Alternatives to Einstein in the Light of the Origin and Development of the Theory of Relativity.

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1.Historical Approach

Both of the Poincare-Lorentz and the Einstein-Minkowski Programmes of Relativity originate from the assumption that Lorentz transformation equations are real.

Lorentz deduced the Lorentz transformation equations as some auxiliary device to solve radiation problems of moving systems of charges and currents in an analogous way as Heaviside and Thomson deduced their auxiliary equations i.e., $x' = \gamma x$, $y' = y$, $z' = z$ [where $\gamma = 1/k$ and $k = \sqrt{1 - u^2/c^2}$, (x,y,z)are the co-ordinates attached with the free space and 'u' is the velocity of the system in the free space] to solve the potential problem of the system of charges and currents moving steadily in free space. If electromagnetic action is considered after the time t of the instant when the coordinate attached to the system coincided with the coordinate attached to free space, the above equation should be changed to the following equation $x' = \gamma(x - ut),\ y' = y,\ z' = z$. The device of the system imaginarily elongated as per the above equation and comprising of x', y', z' was successfully initiated and used by Heaviside (1888,1889) and Thomson (1889) to study the potentials of moving system of charges classically.

Lorentz's auxiliary equations i.e.,[$x' = \gamma(x - ut),\ y' = y,\ z' = z,\ t' = \gamma(t - ux/c^2)$ or $x = \gamma(x' + ut'),\ y = y',\ z = z',\ t = \gamma(t' + ux'/c^2)$] are actually extended Heaviside-Thomson auxiliary equations both of which are solely based on Newton-Maxwell interpretation of the physical world. (Vide the deduction in my paper in PIRT-2000 also given in the Appendix-I & Appendix-II of this paper)

Those auxiliary equations are very useful to solve correctly many electrodynamic as well as radiational problems of moving systems of charges and currents classically. Therefore, all electrodynamic and radiation equations used by relativists are the same as the classical equations. Anybody who uses these equations to explain any electrodynamic phenomena employ classical physics at the outset.

Up to this point of development, Lorentz proceeded classically. But thereafter he left the classical path for the following reasons:

Light propagates with the speed 'c' in free space where Earth moves with a very high velocity. Therefore, the speed of light should change if measured on Earth depending on the direction of the movement of the earth. Michelson in 1881 and Michelson & Morley in 1887 measured the difference of speeds of light in two different directions on Earth and got the null result i.e., there is no direction dependence of the speed of light on the surface of the moving Earth. Moreover it was known at that time that all electrodynamic phenomena on the surface of the moving earth are independent of the movement of Earth. All those perplexing phenomena bewildered Lorentz.

Wrong Route Traversed by Lorentz

To overcome the difficulty, especially to explain the null result of the Michelson-Morley Experiment, FitzGerald in 1889 suggested the real contraction of moving bodies.

As discussed previously, by proceeding from the Heaviside-Thomson auxiliary space equations [*i.e.*, $x' = \gamma(x - ut)$, $y' = y$, $z' = z$] Lorentz developed his auxiliary time equation $t' = \gamma(t - ux/c^2)$ to solve radiation problems of moving bodies classically. But, Lorentz could not explain the null result of the Michelson-Morley experiment from any electro-dynamic principle. So he accepted the doctrine of FitGerald that moving bodies really contract *i.e.*, the equations $x' = \gamma(x - ut)$, $y' = y$, $z' = z$, are real for moving electromagnetic bodies as well as moving mechanical bodies. This view was endorsed by Larmor.

From this consideration, Earth is also really dilated to its direction of motion when measured on Earth.

Now, if $x' = \gamma(x - ut)$ is a real equation for the moving Earth, then x', y', z' are not some arbitrary auxiliary elongated unreal Cartesian co-ordinates, and $\mathbf{E}'$ and $\mathbf{B}'$ will not be auxiliary fields of similar nature, invented to solve some problems, as classical electro-magneticians did. Instead, x', y', z' will be the real co-ordinates of the moving Earth, and $\mathbf{E}'$ and $\mathbf{B}'$ are the real fields measured on the moving Earth. Thus, when a stick on the moving Earth is kept parallel to the direction of motion of Earth and is measured on Earth, its length, according to Lorentz, will be greater on Earth than its length if measured from the free space. Fitzgerald, Lorentz and Larmor have interpreted this as meaning that moving objects contract towards their directions of motions.

Lorentz, however, considered that his time equation is auxiliary and unreal. Thus, to Lorentz, the Cartesian co-ordinate derivative part of the auxiliary Maxwell equation 2 of the Appendix-II is real, while the time derivative part of the same equation is auxiliary and unreal and the auxiliary Maxwell equation 2 of the Appendix-II is quasi-real to him.

Max Abraham contradicted correctly the real contraction of moving objects. Thus the Lorentz transformation equations, though derived from classical electro-dynamics, when infused with the idea of real contraction while moving violated classical mechanics. These masterpieces of Lorentz, although immensely effective in calculating the radiation problems of moving point charges, were illegitimate from the standpoint of mechanics. Lorentz was fully aware of this.

Einstein's Assumptions

Einstein assumed with a further novelty that the time equation of Lorentz was also real, in addition to the reality of the transformation equations of Heaviside-Thomson. So to him the equation 2 of Appendix-II was not quasi-real as it was to Lorentz; it was fully real to Einstein. Lorentz did not proceed to prove the real contraction of his transformation equation from any electro-dynamic or general principle. It was accepted by him as an *ad-hoc* basis to explain the null result of the Michelson-Morley Experiment.

Einstein's step was however to justify by some arbitrary principles the reality of the useful Lorentz transformation equations, and to qualify that these principles are absolutely real as such, and that Lorentz transformation equations derived inversely from those principles are also absolutely real. Thus, Einstein justified the Eq.1 and Eq.2 of Appendix-II *i.e.*, equations (1a) and (2a) of the same appendix, by the principle that *the velocity of light is the same for all inertial frames by which he means (with some philosophy) the dyad of equations i.e.,* $x^2 + y^2 + z^2 = c^2t^2$ and $x'^2 + y'^2 + z'^2 = c^2t'^2$, and, thereafter to justify the sets of equations 5a-c & 6a-c of the appendix-II, he principled *that all physical laws are covariant to all inertial frames, by which he means (obviously with some philosophy)* $x' = \gamma(x - ut)$, $x = \gamma(x' + ut')$, *and* $y' = y$ *and* $z' = z$, *where* γ *is an arbitrary constant.* Thus, these two sets of two equations when solved will give Lorentz transformation equations, and if those principles are real, then all the four Lorentz Transformation Equations will also be real.

In his interpretation Einstein has removed the question of real contraction of moving electron as advocated both by Lorentz and Poincare. According to Einstein's interpretation , the shape and size of the electron remain the same whereas different observers at different velocities would observe it differently. Therefore, Einstein's interpretation is superior to Lorentz's though both have deluded to the same false route.

Thereafter Einstein has further proceeded to extend his beloved covariant principle to the cases of accelerated motions which is according to him equivalent to gravitation.

Therefore, from historical point of view, we may say that both of the Poincare-Lorentz and Einstein-Minkowski Programmes originate from the assumptions that the auxiliary transformation equations of Lorentz are real.

Right Track

Therefore, if any school of thought could explain the null result of the Michelson-Morley Experiments and the allied phenomena from classical point of view, both of the Poincare-Lorentz and Einstein-Minkowski Programmes would be overthrown from the domain of electrodynamics and consequently from the domain of mechanics at a stroke.

To do so we have proposed as under : Equations of electro-dynamics describe interactions of charges with electric and magnetic fields, as well as propagation of electromagnetic disturbances in free space. Electromagnetic fields possess momentum and energy that could be experienced by our sense organs. It follows from these facts that electric and magnetic fields are real material entities to the same extent as charged bodies are real material entities. Now, all material bodies are subject to gravitation. Therefore, it is likely that electromagnetic fields too are subject to gravitation.

Lorentz being one of the greatest electromagnetician of the age was expected to imply that Earth carries electric and magnetic fields along with its surroundings just like it carries all other physical objects with it. This simple consideration along with classically-developed Heaviside's electrodynamics as stated above would immediately clear the problem.(Vide our paper in PIRT-2000)

He might then extend this consideration to the cases of charges and electromagnetic energy too and reach to some interesting results as given below

Gravitational Problem

Lorentz could have presented a deduction of the exact equation of planetary motion following Maxwell and Newton, considering that a planet contains mass and charges and assuming that charges are not only subject to gravitation, they have the same acceleration as that of material bodies in the same gravitating field. This would at once explain the advance of perihelion of Mercury, and would simply explain gravitational red shift and bending of light rays grazing the surface of the sun, if the same consideration is further extended to the cases of electromagnetic energy. [vide my deduction in PIRT-2004 also given in Appendix-III of this paper.

Unfortunately, Lorentz could not foresee any such simple solution. His un-judicious assumption paved the way for establishing the supremacy of pure thought and thought-experiments over appropriate experiments and their rational interpretation in physics.

2. Philosophical Approach

Isaac Newton described his physics within the framework of absolute space and absolute time, independent of each other. According to Newton, even if the Universe were destroyed, absolute space and absolute time would still exist [1]. But Newton's views were very strongly contested by Leibnitz, who viewed space as the order of co-existent phenomena, and time as the order of successive phenomena. Both space and time would therefore be relative: if there were no phenomena, neither would exist [1]. Yet there exists the centrifugal force acting at the surface of our Earth. According to Newton, this indicates that the rotation of Earth is absolute. Hence, absolute space obviously exists. These opinions battled for centuries. Euler, Neumann, Maxwell [2] were influenced by the thoughts of Newton, but Berkeley [3], Boscovich, Stallo and Mach [4] shared the view of Leibnitz.

Mach argued that Earth rotates with respect to the fixed stars, and if a person does not see the fixed stars, he cannot settle whether Earth is rotating or not. Mach proposed that Newton should stop first the rotation of Earth. Then he should rotate the heaven of fixed stars around it. Now, in such a situation, if there would be no centrifugal force acting at the surface of Earth, one might conclude that the centrifugal force originates from the rotation of the Earth. As those feats could not be done, it cannot be said with certainty that the centrifugal force originates from the rotation of this planet. Mach believed that a new system of mechanics could also be developed to explain the centrifugal force acting on the surface of the Earth by the action of rotating stars by considering the Earth to be stationary. But he did not formulate such a system of mechanics.

After Mach, it was clear that the absolute space and absolute time of Newton are only his assumptions – the mental construction of Newton. But because Newton created a system of mechanics that can explain physical phenomena correctly, everyone in the scientific communities rightly believed in Newton's absolute space and absolute time. But from the end of 19th century, it was known that a good many experiments of electrodynamics were not in harmony with Newton's mechanics, *e.g.* **1)** when a charge moves, its mass increases with speed, which is not compatible with Newton's understanding of Nature; **2)** when a charge is accelerated in an electromagnetic field, Newton's sacred law $\mathbf{F} = m\mathbf{a}$ is violated, and the charge cannot exceed the speed of light under any circumstance; **3)** light propagates with speed c in free space, where Earth is moving with a very high velocity; therefore, the speed of light should change if measured on Earth, depending on the direction of Earth's movement. But with repeated experiments it was settled that the speed of light is the same c in all directions, if measured from Earth; **4)** from the consideration of Newton, the laws of reflection, refraction, diffraction and interference of terrestrial and astral light should differ. But no such difference was observed. **5)** As measured by the scientists, the speed of light propagating in a moving medium is $V = c/n + u(1 - 1/n^2)$, where n is the refractive index of the medium, and u is the speed of the medium. This is not in conformity with Newton's understanding of velocity.

To explain all these perplexing electromagnetic results on an *ad hoc* basis, H.A. Lorentz developed some real equations that are not Newtonian in appearance. The transformation equations between two inertial frames S and S' with translation at relative speed u along their x axes are: **i)** $x' = \gamma(x - ut)$, **ii)** $y' = y$, **iii)** $z' = z$, **iv)** $t' = \gamma(t - ux/c^2)$, where $\gamma = 1/\sqrt{1 - u^2/c^2}$.

It was Albert Einstein who deduced those same four equations from assumptions allowing space and time to be relative, and time to be a linear function of coordinates. His assumptions were: **i)** $x^2+y^2+z^2=c^2t^2$, **ii)** $x'^2+y'^2+z'^2=c^2t'^2$, commonly known as the principle of light-speed constancy in all inertial frames; **iii)** $x'=\gamma(x-ut)$, **iv)** $x=\gamma(x'+ut')$, commonly known as the principle of covariance of all physical laws; with **iv)** $y'=y$, **v)** $z'=z$, and with γ then derived from the stated principles. Thus within the framework of relative space and relative time, Einstein was able to create a new system of mechanics, which was the dream of philosophers and physicists for centuries. Therefore, Albert Einstein has been considered as one of the greatest philosophers and scientists of the 20th century.

But the first of Einstein's assumptions one is a derivation from Maxwell's wave equation, which is applicable only in free space, but not at all in other situations. The next three Einstein assumptions are absurd from any realistic viewpoint. Moreover, Einstein could not satisfy Mach's criterion; *e.g.*, when a radiating dipole moves on Earth, and an observer is at rest on Earth, there is transverse Doppler effect, which has been confirmed by experiments. But if the radiating dipole is at rest on Earth, and an observer moves in the same opposite motion, then from consideration of the Einsteinian idea of relative space, there should also be the same transverse Doppler's effect. But this could not be shown by any experiment.

Therefore, it is clear that, just as absolute space and absolute time were the mental creation of Newton, so were the relative space and relative time the mental creation of Einstein. The only difference is that Newton's assumptions are easy, independent, and plausible, whereas Einstein's assumptions are complicated, interrelated and absurd.

Moreover, all those perplexing results stated in the third paragraph of this section could easily be shown to conform to the physics of Newton and Maxwell.(Vide our paper in PIRT-2000)

3.Methodological Approach

Einstein uses the principle of co-variance for all physical laws to develop the theory of relativity. The principle is intrinsically hollow and can not create any new physical system. It can only trim the Newton-Maxwell laws which are the only laws of nature. Therefore its role in physics if any is corrective. The principle of covariance was used by Heaviside, Thomson and their contemporaries in the eighties and nineties of the past century to study electrodynamics of charges in order to match electrodynamic potential with auxiliary potential .It was used as a mathematical device to calculate dynamic potential in terms of auxiliary potential which is related with static potential. Einstein has extended this principle to the domain of mechanics and gravitation. The principle can be used in the study of point charge or line current electrodynamics as equally as classical electrodynamics. There is not a single even indirect proof to demonstate that the principle could be used for big charge, surface current or volume current electrodynamics or mechanical systems. It is accepted by institutional physicists like "God is everywhere principle".

4.Neo-Relativistic Approach

Marmet uses "the increase of length of matter" and "slowing down of clocks" – which according to him are natural consequences of mass energy conservation, and he likes to combine these with classical physics to explain electromagnetic phenomena [5]. This is nothing but special relativity in a new format. Selleri has invented some other relativistic transformation equations from "empirically based assumptions", one of which reads, "the two way velocity of light is the same in all inertial frames" [6]. It is only verified from experiments that the two-way velocity of light is the same in all directions on the surface of the moving Earth, which is a very large body. It has not been verified at all in any other inertial frames. Therefore, Selleri's theory in essence is likely to be neither correct nor different from special relativity.

5.Anti-Relativistic Approaches

Analyses of AG Kelly and excellent papers of Jo'zef Wilczyn'ski and C.A. Zapffe published in the 80's and 90's in the Indian Journal of Theoretical Physics and in the Toth-Maatian Review are rich with insight and help us much to understand nature, physics and relativity.

5. Experimental Foundation

The Lorentz transformation equations are actually the relation of co-ordinates and time between the imaginary auxiliary state and real dynamic state of electro-dynamics. The equations are never real even in the domain of electro-dynamics.

Einstein made an effort to justify the reality of those relations with the assumption (or with some principles originating from this assumption) that auxiliary state itself is real. Therefore, the transformation equations are general laws of Nature.

But, to the electro-magneticians, the reality of these relations is artificial and meaningless. Incidentally, for point-charge electro-dynamics, the calculations of both electro-magneticians and relativists concur owing to the same geometry of a point charge in the static as well as the auxiliary state.

In big-charge electro-dynamics, electro-magneticians will correlate between S and S_0 through the auxiliary state $S^{\prime}$. Whereas, relativists will correlate between S and $S^{\prime}$, $S^{\prime}$ having been assumed as the real system by them. This leaves no room for S_0. Therefore, for big-charge electro-dynamics, this difference of approach leads to different results. So, any effort to explain electro-dynamics by means of special relativity can only be partly successful in case of big-charge experiments. However, this was never gone into.

In a situation where line current flows in any arbitrary direction, and surface current and volume current flow in the direction of the movement of the system, equations (32a) and (32b) of the Appendix-I work for both electro-dynamics as well as special relativity. In this situation too, electro-dynamic calculations will differ much from relativistic calculations, excepting a few special cases for the reasons stated in the previous paragraph.

In the surface current and volume current electro-dynamics, when these currents flow in any arbitrary direction, equations (32a) and (32b) of the Appendix-I fail to work for electro-dynamics, whereas, these equations work for special relativity. Therefore one wonders why relativists do not make any effort to substantiate their claim by conducting experiments relating to surface current and volume current within the moving system in any arbitrary direction. [7]

To justify the theory that the auxiliary state itself represents reality for all physical phenomena, relativists make use of the situation that for all mechanical bodies the quantity $\sqrt{1-u^2/c^2}$ is approximately equal to 1. This, however, does not in any sense prove the reality of the auxiliary state in mechanics.

In order to demonstrate the relativity of space, one should be able to produce results like a transverse Doppler effect when the source is at rest and the observer is in opposite motion, and so on.

Out of four famous assumptions of special relativity; *i.e.*, **i)** $x^2+y^2+z^2= \ c^2t^2$, **ii)** $x'^2+y'^2+z'^2=c^2t'^2$, (commonly known as the principle of constancy of the velocity of light in all inertial frames), **iii)** $x'=\gamma(x-ut)$; (iv) $x=\gamma(x'+ut')$ (commonly known as the principle of covariance of all physical laws), the first one is applicable only in free space and not in other inertial frames. The three other assumptions are absurd from any realistic viewpoint.

Conclusion

From historical study it stands that the path adopted either by the Poincare-Lorentz or by the Einstein-Minkowski is wrong and easily be corrected by the wise and simple application of classical physics. From philosophical point of view, the concept of time and space of the relativists is very interesting and grandiose-thought-provoking but not superior to that of Newton. Experiments cited by the relativists in their favor uphold equally the classical view. Methodology adopted by the relativists may have only partial validity, if any. Scientifically its role in physics, if any, is only corrective .All these demand the necessity of total rejection of the theory in favor of a more plausible, simple and economic understanding of the physical world preferably in the Newton-Maxwell framework. Einstein is the Hegel of the twentieth century physics. We are in need of a Marx in this century to replace monumental subjectivism of the past century physics and there will be no dearth of philosophy and physics to justify this.

References:

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[4] E. Mach, **Science of Mechanics**, pp. 280-288 (The Open Court, Chicago, 1942),.

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[**Appendix-I**

Derivation of the Auxiliary Heaviside-Thomson Transformation Equations classically :

Maxwell elegantly explained the nature of propagation of electromagnetic disturbances in free space. Oliver Heaviside, Thomson, and their contemporaries, proceeded to cast the potential problems of a system of moving charges into Poisson's form by the elongation of the coordinate axis in the direction of movement of the system of charges, taken to be the OX axis. Thus they developed a way of solving dynamic potential problem in a static format by using a static auxiliary equation in the form of Poisson's potential equation. In short, they correlated between the static and dynamic states through an auxiliary state.

The E-field and B-field originating from a system of charges and currents when the system is stationary in free space are governed by Poisson's equations; *viz.*

$$\partial^2\Phi/\partial x^2+\partial^2\Phi/\partial y^2+\partial^2\Phi/\partial z^2=-\rho/\varepsilon_0 \qquad (1)$$

and

$$\partial^2\mathbf{A}/\partial x^2+\partial^2\mathbf{A}/\partial y^2+\partial^2\mathbf{A}/\partial z^2=-\mathbf{J}/\varepsilon_0 c^2 \qquad (2)$$

where Φ and ρ are the scalar potential and charge density, and $\mathbf{A}$ and $\mathbf{J}$ are the vector potential and current density of the system stationary in free space, ε_0 and μ_0 are the permittivity and permeability of free space,

$c = 1/\sqrt{\mu_0 e_0}$, and the introduced Cartesian coordinate system is in free space. From the above equations we have:

$$E_x = -\partial\Phi/\partial x,\;\; E_y = -\partial\Phi/\partial y,\;\; E_z = -\partial\Phi/\partial z \qquad (3)$$

and
$$\mathbf{B} = \nabla\times\mathbf{A} \qquad (4)$$

When this system of charges and currents is stationary in free space, we may call it 'the stationary system S_0'. But if the system moves with constant translational velocity, what will the E-field and the B-field in the free space be? We know that when this system moves in free space, the E-field in free space will induce a B-field $\mathbf{B}^*$, which, if varying, will induce an electric field. Therefore, we may say qualitatively that when the system moves, the E-field in free space changes its magnitude and direction, and an induced B-field $\mathbf{B}^*$ emerges. The same happens for the B-field: it will also change its magnitude and direction, and an induced E-field $\mathbf{E}^*$ will emerge.

Let us now proceed to deduce the exact formulas for the E-field and the B-field while the system moves in free space with constant translational velocity $\mathbf{u}$, simply using Maxwell's equations, in the way first exemplified by Oliver Heaviside, Thomson and their contemporaries. As is well known, when the system translates in free space with a constant translational velocity, the E-field and the B*-field are governed by d'Alembert's equation, *viz.*,

$$\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + \partial^2\Phi/\partial z^2 - \partial^2\Phi/c^2\partial t^2 = -\rho/\varepsilon_0 \qquad (5)$$

and

$$\partial^2\mathbf{A}^*/\partial x^2 + \partial^2\mathbf{A}^*/\partial y^2 + \partial^2\mathbf{A}^*/\partial z^2 - \partial^2\mathbf{A}^*/c^2\partial t^2 = -\rho\mathbf{u}/\varepsilon_0 c^2 \qquad (6)$$

Assuming $\mathbf{u}$ in the OX direction and comparing (5) and (6), we have

$$A_x^* = u\Phi/c^2,\;\; A_y^* = 0,\;\; A_z^* = 0 \qquad (7)$$

(u_y and u_z being zero). Here

$$E_x = -\partial\Phi/\partial x - \partial A_x^*/\partial t,\; E_y = -\partial\Phi/\partial y,\; E_z = -\partial\Phi/\partial z \qquad (8)$$

(A_y^* and A_z^* being zero), and

$$\mathbf{B}^* = \nabla\times\mathbf{A}^* \qquad (9)$$

We may call such a system 'the dynamic system S', where charges (and currents) inside the system move with the system in free space with a constant translational velocity $\mathbf{u}$ in the OX direction. Here $\Phi(x,t)$ is a function of two independent variables, x and t. From the definition of two independent variables we have

$$d\Phi = (\partial\Phi/\partial x)dx + (\partial\Phi/\partial t)dt = (\partial\Phi/\partial x)u\,dt + (\partial\Phi/\partial t)dt$$

(dx being equal to udt). Moreover, in such a situation, the potentials at the point (x,y,z) at the instant t and the potentials at the point ($x+udt,\ y,\ z$) at the instant $t+dt$ in free space will be the same; *i.e.*,

$$\Phi(x,y,z,t) = \Phi(x+udt, y, z, t+dt) = \Phi(x,y,z,t) + d\Phi$$

From the above two equations we have

$$\Phi = \Phi + (\partial\Phi/\partial x)u\,dt + (\partial\Phi/\partial t)dt \qquad (10)$$

$$\therefore \partial\Phi/\partial t = -u\partial\Phi/\partial x \quad \text{and} \quad \partial^2\Phi/\partial t^2 = +u^2\,\partial^2\Phi/\partial x^2 \quad (11,12)$$

Similarly, using Eqs. (7) and (11),

$$\partial A_x^* / \partial t = -(u^2 / c^2)\partial\Phi / \partial x \qquad (13)$$

Now, Eq. (5) could be written as

$$(1 - u^2 / c^2)\partial^2\Phi / \partial x^2 + \partial^2\Phi / \partial y^2 + \partial^2\Phi / \partial z^2 = -\rho / \varepsilon_0 \qquad (14)$$

Attach the co-ordinates x', y', z' to the moving system, and let these coincide with the co-ordinates x, y, z of the free space at $t = 0$. Now imagine an elongation of the moving system as below towards –OX, which is the direction of motion of the system:

$$x' = x\Big/\sqrt{1 - u^2 / c^2}\,,\ y' = y,\ z' = z \qquad (15)$$

for which the transformation of volume charge density ρ becomes

$$\rho' = \rho k \text{ with } k = \sqrt{1 - u^2 / c^2} \qquad (15a)$$

and Eq. (14) takes the form

$$\partial^2\Phi / \partial x'^2 + \partial^2\Phi / \partial y'^2 + \partial^2\Phi / \partial z'^2 = -\rho / \varepsilon_0 \qquad (16)$$

which is again a Poisson equation.

If electromagnetic action is considered after the time t of the instant when the coordinate attached to the system coincided with the coordinate attached to free space, Eq. (15) should take the form

$$x' = \gamma(x - ut),\ \ y' = y,\ \ z' = z) \qquad (17)$$

where $$\gamma = 1 / k = 1\Big/\sqrt{1 - u^2 / c^2} \qquad (18)$$

Eq. (17) conjointly with Eq. (18) may be called ‘the auxiliary transformation equation of Heaviside and Thomson’.

Below, the value of Φ in the S system will be connected to the potential Φ' of a stationary auxiliary system S' in which all the coordinates parallel to OX have been changed in the ratio determined by Eq. (15). This transformed coordinate S' (x', y', z') system, which is obviously imaginary, will be used as a mathematical tool to correlate electromagnetic phenomena between the static S_0 and the dynamic S [1(i),1(iii),2,3,4,5]. This transformed auxiliary elongated coordinate system will be called ‘Auxiliary system S'’. The electrodynamic role of S' was first illustrated by Thomson.

In the Auxiliary system S', we have

$$\partial^2\Phi' / \partial x'^2 + \partial^2\Phi' / \partial y'^2 + \partial^2\Phi' / \partial z'^2 = -\frac{\rho'}{\varepsilon_0} = -\frac{\rho}{\varepsilon_0}\sqrt{1 - u^2 / c^2} \qquad (19)$$

where Φ' electrostatic potential in Auxiliary system S'. By comparing (16) and (19) we have, $\Phi' = \Phi k$, or

$$\Phi = \gamma\Phi' \qquad (20)$$

Therefore, [using the Eqs. (13) & (20)]

$$\begin{aligned} E_x &= -\frac{\partial\Phi}{\partial x} - \frac{\partial A_x^*}{\partial t} = -\frac{\partial\Phi}{\partial x} + \frac{u^2}{c^2}\frac{\partial\Phi}{\partial x} = -\frac{\partial\Phi'}{\partial x'} = E_x' \\ E_y &= -\partial\Phi / \partial t = -\gamma\partial\Phi' / \partial y' = \gamma E_y' \ \ (A_y \text{ being } 0) \\ E_z &= \gamma E_z' \ \ (A_z \text{ similarly being } 0) \end{aligned} \qquad (21)$$

From $\mathbf{B}^* = \nabla \times \mathbf{A}^*$, we have

$$\begin{aligned} B_x^* &= 0,\ B_y^* = -\,uE_z / c^2 = -\gamma uE_z' / c^2 \\ B_z^* &= \,uE_y / c^2 = \gamma uE_y' / c^2 \end{aligned} \qquad (22)$$

Eqs. (21) & (22) are general, and may be used to determine the electric fields and the induced magnetic fields of moving charges of any shape and size.

For the independent magnetic field originating from a system of current when the system is moving with a constant translational velocity $\mathbf{u}$, we have,

$$\partial^2 A_x / \partial x^2 + \partial^2 A_x / \partial y^2 + \partial^2 A_x / \partial z^2 - \partial^2 A_x / c^2 \partial t^2 = -\rho V_x / \varepsilon_0 c^2 \quad (23)$$

and

$$\partial^2 A_y / \partial x^2 + \partial^2 A_y / \partial y^2 + \partial^2 A_y / \partial z^2 - \partial^2 A_y / c^2 \partial t^2 = -\rho V_y / \varepsilon_0 c^2 \quad (24)$$

and the similar equation for the z-component. Replacing Φ by $\mathbf{A}$ in Eq. (10), Eqs. (23) and (24) could be transformed (in the way previously shown) to the following:

$$\partial^2 A_x / \partial x'^2 + \partial^2 A_x / \partial y'^2 + \partial^2 A_x / \partial z'^2 = -\rho V_x / \varepsilon_0 c^2 \quad (25)$$

and

$$\partial^2 A_y / \partial x'^2 + \partial^2 A_y / \partial y'^2 + \partial^2 A_y / \partial z'^2 = -\rho V_y / \varepsilon_0 c^2 \quad (26)$$

and the similar equation for the z' component. Here $\rho\mathbf{V}$ is the current density in the S system.

In the auxiliary system S′, for line currents flowing within the moving system in any arbitrary directions, and for surface currents and volume currents flowing within the moving system in the direction of movement of the system (*i.e.*, when the magnetic field depends on the length of the current element but not on its cross section), we have

$$\frac{\partial^2 A'_x}{\partial x'^2} + \frac{\partial^2 A'_x}{\partial y'^2} + \frac{\partial A'_x}{\partial z'^2} = -\frac{\rho' V_x}{\varepsilon_0 c^2} = -\frac{\rho V_x}{\varepsilon_0 c^2}\sqrt{1 - u^2 / c^2} \quad (27)$$

$$\partial^2 A'_y / \partial x'^2 + \partial^2 A'_y / \partial y'^2 + \partial^2 A'_z / \partial z'^2 = -\rho V_y / \varepsilon_0 c^2 \quad (28)$$

and the similar equation for the z'-component. By comparison of (25) and (27), (26) and (28), we have,

$$A_x = \gamma A'_x,\ A_y = A'_y,\ A_z = A'_z \quad (29)$$

whence

$$\begin{aligned} B_x &= [\partial A_z / \partial y - \partial A_y / \partial z] = [\partial A'_x / \partial y' - \partial A'_y / \partial z'] = B'_x \\ B_y &= [\partial A_x / \partial z - \partial A_z / \partial x] = [\gamma \partial A'_x / \partial z - \gamma \partial A'_z / \partial x'] = \gamma B'_y \quad (30) \\ B_z &= [\partial A_y / \partial x - \partial A_x / \partial y] = [\gamma \partial A'_y / \partial x' - \gamma \partial A'_x / \partial y'] = \gamma B'_z \end{aligned}$$

where A'_x, A'_y and A'_z are the components of the Auxiliary magnetic potential in the Heavisidean imaginary elongated system S′. For the induced vector, we have the relation $\mathbf{E}^* = -\mathbf{u} \times \mathbf{B}$, from which we have,

$$E^*_x = 0,\ E^*_y = uB_z = \gamma u B'_z,\ E^*_z = -uB_y = -\gamma u B'_y \quad (31)$$

These equations correlate between the induced electric vector in the moving system S and the auxiliary magnetic vector in S′.

Now, if the sources of an independent electric field (originating from charges of any shape and size) and an independent magnetic field (originating from line currents flowing within the system in any arbitrary directions), move with the system at the constant translational velocity $\mathbf{u}$ in free space, then from consideration of Eqs. (21), (22), (30), and (31), we can derive the following auxiliary field equations:

$$\begin{aligned} &E_x = E'_x,\ E_y = \gamma[E'_y + uB'_x],\ E_z = \gamma[E'_z - uB'_y] \\ &B_x = B'_x,\ B_y = \gamma[B'_y - uE'_z / c^2],\ B_z = \gamma[B'_z + uE'_y / c^2] \end{aligned} \quad (32a)$$

or

$$E'_x = E_x,\ E'_y = \gamma\left[E_y - uB_x\right],\ E_z = \gamma\left[E'_z + uB'_y\right]$$
$$B'_x = B_x,\ B'_y = \gamma\left[B_y + uE_z / c^2\right],\ B'_z = \gamma\left[B_z - uE_y / c^2\right] \quad (32b)$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and the magnetic fields of the system of charges and currents having the constant translational velocity $\mathbf{u}$ in free space, and $\mathbf{E}'$ and $\mathbf{B}'$ are corresponding auxiliary quantities.

Thus, we see that the electromagnetic quantities in our moving system S are not connected with the same quantities of the same system at rest (S_0). These quantities of the moving system S are connected by the equation (32a) with the corresponding quantities of the system (S') in which the co-ordinates parallel to the OX axis lying along the movement of the system have been elongated by the Eq. (15).

Those Eqs. (32) are also valid for induced electromagnetic fields when the inductor or the inducted body moves with respect to free space. For dealing with steadily moving point charges, we may use the following two corollaries, the first of which correlates between the static system S_0 and the auxiliary system S', and the second of which describes the validity of Maxwell's equation for Heaviside's fields; *viz.*,

Corollary 1. In a stationary system S_0, if $\mathbf{E}_0$ is the source of $\mathbf{B}_0$, and $E_0 = bB_0$, where b is constant, then in the auxiliary system S', $E' = bB'$.

Corollary 2. It can easily be shown that Heaviside's fields obey Maxwell's equations just like Coulomb's fields do. Therefore, if a stationary dipole radiates in free space, it will also radiate while moving in free space at constant translational velocity.

Heaviside's Electrodynamics in Free Space

The first free-space application deals with the electric field $\mathbf{E}$ and the induced magnetic field $\mathbf{B}^*$ of a point charge Q moving with a constant translational velocity $\mathbf{u}$. Following Heaviside, the fields could be calculated at a point P (x, y, z) in free space (considering at a particular instant, the point charge as origin and OX as the direction of motion of the charge) in the following way:

$$E'_x = \frac{Qx'}{4\pi e_0 r'^3},\ E'_y = \frac{Qy'}{4\pi e_0 r'^3},\ E'_z = \frac{Qz'}{4\pi e_0 r'^3} \quad (33)$$

where E'_x, E'_y, and E'_z are the components of auxiliary electric field in the Heavisidean auxiliary system S' at P' (x', y', z')

Now, the distance of the point P' from the origin in this S' system, $r' = \sqrt{x'^2 + y'^2 + z'^2}$ where P' (x', y', z') of the S' system is the corresponding point P (x, y, z) of the S system. The Coordinate of the corresponding point P is (x, y, z) in Cartesian System and (r, θ, φ) in spherical polar coordinate where r is the distance from the origin, θ is the angle down from the x axis and φ is the angle around from the z axis. Therefore, $x = r\cos\theta,\ y = r\sin\theta\sin\varphi,\ z = r\sin\theta\cos\varphi$, from which $x^2 = r^2\cos^2\theta$ and $y^2 + z^2 = r^2\sin^2\theta$. Therefore:

$$r'^3 = (x'^2 + y'^2 + z'^2)^{3/2}$$
$$= (\gamma^2 x^2 + y^2 + z^2)^{3/2} \text{ [using Eq. (15)]}$$
$$= (\gamma^2 r^2\cos^2\theta + r^2\sin^2\theta)^{3/2} \text{ [in polar coordinates]}$$
$$= \gamma^3 r^3\left[1 - (u^2/c^2)\sin^2\theta\right]^{3/2} \text{ [a result independent of } \varphi\text{]}$$

Now, $E = \sqrt{E_x^2 + E_y^2 + E_z^2}$
$$= \sqrt{E_x'^2 + \gamma^2 E_y'^2 + \gamma^2 E_z'^2} \text{ [using Eq. (21)]}$$
$$= \frac{Q}{4\pi\varepsilon_0 r'^3}\sqrt{x'^2 + \gamma^2 y'^2 + \gamma^2 z'^2} \text{ [using Eq. (33)]}$$
$$= \frac{Q}{4\pi e_0 r'^3}\gamma r \text{ [using Eq. (15)]}$$

Thus, independent of φ, we have:

$$E = \frac{Q}{4\pi\varepsilon_0}\frac{\gamma r}{\gamma^3 r^3}\left[1-(u^2/c^2)\sin^2\theta\right]^{-3/2} = \frac{Qk^2}{4\pi\varepsilon_0 r^2}\left[1-(u^2/c^2)\sin^2\theta\right]^{-3/2} \quad (34)$$

The auxiliary $\mathbf{E}'$ is directed along OP'. Therefore, the real field $\mathbf{E}$ is directed along OP.

Now, from $\mathbf{B}^* = \nabla\times\mathbf{A}^*$, we have,

$$B_x^* = 0,\ B_y^* = -\frac{u}{c^2}E_z = -\gamma\frac{u}{c^2}E_x',\ B_z^* = \frac{u}{c^2}E_y = \gamma\frac{u}{c^2}E_y' \quad (35)$$

from which the induced magnetic field is

$$\mathbf{B}^* = \mathbf{u}\times\mathbf{E}/c^2 \quad (36)$$

With a little analysis, it can be shown that Eq. (34) and (36) are the same for a charged ellipsoid having its axes with ratios $k{:}1{:}1$ moving with a constant translational velocity $\mathbf{u}$ in free space, k being in the direction of motion of the ellipsoid. Thus Oliver Heaviside (1850 - 1925) the greatest electromagnetician after Maxwell (1831- 1879) has shown that a charged ellipsoid having its axes with ratios $k{:}1{:}1$ while moving with a constant translational velocity $\mathbf{u}$ in free space produces the same external effect as that of a similarly moving point charge [4,5], k acting in the direction of motion of the charge.

Electromagnetic Momentum

The electromagnetic momentum of a Heaviside's ellipsoid (with the axes δRk: δR: δR) while moving rectilinearly with a velocity $\mathbf{u}$ in the OX direction in free space is

$$P_x = \int\left(D_yB_x^* - D_zB_y^*\right)d\tau = \frac{u}{c^2}\varepsilon_0\int\left(\gamma^2E_y'^2+\gamma^2E_z'^2\right)k\,d\tau'$$
$$= \gamma\frac{u}{c^2}\varepsilon_0\int\left(E_y'^2+E_z'^2\right)\mathrm{d}\tau' \qquad cf.\ \text{Eqs. (21)\&(22)}$$

where $d\tau$ is the volume element in the S system and $d\tau'$ is the corresponding volume element in the S' system. Here in the S' system Heaviside's ellipsoid is exactly a sphere. Therefore, when evaluated between δR and ∞, $\int E'^2 d\tau' = q^2/4\pi\varepsilon_0^2\delta R$, and each integral in the previous equation is equal to $q^2/12\pi\varepsilon_0^2\delta R$. Therefore,

$$P_x = q^2\mathbf{u}/6\pi\varepsilon_0 c^2\delta Rk = m\mathbf{u} \quad (37)$$

which is the electromagnetic momentum of a point charge moving rectilinearly in free space with velocity $\mathbf{u}$, where m_0 and m are the electromagnetic masses of the point charge moving with the velocities near zero and $\mathbf{u}$ respectively in free space such that

$$q^2/6\pi\varepsilon_0 c^2\delta R = m_0 \text{ and } m_0/k = m \quad (38)$$

(Consult Hajra 1998 [6] and Searle 1897 [7] for an alternative deduction of electromagnetic momentum of a steadily moving point charge.)

Electromagnetic Force acting on a Point Charge moving in Free Space

a) in a direction parallel to the direction of the uniform electric field operating in free space,

$$F_{||} = (dP/du)a_{||} = (m_0/k^3)a_{||} \quad (39)$$

where $a_{||}$ is the acceleration of the point charge in the direction parallel to the field.

b) at a direction perpendicular to the direction of the uniform electric field operating in free space.

$$F_\perp = (|\mathbf{P}|/|\mathbf{u}|)a_\perp = (m_0/k)a_\perp \quad (40)$$

where $a_\perp$ is the acceleration of the point charge in the direction perpendicular to the field.

(A general treatment will show $\mathbf{F} = m\,d\mathbf{u}/dt + \mathbf{u}(\mathbf{F}\cdot\mathbf{u})/c^2$.)

Similarly, evaluated between δR and ∞, the energy of a point charge having a constant translational velocity in free space is

$$\varepsilon = \int (\partial P / \partial t) dx = m_0 c^2 / k = mc^2 \qquad (41)$$

[1] H.A. Lorentz, **The Theory of Electron** (i) pp. 35-43, (ii) pp. 57-61, (iii) p. 245, (Dover Publications. Inc. New York, 1909, 1915),
[2] E.T. Whittaker, **A History of the Theories of Aether and Electricity**, vol. 1, pp. 307-309 (Thomas Nelson and Sons, Ltd., 1951).
[3] A.I. Miller, **Albert Einstein's Special Theory of Relativity**, pp. 98-99 (Springer, 1981),.
[4] O. Heaviside, i) Electrician, Dec. 7, p. 148 (1888); ii) The Phil. Mag., pp. 324–339 (1889).
[5] J.J. Thomson, The Phil. Mag., pp. 1-14 (1989)..
[6] S. Hajra, pp. 149-157 in **PIRT, 1998, Supplemental Papers**, edited by Dr. M.C. Duffy.
[7] G.F.C. Searle, The Phil. Mag., pp. 329-339 (1897).

Appendix II

Derivation of Lorentz Auxiliary Transformation Equations classically:

The problem Heaviside and Thomson addressed was to model the potentials for charges having a constant translational velocity in free space. They solved this problem by transforming the D' Alembert's equation in an invariant form with Poisson's in the auxiliary system. Lorentz's problem was to model the radiation from moving bodies. He solved this problem by transforming Maxwell's equations for the moving radiating body to the same form for the auxiliary system, which correlates between the static and dynamic radiation states.

By dint of Corollary 2 of the Appendix-I, we have,

$$\partial^2 \mathbf{E}/\partial x^2 + \partial^2 \mathbf{E}/\partial y^2 + \partial^2 \mathbf{E}/\partial z^2 - \frac{1}{c^2}\partial^2 \mathbf{E}/\partial t^2 = 0 \quad (1)$$

where $\mathbf{E}$ is the electric field of a radiating dipole moving with a constant translational velocity $\mathbf{u}$ in free space. From this we get

$$x^2 + y^2 + z^2 = c^2 t^2 \qquad (1a)$$

To solve radiation problems in a way analogous to that shown in Section 2, we are to keep the Maxwell's equation in the same form in the S' system; *i.e.*, it is now required that have,

$$\partial^2 \mathbf{E}'/\partial x'^2 + \partial^2 \mathbf{E}'/\partial y'^2 + \partial^2 \mathbf{E}'/\partial z'^2 - \frac{1}{c^2}\partial^2 \mathbf{E}'/\partial t'^2 = 0 \qquad (2)$$

where $\mathbf{E}'$ is the auxiliary electric field of Heaviside and Thomson in the S' system.

That is,

$$x'^2 + y'^2 + z'^2 = c^2 t'^2 \qquad (2a)$$

where x', y', z' are defined in the section 2 and t' is to be chosen appropriately.

Subtracting Eq. (1a) from Eq. (2a) and using the Heaviside-Thomson auxiliary equations

$$x' = \gamma(x - ut), \;\; y' = y, \;\; z' = z \qquad (3)$$

to replace the primed variables, we get

$$c^2 t'^2 = (x - ut)^2 / (1 - u^2/c^2) - x^2 - c^2 t^2$$

or $$t' = \gamma(t - ux/c^2) \qquad (4)$$

the famous auxiliary time equation of Lorentz.

An interesting fact about the equations is that the inverse Lorentz Transformation equations (which could be deduced from Lorentz Transformation equations) have the same form as the Lorentz Transformation equations themselves; *i.e*., if

$x' = \gamma(x - ut),\ y' = y,\ z' = z,\ t' = \gamma(t - ux/c^2)$ (5 a-d)

then

$x = \gamma(x' + ut'),\ y = y',\ z = z',\ t = \gamma(t' + ux'/c^2)$ (6 a-d)

Therefore, Lorentz transformation equations can be deduced in reverse from the dyads of Eqs. (1a) & (2a) and (5a-c) &(6a-c). (These transformation equations were later observed and used by A. Einstein in his theory).

It should be mentioned here that all the quantities x', y', z', t', $\mathbf{E}', \mathbf{B}'$, *etc*., are auxiliary, so Eq. (2) is auxiliary. Thus, from the standpoint of electrodynamics, Lorentz Transformation Equations are tactical, just like the tactical equations of Heaviside and Thomson. These auxiliary equations are very useful to solve correctly the radiation problems associated with steadily moving electromagnetic bodies.

Thus Lorentz's auxiliary equations are actually extended Heaviside-Thomson auxiliary equations which are solely based on Newton-Maxwell interpretation of physical world. Anybody who uses these equations to explain any electrodynamic phenomena employ classical methodology at the outset .

Appendix-III

Derivation of the Exact Equation of the Planetary Motion, Gravitational Red Shift, Bending of Light Rays Grazing the Surface of the Sun classically

When a point charge moves in free space with a velocity 'u' through an interacting field (e.g., an electric field), laws of interaction of the charges with the field could be written as follows from Maxwell as per the equations (39) & (40) of the Appendix-I

$(F_E)_{//} = \gamma^3 m_0 a_{//} = m^{/} a_{//}$ (1)

$(F_E)_{\perp} = \gamma\ m_0 a_{\perp} = m^{//} a_{\perp}$ (2)

when $(F_E)_{//}$ is the force (say, electric) acting on the point charge 'q' moving towards the direction of the interacting force, $a_{//}$ is the corresponding acceleration, $(F_E)_{\perp}$ is the force (say, electric) acting on the charge moving perpendicular to the direction of the interacting force and $a_{\perp}$ is the corresponding acceleration

$q^2 / 6\pi\varepsilon_0 c^2 \delta R = m_0$ (3)

$k = \sqrt{1 - u^2/c^2}$ (4)

$\gamma = 1/k$ (5)

longitudinal electromagnetic mass

$= m^{/} = \gamma^3 m_0$

transverse electromagnetic mass (6)

$= m^{//} = \gamma m_0$ (7)

When u $\rightarrow$ 0, both the masses are the same and equal to

$q^2 / 6\pi\varepsilon_0 c^2 \delta R = m_0$ (8)

m_0 being called the rest electromagnetic mass. , δR is the radius of the extremely small spherical charge, μ_0 and ε_0 are the permeability and permittivity of free space such that $c = 1/\sqrt{\mu_0 \varepsilon_0}$.
The laws are applicable to the motion of any amount of charge, large or small, concentrated in a very small spherical volume with a very negligible small radius δR while moving in an interacting field.
Now if a polar co-ordinate is used, we may rewrite the equations (1) and (2) as follows:

$$(F_E)_r = \gamma^3 m_0 \left(\ddot{r} - r\dot{\theta}^2 \right) \qquad (9)$$

$$(F_E)_\theta = \frac{1}{r}\frac{d}{dt}\left(\gamma m_0 r^2 \dot{\theta} \right) = 0 \qquad (10)$$

when the moving point charge is at (r, θ) with polar co-ordinate, pole being the source of the field: $(F_E)_r$ is the radial component and $(F_E)_\theta$ is the transverse component of the field

1. Interaction of Charges with Gravitating Field

Let a gravitating body with the material mass M_s be at rest at the origin and the point charge 'q' be moving with a velocity 'u' across the gravitating field.
Now assume that charges, too, are subject to gravitation and they have the same acceleration due to gravity as that of material bodies when both of them are subject to the same gravitating field..
Therefore, for all charges we may write

$$-\frac{GM_s}{r^2} = \ddot{r} - r\dot{\theta}^2$$

G, being the gravitational constant.
(i.e., as the equation of motion of material bodies towards the center of the gravitating body is independent of their masses, so the equation of motion of the charges towards the center of the gravitating body must be independent of the quantity of the moving point charges).
Now comparing the above equation with the equation (9) and replacing $(F_E)_r$ by $-\frac{GM_s}{r^2}\gamma^3 m_0$ we have,

$$-\frac{GM_s}{r^2}\gamma^3 m_0 = \gamma^3 m_0 \left(\ddot{r} - r\dot{\theta}^2 \right)$$

i.e., the longitudinal electromagnetic mass of a moving point charge is proportional to its mass in a gravitating field.
Thus, in a gravitating field, a point charge should behave like a mass point,
Therefore, the equation of motion of a point charge 'q' moving across a gravitating field could be written as follows:

$$(F_G)_r = \gamma^3 m_0 \left(\ddot{r} - r\dot{\theta}^2 \right) \qquad (11)$$

$$(F_G)_\theta = \frac{1}{r}\frac{d}{dt}\left(\gamma^3 m_0 r^2 \dot{\theta} \right) = 0 \qquad (12)$$

Where $(F_G)_r$ and $(F_G)_\theta$ are the radial component and the transverse component of the gravitating force acting on the charge, $\gamma^3 m_0$ is the mass of the charge when the ratio of proportionality of longitudinal electromagnetic mass of the charge and the mass of the charge is taken as unity. Transverse electromagnetic mass $m^{//}$, however, having no role as per our assumption is inoperative in such a situation. Therefore, we have nothing to do with the equation (10).
From (12) we have,
Mechanical angular momentum of the charge

$$A = \gamma^3 m_0 r^2 \dot{\theta} = \text{constant} \qquad (13)$$

whence we get,

$$\gamma^3 r^2 \dot{\theta} = \text{H, a constant} \qquad (14)$$

Now, we can replace equation (11) as under :

$$\left(F_G\right)_r = -\frac{GM_s\gamma^3 m_0}{r^2} = \gamma^3 m_0\left(\ddot{r} - r\dot{\theta}^2\right) \qquad (15)$$

or, $-\dfrac{GM_s}{r^2} = \ddot{r} - r\dot{\theta}^2$ (16)

Let $U = 1/r$ (17)

From (14) we have

$$\dot{\theta} = HU^2k^3 \qquad (18)$$

$$\dot{r} = -\frac{1}{U^2}\frac{dU}{d\theta}\dot{\theta} = -Hk^3\frac{dU}{d\theta} \qquad (19)$$

$$\ddot{r} = -Hk^3\frac{d^2U}{d\theta^2}\dot{\theta} = -H^2k^6U^2\frac{d^2U}{d\theta^2} \qquad (20)$$

Substituting in (16), we have,

$$\frac{d^2U}{d\theta^2} + U = \frac{GM_s}{H^2}\gamma^6 = \frac{GM_s}{H^2}\left(1 + 3\frac{u^2}{c^2}\right)$$

(when ‘u’is very small in comparison to ‘c’)

$$= \frac{GM_s}{H^2} + \frac{3GM_s}{H^2}\frac{u^2}{c^2}$$

$$= \frac{GM_s}{H^2} + \frac{3GM_s}{\gamma^6 r^4\dot{\theta}^2}\frac{r^2\dot{\theta}^2}{c^2}$$

[replacing the second H by equation (14) and noting that for circular motion $u = r\dot{\theta}$]

$$= \frac{GM_s}{H^2} + \frac{3GM_s}{c^2r^2}\left(1 + \frac{u^2}{c^2}\right)^3$$

$$= \frac{GM_s}{H^2} + \frac{3GM_s}{c^2r^2} \qquad (21)$$

This law is applicable to the motion of any point charge with any amount of charge, large or small, in a central inverse square gravitational force field. A point charge may be considered here as any amount of charge concentrated in very small sphere with the radius δR

2. Exact Equation of Planetary Motion:

The equation (21) may aptly be used to explain the advance of the perihelion of Mercury, if we assume that all material bodies are fully electromagnetic in origin. However, we are not in a position to accept such a doubtful proposition.

It will be rather innocuous to say that though both material bodies and charges are subject to gravitation and material bodies are always seen to contain some amount of charges with them, yet material bodies are quite distinctly different from electromagnetic bodies.

We know from our previous discussion [1] that when a charged body moves, its longitudinal electromagnetic mass $m^{/}$ increases with velocity. Now, if acceleration of a point charge due to gravity be the same as that of a material body in the same gravitational field, the mass of the point charge must be proportional to its electromagnetic mass m. Therefore, the mass of the charge, too, must increase with velocity.

But, on the contrary, from the consideration of Newton, the mass of a material body must not change in motion.

We do think that Newton's laws of motion and the law of gravitation for material bodies are the correct description of nature. Therefore, we shall proceed as under.

Suppose that a planet of mass m_p contains 'q' amount of positive and negative charges in total (ignoring the sign of the charges) and for simple calculation let us assume that the total charges are concentrated at the center of the planet. In this situation, the equation of planetary motion will be [cf.eq.(11)] as under:

$$(F_G)_r = \left(m_p + \gamma^3 m_0\right)\times\left(\ddot{r} - r\dot{\theta}^2\right)$$

$$or, -\frac{GM_s}{r^2}\left(m_p + \gamma^3 m_0\right) = \left(m_p + \gamma^3 m_0\right)\times\left(\ddot{r} - r\dot{\theta}^2\right)$$

$$\text{or, } -\frac{GM_s}{r^2} = \left(\ddot{r} - r\dot{\theta}^2\right) \qquad (22)$$

where $\gamma^3 m_0$ is the mass of the charge associated with the planet and m_p is the material mass of the planet

$$\text{and} (F_G)_\theta = \frac{1}{r}\frac{d}{dt}\left[\left(m_p + \gamma^3 m_0\right)\times r^2\dot{\theta}\right] = 0 \qquad (23)$$

from which mechanical angular momentum of the planet,

$$A = \left(m_p + \gamma^3 m_0\right)\times r^2\dot{\theta}$$

$$= \left(m_p + \frac{m_0}{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}\right)\times r^2\dot{\theta}$$

$$= \left[\frac{m_p\left(1 - \frac{u^2}{c^2}\right)^{3/2} + m_0}{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}\right]\times r^2\dot{\theta}$$

$$= \left[\frac{m_p - \frac{3}{2}\frac{u^2}{c^2}m_p + m_0}{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}\right]\times r^2\dot{\theta}$$

And when 'u' is very small with respect to 'c' and 'm_0'is much larger than $\frac{3u^2}{2c^2}m_p$ such that $\frac{3u^2}{2c^2}m_p$ is negligibly small in comparison to 'm_0'

$$A = \frac{m_p + m_0}{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}\times r^2\dot{\theta} \qquad (24)$$

from which we have,

$$\gamma^3 r^2\dot{\theta} \approx H = \text{constant} \qquad (25)$$

From equs. (22) & (25) we have as deduced from the equations (14) & (16),

$$\frac{d^2U}{d\theta^2}+U=\frac{GM_s}{H^2}+\frac{3GM_s}{c^2r^2} \qquad (26)$$

The equation (26) is widely believed to have been confirmed by experiments. Therefore, we may conclude that charges are subject to gravitation and their acceleration due to gravity as per equation (22) have the same acceleration as that of the material bodies.

3. Interaction of Electromagnetic Energy with Gravitating Field

:Electromagnetic radiation though neutral is seen to be affected by gravitation, which elucidate inherent charge like properties of electromagnetic radiation.

Let us assume that electromagnetic energy, too, is subject to gravitation and they should have the same acceleration due to gravity as that of the material bodies in the same gravitational field. From this consideration, we shall study below the interaction of electromagnetic energy with gravitational fields.

(i) Gravitational Red Shift of Spectral Lines

Suppose that a ray is coming from the surface of a star of radius R_t and of mass M_t to the surface of the earth, r distance away from the star. As per our previous discussion, we assume that electromagnetic energy has the same acceleration due to gravity as that of material bodies in the same gravitational field. Therefore, the velocity of the ray at the surface of the earth

$$V=\left(c-\int_R^r \frac{GM_t}{r_1^2}\frac{dr_1}{c}\right)$$

r_1 being the distance of and arbitrary point on the ray from the star $=c\left(1-\frac{GM_t}{R_tc^2}\right)$ (27)

from which we have,

$$\frac{\omega^{/}}{\omega}=\left(1-\frac{GM_t}{R_tc^2}\right) \qquad (28)$$

ω is the radian frequency of light at the surface of the star and $\omega^{/}$ is the radian frequency of the same light ray at the surface of the earth, which is said to have been verified by many refined experiments.

(ii) Bending of Light Ray Grazing the Surface of the Sun

The above study empowers us to apply equations (12) &(15) to the case of propagation of electromagnetic radiation grazing the surface of the sun having gravitational mass M_s

Therefore, the equation of motion of a light beam passing through a medium (such that the velocity of the light beam u is much smaller than c in the medium) , medium being fixed with the sun's surface will take the form:

$$\frac{d^2U}{d\theta^2}+U=\frac{GM_s}{H^2}+\frac{3GM_s}{c^2r^2}$$

Now for a small volume of radiation energy, the mechanical angular momentum A is finite. Therefore, H for this small volume of radiation energy will be infinite, as 'm_0' for this small volume of radiation energy =0.

In such a situation, the above equation for the propagation of a small volume of radiation energy through the sun's adjacent gravitational field inside a medium wherein $u<<c$ should be:

$$\frac{d^2U}{d\theta^2}+U=\frac{3GM_s}{c^2r^2}. \qquad (29)$$

which is widely believed to have confirmed by "billion dollar experiments"

4.CONCLUSION

We may, therefore, conclude that all electromagnetic quantities are subject to gravitation and they have the same acceleration due to gravity as that of material bodies and this simple consideration is seen to be equivalent to special and general relativity.]