# DERIVATION OF LORENTZ TRANSFORMATION EQUATIONS AND THE EXACT EQUATION OF PLANETARY MOTION FROM MAXWELL AND NEWTON

**Sankar Hajra**

CALCUTTA PHILOSOPHICAL FORUM
Salt Lake, AC-54, Sector-1, Flat No.A1, Kolkata-700064.
E-mail: sankarhajra@yahoo.com

**Abstract:** The scalar and vector potentials of a system of charges and currents stationary in free space are governed by Poisson's equation whereas the similar potentials of a system of charges and currents steadily moving in free space are governed by D'Alembert's equation. Heaviside (1888, 1889) and Thomson (1889) first correctly calculated the scalar and vector potentials of a steadily moving point charge by transforming the D'Alermbert's equation of the potentials for the steadily moving charge into Poisson's form for a static charge by elongating a coordinate axis lying along the direction of the translation of the charge. They thus developed a way to solve dynamic problems like static problems using an auxiliary equation in the form of Poisson's potential equation. We have used this ingenious mathematical treatment to derive the fields of a steadily moving point charge, its electromagnetic momentum, its longitudinal and transverse electromagnetic masses and many useful electrodynamic equations including Lorentz Transformation Equations from Maxwell's field equations alone. In the last part of the paper, starting from the longitudinal electromagnetic mass of a point charge steadily moving in free space, we have presented a deduction of the exact equation of planetary motion following Maxwell and Newton considering that a planet contains mass and charges and assuming that in the gravitational field a point charge acts as a mass point, mass of the mass point being proportional to the longitudinal electromagnetic mass of the charge. This study implies that electromagnetic fields, charges and electromagnetic energy are real physical entities. Just like all other material bodies they are not only subject to gravitation, they have the same acceleration as that of material bodies in the same gravitating field and this simple consideration is equivalent to special and general relativity.

## 1. Introduction

All electrodynamic equations, the surprising fact that all electrodynamic phenomena observed on the surface of our Earth are independent of the motion of the Earth, advance of the perihelion of Mercury, gravitational red shift of astral rays and bending of light rays grazing the surface of the sun are explained by institutional physicists from the consideration of special and general relativity which use many absurd metaphysical principles.

In this paper we shall simply derive all those equations from Maxwell and Newton and establish that all electrodynamic entities are subject to gravitation.

## 2. Heaviside-Thomson Auxiliary Equation

The scalar potential ($\Phi$) of a system of charges moving steadily in free space with a velocity 'u' (say S system) is governed by D'Alembert's equation:

$$\Box^2 \Phi = -\rho / \varepsilon_0 \qquad (1)$$

where $\rho$ is the charge density of the system, $\varepsilon_0$ is the permittivity and $\mu_0$ is the permeability of free space such that $c = 1/\sqrt{\mu_0 \varepsilon_0}$, and x,y,z are the Cartesian co-ordinates introduced in the free space. Assume 'u' in the OX –direction. Then, $\Phi$ is a function of two independent variables, $x$ and $t$. From the definition of two independent variables we have,

$$d\Phi = (\partial\Phi/\partial x)dx + (\partial\Phi/\partial t)dt$$
$$= (\partial\Phi/\partial x)udt + (\partial\Phi/\partial t)dt$$

($dx$ being equal to $udt$). Moreover, in such a situation, the potentials at the point ($x, y, z$) at the instant $t$ and the potentials at the point $(x + udt, y, z)$ at the instant $t + dt$ in free space will be the same; *i.e.*,

$$\Phi(x, y, z, t) = \Phi(x + udt, y, z, t + dt)$$
$$= \Phi(x, y, z, t) + d\Phi$$

From the above two equations we have

$$\Phi = \Phi + (\partial\Phi/\partial x)udt + (\partial\Phi/\partial t)dt$$
$$\therefore \partial\Phi/\partial t = -u\partial\Phi/\partial x \qquad (2)$$

$$\partial^2\Phi/\partial t^2 = +u^2\partial^2\Phi/\partial x^2 \qquad (3)$$

Let us now fix a co-ordinate system $x', y', z'$ to the system of charges moving steadily in free space with a velocity 'u' in the OX direction as defined in the equations (1) and assume that at t=0, both coincide.

Let us now imagine an elongation of the moving system towards its direction of motion by the factor $\gamma$ where $\gamma = 1/k$ and $k = \sqrt{1-u^2/c^2}$.

Then the volume charge density of this imaginarily elongated system should be,

$$\rho' = \rho k$$

and we have, $x' = \gamma\, x,\ y' = y,\ z' = z \qquad (4)$

The system imaginarily elongated by the equation (4) towards OX axis which is the direction of motion of the system of charges may be called the auxiliary system $(S')$ comprising of $x', y', z'$.

The scalar potential of this *"Auxiliary System"* should be $\nabla'^2\Phi' = \rho'/\varepsilon_0 \qquad (5).$

Now substituting

$x' = \gamma\, x, y' = y, z' = z$ as in the equ. (4).

and using the equ.(3), the equation (1) could be transformed to

$$\nabla'^2\Phi = -\rho/\varepsilon_0 \qquad (6).$$

Comparing equations (5) & (6), we have,

$$\Phi' = \Phi k \qquad (7)$$

from which we have

$$E_x = E'_x, E_y = \gamma E'_y,\ E_z = \gamma E'_z. \qquad (8)$$

Whence we can get the induced magnetic field from the relation

$$\mathbf{B}^* = \mathbf{u}\times\mathbf{E}/c^2. \qquad (9).$$

Therefore, the electric and magnetic fields of a steadily moving point charge as has been noted by Heaviside [1,2] should be exactly the same as those of a conducting similarly charged and similarly moving ellipsoid (Heaviside's Ellipsoid) having its axes with the ratio k:1:1, k being in the direction of movement of the charge. For, in the "Auxiliary System", both have the same external fields as those of a stationary similarly charged sphere.

The above device of the "*Auxiliary System*" comprising of the equations (4) was used by Heaviside (1888,1889) , Thomson (1889) and their contemporaries to study the electrodynamics of moving system of charges. Therefore, we may call the equation (4) the *"Heaviside-Thomson Auxiliary Equations"*

## 3. Fields of a Steadily Moving Point Charge

Following Heaviside, the electric field $\mathbf{E}$ and the induced magnetic field $\mathbf{B}^*$ of a point charge $Q$ steadily moving with a velocity $\mathbf{u}$ could be calculated at a point P $(x,y,z)$ in free space (considering at a particular instant, the point charge as origin and OX as the direction of motion of the charge) in the following way:

$$E'_x = \frac{Qx'}{4\pi e_0 r'^3},\ E'_y = \frac{Qy'}{4\pi e_0 r'^3},\ E'_z = \frac{Qz'}{4\pi e_0 r'^3}$$

where $E'_x, E'_y$, and $E'_z$ are the components of auxiliary electric field in the "*Heaviside-Thomson auxiliary system"* S′ at P′ $(x', y', z')$

Now, the distance of the point P′ from the origin in this S′ system, $r' = \sqrt{x'^2 + y'^2 + z'^2}$ where P′ $(x', y', z')$ of the S′ system is the corresponding point P $(x,y,z)$ of the S system. The Coordinate of the corresponding point P is $(x,y,z)$ in Cartesian System and $(r,\theta,\varphi)$ in spherical polar coordinate where $r$ is the distance from the origin, $\theta$ is the angle down from the $x$ axis and $\varphi$ is the angle around from the $z$ axis. Therefore, $x = r\cos\theta$, $y = r\sin\theta\sin\varphi$, $z = r\sin\theta\cos\varphi$, from which $x^2 = r^2\cos^2\theta$ and $y^2 + z^2 = r^2\sin^2\theta$. Therefore:

$$r'^3 = (x'^2 + y'^2 + z'^2)^{3/2}$$

$$= (\gamma^2 x^2 + y^2 + z^2)^{3/2} \text{ [using Eq. (4)]}$$

$$= (\gamma^2 r^2\cos^2\theta + r^2\sin^2\theta)^{3/2} \text{ [in polar coordinates]}$$

$$= \gamma^3 r^3\left[1 - (u^2/c^2)\sin^2\theta\right]^{3/2} \text{ [a result independent of } \varphi]$$

Now, $E = \sqrt{E_x^2 + E_y^2 + E_z^2}$

$$= \sqrt{E_x'^2 + \gamma^2 E_y'^2 + \gamma^2 E_z'^2} \text{ [using Eq. (8)]}$$

$$= \frac{Q}{4\pi\varepsilon_0 r'^3}\sqrt{x'^2 + \gamma^2 y'^2 + \gamma^2 z'^2}$$

$$= \frac{Q}{4\pi e_0 r'^3}\gamma r$$

Thus, independent of $\varphi$, we have:

$$E = \frac{Q}{4\pi\varepsilon_0}\frac{\gamma r}{\gamma^3 r^3}\left[1 - (u^2/c^2)\sin^2\theta\right]^{-3/2}$$

$$= \frac{Qk^2}{4\pi\varepsilon_0 r^2}\left[1 - (u^2/c^2)\sin^2\theta\right]^{-3/2}$$

The auxiliary $\mathbf{E}'$ is directed along OP′. Therefore, the real field $\mathbf{E}$ is directed along OP.

Now, from $\mathbf{B}^* = \nabla \times \mathbf{A}^*$, we have,

$$B_x^* = 0,\ B_y^* = -\frac{u}{c^2}E_z = -\gamma\frac{u}{c^2}E'_x,\ B_z^* = \frac{u}{c^2}E_y = \gamma\frac{u}{c^2}E'_y$$

from which the induced magnetic field is

$$\mathbf{B}^* = \mathbf{u} \times \mathbf{E}/c^2$$

## 4. A Corollary

It can easily be shown that Heaviside's fields obey Maxwell's equations just like Coulomb's fields do. Therefore, if a stationary dipole radiates in free space, it will also radiate while moving in free space at constant translational velocity.

## 5. Electromagnetic Momentum of a Steadily Moving Point Charge

Assuming that the point charge is spherical having the radius $\delta R$, the electromagnetic momentum of a Heaviside's ellipsoid (with the axes $\delta Rk$: $\delta R$ : $\delta R$) while moving rectilinearly with a velocity $\mathbf{u}$ in the OX direction in free space can be calculated as:

$$: P_x = \int\left(D_y B_Z^* - D_Z B_Y^*\right)d\tau$$

$$= \frac{u}{c^2}\varepsilon_0 \int\left(\gamma^2 E_Y'^2 + \gamma^2 E_Z'^2\right)k\,d\tau'$$

$$= \gamma\frac{u}{c^2}\varepsilon_0 \int\left(E_Y'^2 + E_Z'^2\right)d\tau' \quad \text{[cf. eqs.(8\&9)]}$$

where $d\tau$ is the volume element in the moving system (S) and $d\tau'$ is the corresponding volume in the S′ system, **D** is the electric displacement vector in (S). Here, in the S′ system the Heaviside's ellipsoid is exactly a sphere. Therefore,
when evaluated between $\delta R$ and $\infty$, $\int E'^2 d\tau' = Q^2/4\pi\varepsilon_0^2\delta R$ and each integral in the previous equation is equal to. $Q^2/12\pi\varepsilon_0^2\delta R$
Therefore,

$$P_x = Q^2 u/6\pi\varepsilon_0 c^2 \delta Rk = mu \qquad (10)$$

which is the electromagnetic momentum of a point charge moving rectilinearly in free space with velocity $\mathbf{u}$, where $m_0$ and $m$ are the electromagnetic masses of the point charge moving with the velocities near zero and $\mathbf{u}$ respectively in free space such that

$$Q^2/6\pi\varepsilon_0 c^2 \delta R = m_0 \qquad (11)$$

and

$$m_0/k = m \qquad (12).$$

Therefore, **Longitudinal Electromagnetic mass** of a point charge =

$$\partial P/\partial u = \gamma^3 m_0 \qquad (13)$$

and **Transverse Electromagnetic mass**

$$= P/u = \gamma \times m_0 \qquad (14).$$

All electrodynamic equations similar to those of special relativity could readily be deduced from the equation (10) [2]

## 6. Auxiliary Transformation Equations of Lorentz

This Section derives Lorentz's auxiliary equations from those of Heaviside. Following Heaviside and Thomson, Lorentz engaged himself in developing some new auxiliary transformation equations. He liked to solve optical problems of moving bodies as well as electrodynamic problems of charges moving with low and high velocities through those transformation equations. These transformation equations should reduce the equations of moving systems to the form of ordinary formula that hold for systems at rest.

The problem Heaviside and Thomson addressed was to model the potentials for charges having a constant translational velocity in free space. They solved this problem by transforming the D' Alembert's equation in an invariant form with Poisson's in the auxiliary system. Lorentz's problem was to model the radiation from moving bodies. He solved this problem by transforming Maxwell's equations for the moving radiating body to the same form for the auxiliary system, which correlates between the static and dynamic radiation states.

By dint of the 'Corollary' stated in the section 4, we have, $\Box^2\mathbf{E} = 0$

*i.e.*,

$$\partial^2\mathbf{E}/\partial x^2 + \partial^2\mathbf{E}/\partial y^2 + \partial^2\mathbf{E}/\partial z^2 - \frac{1}{c^2}\partial^2\mathbf{E}/\partial t^2 = 0 \qquad (15)$$

where $\mathbf{E}$ is the electric field of a radiating dipole moving with a constant translational velocity $\mathbf{u}$ in free space. From this we get

$$x^2 + y^2 + z^2 = c^2 t^2 \qquad (15a)$$

To solve radiation problems in a way analogous to that shown in Section 2, we are to keep the Maxwell's equation in the same form in the S′ system; *i.e.*, it is now required that $\Box'^2\mathbf{E}' = 0$

*i.e.*,

$$\partial^2\mathbf{E}'/\partial x'^2+\partial^2\mathbf{E}'/\partial y'^2+\partial^2\mathbf{E}'/\partial z'^2-\frac{1}{c^2}\partial^2\mathbf{E}'/\partial t'^2=0 \qquad (16)$$

where $\mathbf{E}'$ is the auxiliary electric field of Heaviside and Thomson in the $S'$ system.

That is,

$$x'^2+y'^2+z'^2=c^2t'^2 \qquad (16a)$$

where $x^/, y^/, z^/$ are defined in the section 2 and $t^/$ is to be chosen appropriately.

Subtracting Eq. (15a) from Eq. (16a) and using the Heaviside-Thomson auxiliary equations

$$x'=\gamma(x-ut),\ \ y'=y,\ \ z'=z \qquad (17)$$

[if electromagnetic action is considered after the time $t$ of the instant when the coordinate attached to the system coincided with the coordinate attached to free space, Eq. (4) should be changed to the equation (17) ]

to replace the primed variables, we get

$$c^2t'^2=(x-ut)^2/(1-u^2/c^2)-x^2-c^2t^2$$

or $$t'=\gamma(t-ux/c^2) \qquad (18)$$

the famous auxiliary time equation of Lorentz.

An interesting fact about the equations is that the inverse Lorentz Transformation equations (which could be deduced from Lorentz Transformation equations) have the same form as the Lorentz Transformation equations themselves; *i.e.*, if

$$x'=\gamma(x-ut),\ \ y'=y,\ \ z'=z,\ \ t'=\gamma(t-ux/c^2) \qquad (19\text{ a-d})$$

then

$$x=\gamma(x'+ut'),\ \ y=y',\ \ z=z',\ \ t=\gamma(t'+ux'/c^2) \qquad (20\text{ a-d})$$

Therefore, Lorentz transformation equations can be deduced in reverse from the dyads of Eqs. (15a) & (16a) and (19a-c) &(20a-c). (These transformation equations were later observed and used by A. Einstein in his theory).

It should be mentioned here that all the quantities $x', y', z', t'$, $\mathbf{E}', \mathbf{B}'$, *etc*., are auxiliary, so Eq. (16) is auxiliary. Thus, from the standpoint of electrodynamics, Lorentz Transformation Equations are tactical, just like the tactical equations of Heaviside and Thomson. These auxiliary equations are very useful to solve correctly the radiation problems associated with steadily moving electromagnetic bodies. The auxiliary equations of Heaviside and Thomson (17) are general, but Lorentz's auxiliary time Eq. (18) is not general. It is only applicable in radiation problems of moving systems and in some special cases. To consider Lorentz's auxiliary time equations as general is contrary to the principles of electrodynamics, and to consider all four Lorentz's transformation Eqs. (17) & (18) as 'real' is an over simplification of electrodynamics and of nature.

## 7. Nature of Electric and Magnetic Fields at the Vicinity of the Earth's Surface

All electrodynamic phenomena on the surface of the moving earth are seen to be independent of the movement of the earth, which implies that the earth carries electric and magnetic fields along with its surroundings just like it carries all other physical objects with it. This simple consideration along with Heaviside's electrodynamics as stated above may serve as an equivalent alternative to special relativity in the domain of electrodynamics within the Newtonian framework [2,3].

We may now extend this consideration to the cases of charges and electromagnetic energy too and reach to some interesting results as given below.

## 8. Interactions of Charges with Gravitating Field

Let a gravitating body with the material mass $M_s$ be at rest at the origin and the point charge '$Q$' be moving with a velocity '$u$' across a gravitating field. Now assume that charges are subject to gravitation and the longitudinal electromagnetic mass of the charge 'q' is proportional to its mass. Thus, in a gravitating field, a point charge will behave just like a mass point. Therefore, the equation of motion of a point charge '$Q$'moving across a gravitating field could be written as follows [when the moving point charge is at $(r,\theta)$ with polar co-ordinate, pole being the source of the field]:

$$(F_G)_r=\gamma^3 m_0\left(\ddot{r}-r\dot{\theta}^2\right),$$

and replacing $(F_G)_r$ by $-\frac{GM_s}{r^2}\gamma^3 m_0$ we have,

$$-\frac{GM_s}{r^2}=\left(\ddot{r}-r\dot{\theta}^2\right) \tag{21}$$

(which implies that point charges have the same acceleration as that of material bodies in the same gravitational field) and

$$(F_G)_\theta=\frac{1}{r}\frac{d}{dt}\left(\gamma^3 m_0 r^2\dot{\theta}\right)=0 \tag{22}$$

where G is the gravitational constant, $(F_G)_r$ and $(F_G)_\theta$ are the radial component and the transverse component of the gravitating force acting on the charge, $\gamma^3 m_0$ is the mass of the charge as well as the longitudinal electromagnetic mass of the charge when the ratio of proportionality of both the quantities is unity. The quantity of charge, large or small is being assumed concentrated in a very small spherical volume with a very negligible small radius δR . From (22) we have, Mechanical Angular Momentum of the charge,

$$A=\gamma^3 m_0 r^2\dot{\theta}=\text{Constant} \tag{23}$$

whence we get,

$$\gamma^3 r^2\dot{\theta}=\text{H, a constant.} \tag{24}$$

Now, we can replace equation (21) as under:

Let $U=1/r$ (25)

From (23) we have

$$\dot{\theta}=HU^2k^3 \tag{26}$$

$$\dot{r}=-Hk^3\frac{dU}{d\theta} \tag{27}$$

$$\ddot{r}=-H^2U^2k^6\frac{d^2U}{d\theta^2} \tag{28}$$

Substituting in (21), we have,

$$\frac{d^2U}{d\theta^2}+U=\frac{GM_s}{H^2}\gamma^6=\frac{GM_s}{H^2}\left(1+3\frac{u^2}{c^2}\right)$$

[when '$u$' is very small in comparison to c and noting that for circular motion $u=r\dot{\theta}$]

$$=\frac{GM_s}{H^2}+\frac{3GM_s}{c^2r^2} \tag{29}$$

## 9. Exact Equation of Planetary Motion

Suppose that a planet of mass $m_p$ contains '$Q$' amount of positive and negative charges in total (ignoring the sign of the charges) and for simple calculation let us assume that the total charges are concentrated at the center of the planet. In this situation, if we assume, as before, that the longitudinal electromagnetic mass of a point charge is proportional to its mass, the equation of planetary motion will be as under: Radial Force=

$$(F_G)_r=\left(m_p+\gamma^3 m_0\right)\times\left(\ddot{r}-r\dot{\theta}^2\right)$$

$$or,-\frac{GM_s}{r^2}\left(m_p+\gamma^3 m_0\right)$$

$$=\left(m_p+\gamma^3 m_0\right)\times\left(\ddot{r}-r\dot{\theta}^2\right)$$

Or,

$$-\frac{GM_s}{r^2}=\left(\ddot{r}-r\dot{\theta}^2\right) \tag{30}$$

where $\gamma^3 m_0$ is the longitudinal electromagnetic mass as well as the mass of the charge associated with the planet, proportionality constant being taken as unity. $M_s$ is the mass of the sun and G is the gravitational constant. Cross-radial Force=

$$(F_G)_\theta=\frac{1}{r}\frac{d}{dt}\left[\left(m_p+\gamma^3 m_0\right)\times r^2\dot{\theta}\right]=0 \tag{31}$$

from which mechanical angular momentum of the planet,

$$A=\left(m_p+\gamma^3 m_0\right)\times r^2\dot{\theta}.$$

In case when '$u$' is very small with respect to 'c' and '$m_0$'is much larger than $3u^2m_p/2c^2$,

$$A=\left(m_p+m_0\right)\times r^2\dot{\theta}/\left(1-u^2/c^2\right)^{3/2}$$

from which we have,

$$\gamma^3 r^2\dot{\theta}\approx H=\text{Constant} \tag{32}$$

From equs. (30) & (32) we may deduce

$$\frac{d^2U}{d\theta^2}+U=\frac{GM_s}{H^2}+\frac{3GM_s}{c^2r^2} \tag{33}$$

## 10. Gravitational Red Shift

Suppose that a ray is coming from the surface of a star of radius $R_t$ and of mass $M_t$ to the surface of the earth, r distance away from the star. As per our previous discussion, we assume that electromagnetic energy has the same acceleration as that of material bodies as well as point charges in the same gravitational field. Therefore, the velocity of the ray at the surface of the earth

$$V = \left( c - \int_{R_t}^{r} \frac{GM_t}{r_1^2} \frac{dr_1}{c} \right)$$

($r_1$ being the distance of an arbitrary point on the ray from the star)

$$= c\left(1 - \frac{GM_t}{R_t c^2}\right) \qquad (34)$$

from which we have,

$$\frac{\omega'}{\omega} = \left(1 - \frac{GM_t}{R_t c^2}\right) \qquad (35)$$

$\omega$ is the radian frequency of light at the surface of the star and $\omega'$ is the radian frequency of the same light ray at the surface of the earth, which is said to have been verified by many refined experiments.

**12. Bending of Light Ray Grazing the Surface of the Sun**

The above study empowers us to apply equations (21) &(22) to the case of propagation of electromagnetic radiation grazing the surface of the sun having gravitational mass $M_s$ and radius $R_s$.

Therefore, equation of motion of a light beam passing through a medium (such that the velocity of the light beam $u$ is much smaller than $c$ in the medium) , medium being fixed with the sun's surface will take the form:

$$\frac{d^2U}{d\theta^2} + U = \frac{GM_s}{H^2} + \frac{3GM_s}{c^2 R_s^2}$$

Now for a small volume of radiation energy, the mechanical angular momentum A is finite. Therefore, H for this small volume of radiation energy will be infinite, as '$m_0$' for this small volume of radiation energy =0.

In such a situation, the above equation for the propagation of a small volume of radiation energy through the sun's adjacent gravitational field inside a medium wherein $u << c$ should be

$$\frac{d^2U}{d\theta^2} + U = \frac{3GM_s}{c^2 R_s^2}. \qquad (36)$$

which is widely believed to have confirmed by "billion dollar experiments"

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Electromagnetic energy is neutral. Still we see that the equ. (29) is fully applicable to the case of electromagnetic energy as good as to the cases of point charges. Therefore, we are constrained to speculate that electromagnetic energy, too, may have point existence consisting of equal amount of opposite point charges.[3]

**13. Conclusion**

Electromagneticians have amply demonstrated that electromagnetic fields, charges and electromagnetic energy are all real physical entities. We have shown in this paper that all those entities are not only subject to gravitation just like all other physical objects, they have the same acceleration as that of physical bodies in the same gravitating field, and this simple consideration is equivalent to special and general relativity.

To write this paper, I have been much influenced by the works of D.B.Ghosh [2] and of Prof. K.C. Kar [4].

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