the force acting on the circuit due to the particle in the laboratory frame K_L, as well as in the rest frame of particle K_q.

Within the adopted accuracy of calculations the magnetic field of moving particle is defined by Eq. (12). This magnetic field is collinear to the axis z on the xy-plane: positive for x>0 and negative for x<0. The force acting on a linear element dx of the segment with the current I is defined by Eq. (13). We can see that this force induces a torque on the sides 1 and 3 in the negative z-direction, and drives the sides 2 and 4 along the axis x. Omitting the simple calculations, we present the resultant force acting on the circuit along the axis x:

$$F'_x = F'_{2x} + F'_{4x} = \frac{Iqv}{\pi\varepsilon_0 c^2 L}\left(\frac{h+L}{\sqrt{\left(h^2+L^2\right)+L^2/4}} - \frac{h}{\sqrt{h^2+L^2/4}}\right). \quad (28)$$

The resultant torque, acting on the sides 1 and 3 along the axis z is

$$M'_z = M'_{z1} + M'_{z3} = -\frac{Iqv}{2\pi\varepsilon_0 c^2}\left[\left(\ln\frac{\left(L+\sqrt{L^2/4+h^2}\right)(L+h)}{\left(L+\sqrt{L^2/4+(L+h)^2}\right)h} - \ln\frac{\sqrt{(L+h)^2+L^2/4}}{\sqrt{h^2+L^2/4}}\right)\right]. \quad (29)$$

Therefore, the circuit acquires a combined motion: clock-wise rotation around the axis z and progressive motion in the positive x-direction.

Now let us compute the forces acting on the circuit in the rest frame of particle K_q. In this frame at t=0 the circuit moves at the velocity v along the axis y (Fig. 5). In such a case the side segments 2 and 4 have the non-zero charge densities with the reverse sign, which can be found from transformation of four-current density:

$$\rho_2 = -\rho_4 \approx jv/c^2 . \quad (30)$$

Since the segments 1 and 3 remain electrically neutral in the frame K_q, and the magnetic field of resting particle is equal to zero, that the total force, acting on the square circuit, is fully determined by the electric forces exerted on the segments 2 and 4. This force is central, and it drives the circuit along the axis x, as well as rotates it in the counter clock-wise direction (i.e., the torque along the axis z is positive). This result is in a strong contradiction with that in the laboratory frame K_L, where the torque along the axis z was negative! Now let us make the corresponding numerical estimations.

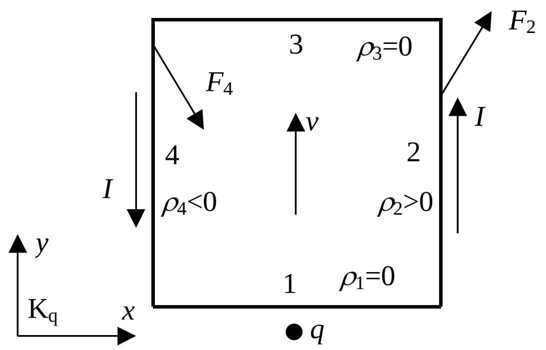

Fig. 5. The same problem, like in Fig. 4, as it is seen from the rest frame of charged particle K_q. The segments 1 and 3 remain electrically neutral, the segment 2 has a positive charge density ρ_2, while the segment 4 has a negative charge density ρ_4. Approximate directions of the forces F_2 and F_4, acting on the segments 2 and 4, are shown.

A resting charged particle produces the electric field according to Eq. (21). Hence, the force acting on each element *dy* of the segment 2 is

$$d\vec{F} = \rho_2 \vec{E} S dy = \frac{q\rho_2 S dy \vec{r}}{4\pi\varepsilon_0 r^3}.$$

The corresponding force, acting on the infinitesimal element of the segment 4 with the same *y* co-ordinate has the same *x*-component and the reverse *y*-component due to the reverse sing of charge density. From there the total force, acting on the circuit along the axis *x* is

$$F_x = 2\int_0^L dF \frac{L}{2r} = \int_0^L \frac{q\rho_2 S L dy}{4\pi\varepsilon_0 r^3} = \frac{qvIL}{4\pi\varepsilon_0 c^2}\int_0^L \frac{dy}{\left[(h+y)^2 + L^2/4\right]^{3/2}} =$$
$$= \frac{qvI}{\pi\varepsilon_0 c^2 L}\left(\frac{h+L}{\sqrt{\left(h^2+L^2\right)+L^2/4}} - \frac{h}{\sqrt{h^2+L^2/4}}\right). \qquad (31)$$

We see that to the adopted accuracy of calculations Eq. (31) coincides with Eq. (28): the force, experienced the circuit along the axis *x* is the same in both K_L and K_q frames.

The torque, acting on the circuit along the axis *z* can be written as

$$M_z = L\int_0^L dF_y, \qquad (32)$$

where

$$dF_y = \rho_2 E_y S dy = \rho_2 E \frac{h+y}{r} S dy = \frac{q\rho_2 S dy (h+y)}{4\pi\varepsilon_0 r^3}. \qquad (33)$$

Combining Eqs. (32), (33) and (30), we obtain

$$M_z = \frac{qvIL}{4\pi\varepsilon_0 c^2}\int_0^L \frac{dy(h+y)}{\left[(h+y)^2+L^2/4\right]^{3/2}} = \frac{qvIL}{4\pi\varepsilon_0 c^2}\left(\frac{1}{\sqrt{h^2+L^2/4}} - \frac{1}{\sqrt{(h+L)^2+L^2/4}}\right). \qquad (34)$$

We see that the value of torque (34) is not equal to the value from Eq. (29) and, moreover, the torque has the reverse sign. Such is an essence of the paradox.

6. A charged flexible belt, driven by two rotors, and resting charged particle

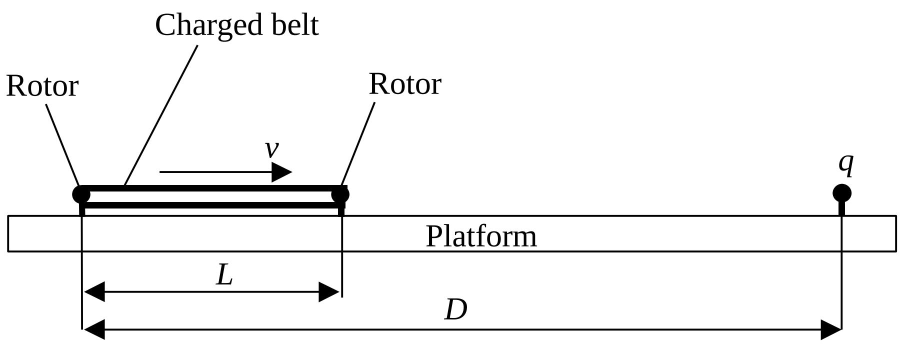

Fig. 6. The charged point-like particle and two identical small rotors are fixed on the platform. The rotors drive a flexible belt with the total charge *Q*, as shown.

Let us consider the problem as follows. Let these is a charged particle *q*, fixed on a resting platform on the plane *xy*. There are two rotors on the platform with very small radius *r*, rotating around the axis *y*. The rotation frequency of both rotors is ω, and they drive a flexible belt, as shown in Fig. 6. The charge of belt is *Q*, and it is homogeneously distributed over its perimeter. One wants to compute the force between the charged particle and charged belt.

Analyzing this problem, we assume that the rotor radius r is so small that we can neglect by interaction of curved parts of the belt with the particle q, taking the total charge of linear segments of the belt to be equal to Q. Let us take the coordinates of the left rotor $x,y,z=0$, the coordinates of the right rotor $x=L$, $y,z\approx 0$. The coordinates of particle are $x=D$, $y,z=0$.

Further, under calculation of an electric field, produced by moving charges, we proceed from the Heaviside dependence [9] of this field on a velocity of charge

$$\vec{E} = \frac{q\left(1-v^2/c^2\right)\vec{r}}{4\pi\varepsilon_0 r^3\left[1-\left(v^2/c^2\right)\sin^2\vartheta\right]^{3/2}}, \tag{35}$$

where $\vec{r}$ is the radius-vector directed from the source point to the point of observation, and ϑ is the angle of $\vec{v}$ with $\vec{r}$. Then the electric field, produced by the charge λdx of the belt's point P$\{x,0,0\}$ at the location of particle is

$$dE_x = \frac{\lambda dx\left(1-v^2/c^2\right)}{4\pi\varepsilon_0 (D-x)^2\left[1-\left(v^2/c^2\right)\sin^2\vartheta\right]^{3/2}} = \frac{\lambda dx\left(1-v^2/c^2\right)}{4\pi\varepsilon_0 (D-x)^2},$$

where λ is the charge per unit length in the driven belt. Here we take into account that for $r<<D$, $\sin\vartheta\approx 0$. In this approximation the same electric field is produced by the point P'$\{x,0,0\}$ on the lower part of the belt. Hence, the resultant field of the belt at the location of particle is

$$E_x = \int_0^L \frac{2\lambda dx\left(1-v^2/c^2\right)}{4\pi\varepsilon_0 (D-x)^2} = \frac{2\lambda\left(1-v^2/c^2\right)}{4\pi\varepsilon_0}\frac{L}{D(D-L)} = \frac{Q\left(1-v^2/c^2\right)}{4\pi\varepsilon_0 D^2(1-L/D)}, \tag{36}$$

where we take into account that $2\lambda L = Q$. Hence, the electric force, acting on the particle is

$$F_x = qE_x = \frac{qQ\left(1-v^2/c^2\right)}{4\pi\varepsilon_0 D^2(1-L/D)}. \tag{37}$$

Further, the electric field of resting charged particle along the axis x is $E'_x = -q/4\pi\varepsilon_0 x^2$. Hence, the force, acting on the belt due to the charged particle is found as

$$F'_x = \int_0^L 2\lambda E'_x\, dx = -\frac{2\lambda q}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(D-x)^2} = -\frac{2\lambda qL}{4\pi\varepsilon_0 D^2(1-L/D)} = -\frac{Qq}{4\pi\varepsilon_0 D^2(1-L/D)}. \tag{38}$$

We obtain from Eqs. (37), (38) that the sum of both forces is

$$F_\Sigma = F_x + F'_x = -\frac{qQv^2}{4\pi\varepsilon_0 c^2 D^2(1-L/D)}. \tag{39}$$

Thus, we see that an action is not equal to reaction in the problem under consideration. In general, the inequality of active and reactive forces in electromagnetic interactions is not a surprising fact, because the electromagnetic field has its own momentum. However, the problem in Fig. 6 is stationary, and the electric and magnetic fields do not depend on time. Therefore, the momentum of the electromagnetic field also does not change with time, and the Newton third law should be obeyed, on the contrary to obtained Eq. (39). Moreover, we see that the force F_Σ perpetually drives the platform along the axis x, if the friction in the rotor axes and radiation processes for the charged belt are negligible.

7. A charged particle and a charged spinning ring fixed on a platform

This paradox represents a modification of the problem in section 6. There is a thin charged ring, lying in the plane yz, and a resting charged particle q on the axis x, separated by the distance R from the plane of the ring. The radius of the ring is r, its charge is Q, which is homogeneously distributed over its perimeter. The ring spins around the axis x at the constant angular frequency ω (Fig. 7). The rotational axis of ring and the particle are fixed on a platform. One requires to find the forces, acting between the particle and the ring.

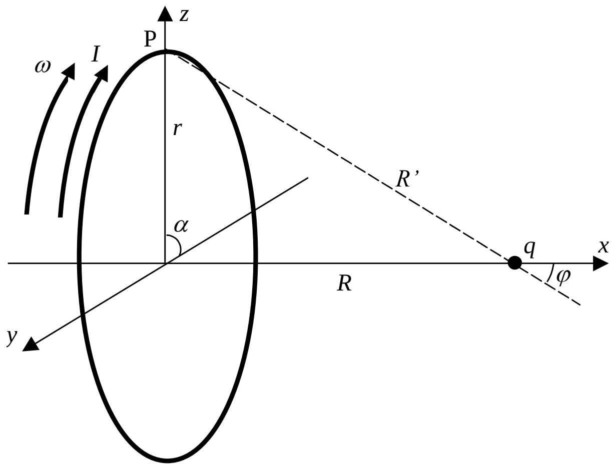

Fig. 7. The spinning charged ring and the charged particle q, resting on the rotational axis of the ring.

First compute the force, exerted on the particle by the rotational ring. Since the particle rests, the magnetic field of the ring is not relevant. Under evaluation of its electric field we again proceed from the Heaviside expression (35) for the electric field of moving charge. Then for any designated point P on the ring with the charge dQ, we have $v=\omega r$, $\sin\vartheta=1$, and

$$d\vec{E}=\frac{dQ\left(1-v^2/c^2\right)\vec{R}'}{4\pi\varepsilon_0 R'^3\left[1-\left(\omega^2r^2/c^2\right)\sin^2\vartheta\right]^{3/2}}=\frac{dQ\vec{R}'}{4\pi\varepsilon_0 R'^3\sqrt{1-\omega^2r^2/c^2}}$$

at the location of particle. (Here $R'=\sqrt{r^2+R^2}$ is the distance from the point P to the particle q). One sees from Fig. 7 that $dE_x=dE\cos\varphi$, $dE_y=dE\sin\varphi\cos\alpha$, $dE_z=dE\sin\varphi\sin\alpha$. Taking into account that $\sin\varphi=\frac{r}{R'}$, $\cos\varphi=\frac{R}{R'}$, we obtain

$$E_y=0,\ E_z=0,\ E_x=\int\limits_Q\frac{dQR}{4\pi\varepsilon_0\left(r^2+R^2\right)^{3/2}\sqrt{1-\omega^2r^2/c^2}}=\frac{QR}{4\pi\varepsilon_0\left(r^2+R^2\right)^{3/2}\sqrt{1-\omega^2r^2/c^2}}.$$

Then the force acting on the particle along the axis x due to the ring is

$$F_x=qE_x=\frac{qQR}{4\pi\varepsilon_0\left(r^2+R^2\right)^{3/2}\sqrt{1-\omega^2r^2/c^2}}. \tag{40}$$

Now let us compute a reactive force, exerted on the ring by the particle. Resting charged particle creates the electric field $\vec{E}=q\vec{R}'/4\pi\varepsilon_0R'^3$ at each point on the ring, with the x-component $E'_x=-E\cos\varphi=-qR/4\pi\varepsilon_0\left(r^2+R^2\right)^{3/2}$. Hence, the total reactive force, acting on the ring due to the charged particle is

$$F'_x=QE'_x=-\frac{qQR}{4\pi\varepsilon_0\left(r^2+R^2\right)^{3/2}}. \tag{41}$$

Then we obtain from Eqs. (40) and (41)

$$F_x+F'_x\approx\frac{qQR\left(1+\omega^2r^2/2c^2\right)}{4\pi\varepsilon_0\left(r^2+R^2\right)^{3/2}}-\frac{qQR}{4\pi\varepsilon_0\left(r^2+R^2\right)^{3/2}}=\frac{qQR\omega^2r^2}{8\pi\varepsilon_0c^2\left(r^2+R^2\right)^{3/2}} \tag{42}$$

within the adopted accuracy of calculations c^{-2}.

Thus, we again derive the inequality of active and reactive forces for the stationary electrodynamic problem. Hence, we again get a perpetual forced motion of this platform along the axis x due to the force (42). (We assume that rotation of the ring does not require any supply of energy, if friction in the rotational axis and radiation processes are negligible).

One can add that Eq. (35) is, in general, applicable to inertial motion only. This is not the case for the problem in Fig. 7. However, the resultant force acting on a platform, has the order of magnitude c^{-2}, and any additional terms, which appear in a rotational reference frame, are negligible (see, e.g. [10]).

8. Conclusion

The paradoxes considered in this paper have a common feature: the author was failed to find their consistent resolution within relativity theory. At the same time, in this contribution I omit their consideration in the covariant ether theories [11]. I just decided to describe the paradoxes, hoping that their statement will stimulate further research of the properties of space-time within classical electrodynamic theory.

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