ESSAY No.2 UPON SPECIAL RELATIVITY THEORY.

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Content:

1.Lorentz-transformations without relativity principle

In the followings,among other things,we refer to "Invariance-Postulate":IP,of Alexander v.Gaal [1,2,3]:
IP:"If one quantity,element of any mathematical formalism,designates one existent quantity in the nature,in that way is necessary that to be independent from transformational-relations" and to his **axioms**,what assures the validity of causal-principle:
AI.:"Exists one singular limit-velocity"
A.II.:"The law of sum of velocities commuts."
A.III"Relative to **v** parameter is valid the mirror-symmetry:the inversion of sign of **v** parameter produces the inverse of transformations."

The Linear Translation" also the Lorentz transformations without of relativity principle [2, 3]
In conformity to the Gaál assertion the causal principle is equivalent to following three basic axioms:

I.Axiom:Exists one singular limit-velocity (see the relation (1.8),
II.Axiom:The law of sum of velocities commuts,
III.Axiom: Relative to parameter v is valid the *mirror-symmetry*: the **inversion** of sign of v produces the **inverse of transformation**."

(Observation: v is the relative velocity of the systems S_0 and S)
Also we consider the transformations T:

$$T\,S_o:\begin{cases} x=ax_o+bt_o & (1.1)\\ t=px_o+qt_o & (1.2)\end{cases}$$

also

$$TS_o = S \qquad (1.3)$$

Also we write the transformations (1.1)-(1.2) into the differential form and making their rate we obtain

$$(dx/dt)=[a(dx_o/dt_o)+b][(p(dx_o/dt_o)+q] \qquad (1.4)$$

Because the derivatives (1.4) represents any constant velocities, also we notate:

$$u=[(au_o+b)/(pu_o+q)]=$$
$$=[((a/q)u_o+(b/q))/((p/q)u_o+1)] \qquad (1.5)$$

In case if $u_o=0$, results that $u=b/q$ is one constant velocity, too; we notate $(b/q)=v$. After any elementary transformations :

$$(p/q)=(p/b).(b/q)=(p/b).v$$

we have

$$u=[(a/q)u_o+v]/[(p/b)u_ov+1] \qquad (1.6)$$

“This a the most general invariant law of addition (sum) of velocities u_o and v.In this form however does not corresponds in all regards to obligatory causal-principle out of following causes:If in (1.6) we introduce $(a/q)=k$, $(p/b) = l$,and we notate their derivatives function from v with k' and l',respectively we obtain from (1.6) the following equation:

$$l'^2 u_o v^2 + (kl' - k'l) u_o^2 v - k' u_o + kl u_o^2 - 1 = 0 \qquad (1.7)$$

and by the finite solutions of (1.7),the relation (1.6) has extremum.This is one paradox : also, if we sum u_o with sufficient large values of v, u not increases, but decreases, in contrast with reality. But this is not valid, if k and l are constant*: (k'=l'=0*). In this further case u_o always **has one limit u_o*** what cannot increases with summing any discretional large value of v .In this case from (1.7) results: $klu_o^2 = 1$, or

$$u_o^* = ((bq/ap))^{1/2} \qquad (1.8)$$

what (in conformity to the Axiome 1.) represents the specific velocity, the **limit-velocity of system** ."

Observation: at Gaál

$$u_o^* = ((bq/ap))^{1/2} = 1 \qquad (1.9)$$

and the limit velocity in the same time is considered as a natural absolut unit of measure of velocity.

Alexander von Gaál in basis of relation (1.6) declares that the law of sum of velocities in general not commuts and in this way continues his idea: the hypothesis "if we pass forward along one direct line **n** meters and **n** meters back, we come back in the origin-point, in generaly is not valid"

So, from Axiom II. it results:

$$(a/q) = 1 \qquad (1.10)$$

From III.Axiom it follows “the validity of symmettrising determinant”

$$\Delta = aq - bp = 1 \qquad (1.11)$$

Finally; from (3.10) results a=q ; from (3.9) and (3.10) , results that: p=b ,but (b/q)=v ,then: b=av and introducing into expression of symmetrising determinant, we obtain:

$$\Delta = aq - bp = a^2 - a^2v^2 = 1,$$

hence

$$a = q = (1-v^2)^{-(1/2)} = \beta \qquad (1.12)$$

where β is the Fitzgerald-Lorentz factor, and

$$b = p = \beta v \qquad (1.13)$$

Introducing (1.12) and (1.13) into (1.1) and (1.2) and (1.6) respectively, we obtain the conventional form of (direct) Lorentz tranformations (evident for $u_o^* = 1$):

$$L: \begin{cases} x = \beta\,(x_o - v\,t_o) & (1.14.) \\ t = \beta\,(t_o - v\,x_o) & (1.15) \end{cases}$$

and

$$u = (u_o - v)/(u_o v - 1) \qquad (1.16)$$

the low of addition of velocities.

Observation: if we continue the rationament of Gaál, we obtain the invers of tranformations and the

corresponding low of addition of velocities (considered by Gaál the direct transformations).
Also :
„From (1.13), take into consideration the mirror-symmetry ± v , after the change of sign of v and rearranged this relation, we obtain the **conventional, two-dimmensional** form of **Lorentz-transformations**:

$$L: \begin{cases} x=\beta\,(x_o-vt) & (1.14.a) \\ t=\beta\,(t_o-vx_o) & (1.15.a) \end{cases}$$

evidently for unitary value of the limit velocity $u_o^* = 1$.
In following we notate the inverse of L: with L^{-1} .Here the measure of v is the limit-velocity: u_o^* and introducing of any conventional measure of velocity, exemple the velocity of light c , cannot possible in basis of our pure logic deduction."
„The form of sum of velocities conformly to relations (1.14.a)-(1.15.a) are after all:

$$u=(u_o-v)/(u_o v -1) \qquad (1.16.a)$$

Observation: here the denominator of the product $u_o v$ is 1, also the square of the limit velocity which in our case is $u_o^* = 1$ m/s .

"The I.-Axiom it is not of quantity nature: consequently postulates not one given value of limit-velocity, the value-interval is whole number - axis .In case of relation

(1.8) or (1.9), their right part appears as nature velocity-unit, but the choice of measure-unit is arbitrary and it is not unconditional identicaly with the limit-velocity. *Also writing the L-E transformations for an arbitrary value of the limit-velocity* u_o*

$$L: \begin{cases} x=(1-v^2/ u_o*^2)^{-1/2} (x_o-vt) \\ t=(1-v^2/ u_o*^2)^{-1/2} (t_o-vx_o/ u_o*^2) \end{cases} \qquad (1.16b)$$

1.2.Galilei-transformations as case special of the L-transformations if the limit-velocity tinds to infinity

If $u_o* \to \infty$ as well if into (1.9) we place p = 0 we obtain the classical Galilei-transformations

$$x = x_o - v_{01}t \qquad (2.1.)$$

and $$t_1 = t_o \qquad (1.17)$$

($v_{o1} = v_{1o} = v$ *is the relative velocity of systems S and* S_o) what in contradictory with L-transformations are unidimensional, because t appears quasi as second parameter. If we measure the velocity of light with conventional **c,** it appears that in G-transformation the velocity of light is infinite large.But this is an erroneous interpretation.: here that it is not one specific-velocity. The limit velocity is a specific-velocity. The limit-velocity is a favoured-velocity in case of G-transformations in correspondence to singular ∞ value.

Consequently the G-transformations satisfies to three axioms, too.
For $u_o^* = 1$ (evidently having the measure unit m/s) we have

$$\mathbf{L}: \quad x = \beta (x_O - vt_O) \quad \text{and} \quad t = \beta (t_O - vx_O) \qquad (1.18)$$

where $\beta=(1-v^2/ u_o^{*\,2})^{-1/2} = (1-v^2/ 1^2)^{-1/2} =(1-v^2)^{-1/2}$
And so the additional law of velocities to be:

$$u = (u_O - v) / (u_O v - 1) \qquad (1.19)$$

3.Form- or value-invariance ? The Cardinal-Theorem; equivalence between L- and G-transformations

Finally in basis of relations (1a) - (13) we write the results having great importance, what correlate the L-transformations with G-transformations adjointing identical v - parameter. (As well as, identical relative velocity).
In this aim we transform the system S_O into S system with L-transformations and into S' with G-transformations. Also x_O, t_O designate the coordinates of any P point in the S_O ,and x, t in the S ,and x',t' in the S', obtained by L-and G-transformations, respectively. The resulting velocities we notate by u and u' respectively.
In this case we have:

$$S = LG^{-1}S' : \begin{cases} x = \beta.x' \\ t = \beta.(1 - v^2).t' - \beta.v.x' \end{cases} \quad (3.1)$$

and

$$S' = GL^{-1}S : \begin{cases} x' = (1 - v^2)^{1/2}\, t \\ t' = \beta.(t + v.x) \end{cases} \quad (3.2)$$

and the resulting velocities are:

$$u = (x/t) =(u') / ((1 - (u' + v).v) \quad (3.3)$$

and

$$u' = (x' / t') = (u.(1 - v^2)) / (u.v + 1) \quad (3.4)$$

These relations result essentially from general relations related in ([2] Szöcs 1995), but we can deduce as directly from the mathematics of Invariant Postulate (see Appendix). It is very important to consider that x,t and x',t' were obtained by L - and G - transformations, respectively. But L and G in virtue of C.III axiom are simmetricals referring to change of sign of v parameter. This fact signifies the possibility of one <u>selection without restriction</u> that the L- and G- transformations with what sign of v we can consider as their inverses, as the L^{-1} - and the G^{-1} -transformations. Also if we write the inverses of L and G, are valid the relations:

$$\beta.(x + v.t) = x_o = x' + v.t' \quad (3.5)$$

and:

$$\beta.(t + v.x) = t_o = t' \qquad (3.6)$$

Consequently is valid the *cardinal-theorem* resulting from v - symmetry, as:

The cardinal theorem**:" The expressions transformed by L- or G-transformations, in case of identical value of v parameter, are equivalents, as well are invariant as value, but are not identical as form." .**

About of consequencies of this theorem, as the solving by Al.v.Gaál the Doppler- antinomy and his consequence : the clock-paradox and the existence of one absolute (coordinate-) system (frame) via rectification (in virtu of the Fermi Theorem) of the rotating experience [4,5,6]

4.The Doppler antinomy [4,5]

Considering previous systems of coordinates we assume one electromagnetic configuration (ex.one series of waves),what propagates along **x** axis.,describable by function $F_o(x_o,t_o)$.Because in this case $x_o = c.t_o$,and here c = 1 the observer S_o viewing S_1 ,finds in that $F(x_1,t_1) = L.F(x_o,t_o)= F(t_o\ \beta_{o1}(1- v_{o1}))$,where β_{o1} it is the Lorentz-Fitzgerald-factor.Is valid generaly:

$$\mathbf{t_1 = t_o\, \beta_{o1}(1 - v_{o1})} \qquad (4.1)$$

Aplying the Galilei-transform,we obtain:

$$\mathbf{t_1' = t_o\, (1 - v_{o1})} \qquad (4.2)$$

which we can deduce directly,or by Cardinal Theorem **R**.In conformity to this we have

$$\mathbf{t_1 = t_1'\beta_{o1}} \qquad \text{(if } x = 0\text{)}$$

In these formulas the **t** time-measure measures the duration of light-phenomenon F(x,t),as exemple if it is one wave,measures its running-time,as well as its wavelenght (because of x = t).Consequently its reciprocal (because of c = 1),is the frequency **f .**By (4.1) and (4.2) S_o measures the events from S_1.Making and rearranging the reciprocal of (4.1),we obtain

$$\mathbf{f_o = f_1\, \beta_{o1}\, (1 - v_{o1})} \qquad (4.3)$$

Consequently here the S_1describes the events from S_o.After this we wiev one -coherently and transversaly flat-wave propagating along x axis.
If this,in conformity with the **L** propagates towards negative **x** axis,so this is described in the S_o system by:

$$\mathbf{E_o = A_{ogf}\, \sin 2\pi\, f_o(t_o + x_o)} \qquad (4.4)$$

If,opposite,the S_1 describes S_o and in this aim we transform (4.4) with **L**,that the result it is,in conformity with the change of indicies:

$$\mathbf{E_1= A_1 \sin 2 \pi f_1\beta_{o1}(1 - v_{o1})(t_1+x_1)} \quad (4.5)$$

what it is the relativistic expression of Doppler's effect,f_o,f_1 being the corresponding frequencies.The periods because of c = 1 and T = 1/f are given by reciprocal of $\mathbf{f_1\beta_{o1}(1 - v_{o1})}$ **:**

$$\mathbf{T_1 = T_o \beta_{o1}(1 - v_{o1})} \quad (4.6)$$

where S_o describes the events of S_1.

5. Transmission of electromagnetic waves through relay-systems

After these we consider one ordered-assembly **A** of coordinate-systems S_i so that whichever element S_i it is linked with all other S_n by the **L**-transformations.This follows from transitivity of these transformations.;"If exist L_i and L_k,so exists $L_iL_k=L_{ik}$, for all i and k."The **A** series it is ordered by increasing values of **v** parameter,which , between -1 and +1 ,in conformity with Dedekind-principle constitutes one continously increasing and realy series .It follows from previous that must exist in the **A** one element in which v = 0.

In following we consider three,**L**-linked systems: S_o,$S_{,1}$,S_2.and let be in the S_o v = 0.In this way between S_o and S_1 and between S_o and S_2 respectively,we obtain for (4.3) in conformity with (4.5) in S_1 and in S_2 respectively:

$$\mathbf{f_o = f_1\beta_{o1}(1 - v_{o1}) \quad and \quad f_o = f_2\beta_{o2}(1 - v_{o2})} \qquad (5.1)$$

Suitably with **G**-transformations we obtain:

$$\mathbf{f_o = f_1'(1 - v_{o1}) \text{ and } f_o = f_2'(1 - v_{o2})} \qquad (5.2)$$

From which follow the corresponding ratios:

$$\mathbf{(f_2\beta_{o2})/(f_1\beta_{o1}) = (1 - v_{o1})/(1 - v_{o2})}$$

$$\mathbf{and \quad f_2'/f_1' = (1- v_{o2})/(1 - v_{o2})} \qquad (5.3)$$

Here it is surprising the validity of **R-**theorem..

Now we place in the S_o one TV-transmitter and in the S_1 one relay which pass the received waves from S_o in the S_2**.,**because these can not propagate <u>directly</u> in the S_2.Also the condition of transmission it is the emission of such waves by S_1 as coming directly from S_o.The question is: what is the frequency of S_2 in this case (considering that S_1 moves in comparison with the S_o with velocity v_{o1}) ?

The response is given by (5.1) and (5.2),respectively, from which follows (5.3).In the left part of all these we have f_o,consequently it follows:

$$\mathbf{f_1\beta_{o1}(1--v_{o1}) = f_2\beta_{o2} (1-v_{o2})}$$

and

$$\mathbf{f_1'(1 -v_{o1}) = f_2' (1 - v_{o2})}$$

respectively.
It follows that S_1 must not change the received frequency,but must transmet towards S_2 with the received frequency from S_o,also with the f_1.

And now we eliminate the S_o,but S_1 emits longer with frequency f_1.The disappearance of S_o it is unobservable for S_2,because before not any signal received directly from S_o.Consequently the ratio of both frequencies remain as (5.3)

$$\mathbf{(f_2\beta_{o2})/(f_1\beta_{o1}) = (1 - v_{o1})/(1 - v_{o2}} \qquad (5.4)$$

But here it is very surprising that this ratio it is given not by the relatively velocity v_{12} of S_1 and S_2,respectively,but only exclusively with the velocities v_{o1} and v_{o2} relative to S_o,but which does not exists.Here exists only S_1 and S_2,and from (5.4) it results that S_2 it is receiving the frequency f_2, instead of $f_1$emitted by the S_1because any velocities v_{o1} and v_{o2} relative to one and the same S_o,but which per definitionem does not exists

In virtue of (5.4) we have:

$$\mathbf{(f_2\beta_{o2})/(f_1\beta_{o1}) = f_2'/f_1' = (1 - v_{o1})/(1 - v_{o2})} \qquad (5.5)$$

By the (14) is expressed the invariance-postulate **IP:** the velocities v_{o1} and v_{o2} can be interpreted as velocities of S_1 and S_2 relative to motionless system S_o in which propagates different waves, as the sound-waves in one elastic-medium and as the electromagnetic waves in "ether".It is very important that this paradox does not solves by means of declaration that whichever element of assembly **A** (as the assembly of **L**-transformation linked coordinate-systems) can play the role of S_o basic-system, because as we proof in following work, exists only one system from **A** in which the velocity **v = 0**.And after these arises the question, that the **L**-group defines one basic-system identicaly with the classical light-ether ?

6.The clock-paradox [4,5]

In virtue of relativity-principle the correlation between the systems S_1 and S_2 it is determined exclusively by theirs relative-velocity: $\mathbf{v_{12}}$.This is one symmetricaly relation ($v_{12} = v_{21}$).The value and the sign of relative - velocity (as the variation in function of time of the distance **H** between systems S_1 and S_2: **dH/dt**) it is pozitive if the distance between S_1 and S_2 increases,and it is negative,if this distance decreases.,but as relative to S_1 and to S_2 is identicaly heaving identicaly sign, too.In opposite with this all homogenous-affin transformations, as **L** transformations, too, between any S_i and S_k for all i and k are assymetricaly because $\mathbf{L \neq L^{-1}}$,where L^{-1} are the inverse-transformations.The result it is the **clock-paradox** unresolved up to this time and which in our case appears as following:

In virtue of relativity-principle from systems S_1 and S_2 not any is favoured ,in other words the relation of S_1 to S_2 is identicaly with the relation of S_2 to S_1,as well as the indicies of systems can invert .In consequence:

$$\text{if:} \quad \mathbf{f_2 = f_1 \beta_{12} (1 - v_{12})}$$

$$\text{so:} \quad \mathbf{f_1 = f_2 \beta_{12} (1 - v_{12})} \qquad (6.1)$$

because of $v_{12} = v_{21}$.

And from (6.1) immediately follows:

$$\mathbf{f_1/f_2 = f_2/f_1} \qquad (6.2)$$

as well as the clock-paradox.

7.The solution of the clock-paradox [4,5]

Let $L_{\mathbf{ip}}$ denote the transforming from system $S_{\mathbf{i}}$ into $S_{\mathbf{p}}$ by L, and similarly let $L_{\mathbf{pk}}$ denote the transforming from $S_{\mathbf{p}}$ into $S_{\mathbf{k}}$. L is transitive so there exists a transformation $L_{\mathbf{ik}} = L_{\mathbf{ip}} L_{\mathbf{pk}}$. S_o the following relation - a double connection, shows the above transmission of time measure from S_1 to S_2

$$L_{12} L_{21} = L_{11} \qquad (7.1)$$

the resultant of which is the unit transformation L_{11}. So there is no antinomy in case of correct interpretation.

It will be clear if we show it explicitly way: The general form of the solution is $L_{12} L_{23} = L_{13}$, the parameters v_{12} and v_{23} are usually different:

$$x_3 = (1+v_{12}v_{23})\beta_{12}\beta_{23}x_1-(v_{12}+v_{23})\beta_{12}\beta_{23}t_1 \quad (7.2a)$$

$$t_3 = (1+v_{12}v_{23})\beta_{12}\beta_{23}t_1-(v_{12}+v_{23})\beta_{12}\beta_{23}x_1 \quad (7.2b)$$

So $L_{12}L_{23} = L_{13}$. Since in our case x_1=0:

$$t_3 = (1+v_{12}v_{23})\beta_{12}\beta_{23}t_1 \quad (7.3)$$

If we put here $(1+v_{12}v_{23})\beta_{12}\beta_{23} = \beta_{13}$, so if we interpret $L_{12}L_{23}$ as L_{13} with one parameter we immediately realize that in this case the parameter of L is:

$$v_{13} = \frac{v_{12} + v_{23}}{v_{12}v_{23} + 1}$$

that is the L-composition of v_{12} and v_{23}.
If now we consider $L_{23} = L_{21}$,we have :

$$t_3 = (1+v_{12}v_{21})\beta_{12}\beta_{21}t_1 \quad (7.4)$$

So in the above $L_{12}L_{21}$, L_{21} is the inversion of L_{12} ,as well $L_{21} = L^{-1}_{12}$, that is $v_{21} = -v_{12}$ and $\beta_{12} = \beta_{21}$ .If we substitute this result into (7.4) we get:

$$t_3 = (1-v^2_{12})\beta^2_{12}\, t_1 \quad (7.8)$$

Since $$\beta^2_{12} = \frac{1}{1-v^2_{12}}$$

this immediately implies that

$$t_3 = t_1 \qquad (7.9)$$

So there is <u>no question of paradox</u> in the case of correct computation.

8.Any consequencies of the generalized linear transformations *(Probably this problem will be the subject of one late or supplementary paper)*

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