ON THE FUNDAMENTALS OF THE CC RELATIVITY THEORY

Lyubomir T. Gruyitch – University of Technology Belfort–Montbeliard (UTBM),
Site de Belfort, Rue Thierry Mieg - 90010 Belfort Cedex - France

Abstract.

The goal of the paper is to present the inherent characteristics of *time* and their implications in and influences on the relativity theory, which lead to a novel relativity theory.

Time is the unique physical variable such that its value has been changing continuously monotonously increasingly equally and uniformly in all directions independently of anybody and anything. Its value has been increasing independently of space [as recognised in classical physics, classical mechanics, mathematics, systems and control (science and engineering), but rejected in the Einsteinian relativity theory].

Recent research results, which started with the properties of *time*, have showed that Newton's explanations of *time* and of its relative sense express well the features of *time* and incorporate Einstein's explications of *time* and of its relativity. They proved that Lorentz transformations and from them obtained results in the Einsteinian relativity theory are restricted not only to a special case, but only to a singular case. This is due to the constraints a priori accepted by Lorentz and Einstein and adopted in the Einsteinian relativity theory. The new results discovered also its inconsistencies and incompatibilities.

The drawbacks of the Einsteinian relativity theory have been recently overcome by the fundamentals of a new relativity theory that is consistent, and which provides compatible results. It is non-Einsteinian relativity theory. It is called *the consistent and compatible relativity theory*, for short: *CC relativity theory*. Its fundamentals are explained in the paper. They concern temporal and spatial co-ordinate transformations, as well as velocity and acceleration transformations in the general case, in the special case and in the singular case.

The general case is free of all constraints a priori accepted in the Einsteinian relativity theory. Consequently, new formulas are essentially different from those in the Einsteinian relativity theory. In the special case the formulas become of the form from the Einsteinian relativity theory, but more general. They reduce to the well known results of the Einsteinian relativity theory in the singular case.

Various speeds of the propagation of different phenomena and processes motivated humanity to introduce and use different *time* scales and *time* units. Different *time* scales were also introduced in the framework of dynamical systems, the parts of which generate motions with essentially different speeds of their propagation. They are known as systems with multiple *time* scales, which incorporate singularly perturbed systems. The new results of the relativity theory show conditions under which several *time* scales can be attributed to dynamical systems.

The paper opens new avenues for research in physics, in the relativity theory, in mathematics and in the theory of dynamical systems with multiple *time* scales.

Keywords: Basic physics, co-ordinate transformations, dynamical systems Lorentz transformations mathematical physics, relativity theory, systems with multiple *time* scales, *time*.

1. INTRODUCTION

The recent paper [19] stressed out the importance of compatibility of the co-ordinate transformations. Temporal (or, spatial) co-ordinate transformations are *compatible* if and only if they lead to an identity when they are combined (i. e. when the inverse transformation is applied to the transformation). *Pairwise compatibility* of the transformations expresses both compatibility between the temporal co-ordinate transformations and compatibility between the spatial co-ordinate transformations. *Entire compatibility* of the transformations means that they altogether lead to an identity when temporal co-ordinate transformations and spatial co-ordinate transformations are mutually combined. *Complete pairwise (entire) compatibility* of the transformations means, respectively, their pairwise (entire) compatibility in arbitrary case independently of both a position and speed of an arbitrary point. Their *partial* (*restrictive*) *pairwise (entire) compatibility* means, respectively, their pairwise (entire) compatibility only under the condition that an arbitrary point moves with a restricted speed, e. g. with the speed of light. It was shown in [19] that the Lorentz transformations are restrictively pairwise compatible. They are not completely pairwise compatible. This opened a new problem in the relativity theory - the problem of *complete (pairwise, entire) compatibility of the transformations*, and posed the following questions:

Q1 Under which conditions the temporal co-ordinate transformations and the spatial co-ordinate transformations are completely (pairwise, entire) compatible?

Q2 Under which conditions the velocity transformations are completely (pairwise, entire) compatible?

The new results obtained in [19] provoked also the next questions:

Q3 What are implications of different *time* scales linked with the Gaussian generalisation of the Lorentz transformations on the relativity theory?

Q4 What are implications on dynamical systems with multiple *time* scales?

Q5 What are implications on speed, mass and energy?

The paper [19] contributed with full replies to these questions in the framework of the uniform transformations. Uniformity of the transformations expresses their property that the same initial moment, the same *time* scale and *time* unit, i. e. the same *time* set and *time* axis, hold over (cover) the whole space. The new transformations in [19] represent Newtonian generalisations of the Lorentz transformations. They are based on the properties of *time*, which we discover in the physical reality.

Newton's explanation in [34] of the meaning and the sense of both *absolute time* and *relative time* was shown to be essentially correct [10], [11], [14] – [20]. It incorporates Einstein's meaning of *time*, which is explained in [10], [11], [14] – [19].

An inherent statement of Einstein's relativity theory is the postulate on the constancy of the value of the speed of light in vacuum relative to inertial frames. This is surprisingly interpreted in some literature as invariance of the (numerical) value of the light speed with respect to a choice of *time* unit and length unit. Surprisingly, because the (numerical) value of the speed of anybody or of anything depends on the units of both *time* and length. Although this holds also for the speed of light, it has not been a priori taken into account in the Lorentz – Einstein relativity theory [2] – [6], [22] – [27]. Final results show that the value of the speed of light does not change when the Lorentz transformations are applied. This is the consequence of the equality of the *time* and space scaling coefficients in the Einsteinian relativity theory. Their equality ensures the changes of a *time* unit and of a length unit in the same ratio. Their changes do not influence the value of the quotient of the length over *time*.

The Lorentz - Einstein relativity theory starts with the a priori demands for equality of *time* scaling coefficients and for equality of space scaling coefficients in the Lorentz transformations. Besides, their values are assumed constant a priori, as well, which was achieved by determining them for a priori accepted speed of an arbitrary point to be the light speed [2] – [6], [19] – [23].

The aim of this paper is to establish fundamentals of a new relativity theory. It will be relaxed of the accepted restrictions that are accepted a priori in the Einsteinian relativity theory. The new relativity theory is compatible and consistent. It is called *compatible and consistent* (for short: *CC*) *relativity theory*. It establishes new solutions to the posed problems.

2. NOTATION

We may accept any *time* scale for a *time* axis T. Different *time* scales can be associated with different *time* axes: "an original *time* scale" T that is not indexed and "k"-*time* scale T_k. A *time* value (instant, moment) measured in T-scale and T_k-scale is designated by t and t_k, respectively, $t \in T$ and $t_k \in T_k$, k = 1, 2, …, *s*. The indices i, j are arbitrary and fixed, $i, j \in \{1, 2, \ldots, s\}$. They may be equal or different. If they are equal, which is the trivial case, the equations become identities.

The Cartesian product set $T \mathrm{x} R^n$ is denoted by I, $I = T \mathrm{x} R^n$. It is the *(n+1)-dimensional real integral vector space*, for short *the integral space.* A pair (t, x) is called *an event in I* [21]. It can occur exactly once due to the properties of *time*. The integral space corresponding to the *time* axis T_k and to the frame R^n_k is I_k: $I_k = T_k \mathrm{x} R^n_k$, $k = 1, 2, \ldots, s$.

The vector $\mathbf{u}$ is an arbitrarily chosen and fixed constant unity vector: $\|\mathbf{u}\| = 1$, where $\|.\|$ is the Euclidean (or any other scalar) norm.

Let origin O_k of the co-ordinate system R^n_k, hence R^n_k itself, move with a constant velocity $\mathbf{v}_{Ok}$ relative to the origin O of R^n and measured with respect to $t \in T$. The norm $\|.\|$ of the velocity $\mathbf{v}_{Ok} = v_{Ok}\mathbf{u}$ is denoted by v_{Ok}, $v_{Ok} = \|\mathbf{v}_{Ok}\|$ if measured relative to $t \in T$, $k = 1, 2, \ldots, s$. It is the speed of the origin O_k relative to the origin O of R^n.

The value of the light speed in vacuum is denoted as usually by c [3: p. 15], [4: p. 26]. It is the light speed value with respect to the vacuum and measured relative to I, for short: *the light speed value*. We can accept to consider a light signal and a translation of O_i together with R^n_i in a direction and sense of the unity vector $\mathbf{u}$. The constant unity vector $\mathbf{u}$ is used to represent symbolically also the direction and the sense of a movement of the space R^n_j. Without losing in generality, we represent a position of an arbitrary point

P relative to the origins O and O_i of R^n and R_i^n at the same moment by vectors $\mathbf{r}_P \in R^n$ and $\mathbf{r}_P^i \in R_i^n$ as $\mathbf{r}_P = \rho_P\mathbf{u}$, and $\mathbf{r}_P^i = \rho_P^i\mathbf{u}$, i = 1, 2, …, *s*. Their lengths are expressed by their norms $\rho_P = \|\mathbf{r}_P\|$ and $\rho_P^i = \|\mathbf{r}_P^i\|$, respectively, which may vary in *time*:

$\mathbf{r}_P(t_{(.)};t_{(.)0}) = \rho_P(t_{(.)};t_{(.)0})\mathbf{u}$ is the position vector of the point P with respect to $O_{(.)}$ at $t_{(.)}$, (.) = i, j, which will be denoted also by $\mathbf{r}_i(t_i;t_{i0}) = \rho_i(t_i;t_{i0})\mathbf{u}$: $\mathbf{r}_P(t_i;t_{i0}) = \rho_i(t_i;t_{i0})\mathbf{u}$.

Their velocities depend on the reference integral space in general:

c_j^i is the scalar value of the light speed measured with respect to the origin O_j of R_j^n and relative to t_i (rather than relative to t_j if $T_i \neq T_j$),

c_{ij} denotes both c_i^i and c_i^j if and only if $c_j^j = c_i^i$: $c_{ij} = c_{ji} = c_i^i = c_j^j$, which is denoted simply by c in the Lorentz transformations (L1) through (L4) because it has been considered in the Einsteinian relativity theory as invariant with respect to a choice of the integral space,

$\mathbf{v}_P^{O_i}(t_i;t_{i0}) \equiv \mathbf{v}_P^i(t_i;t_{i0}) \equiv v_P^i(t_i;t_{i0})\mathbf{u}$ is the instantaneous velocity (the speed vector) of the point P with respect to O_i at t_i and measured relative to t_i provided the initial moment was t_{i0}; $\mathbf{v}_P^i(t_i;t_{i0}) \equiv \mathbf{v}_P^i \equiv v_P^i\mathbf{u}$ if and only if the speed vector $\mathbf{v}_P^i(t_i;t_{i0})$ is constant vector, and

$\mathbf{v}_P^i(t_i;t_{i0}) \equiv \mathbf{v}_P^i(t_i) \equiv v_P^i(t_i)\mathbf{u}$ if and only if t_{i0} is known and fixed,

$\bar{\mathbf{v}}_P^{O_i}(t_i;t_{i0}) \equiv \bar{\mathbf{v}}_P^i(t_i;t_{i0}) \equiv \bar{v}_P^i(t_i;t_{i0})\mathbf{u}$ is the average speed vector of P over $[t_{i0}, t_i]$ with respect to O_i at t_i and measured relative to t_i:

$$\bar{\mathbf{v}}_P^i(t_i;t_{i0}) = \frac{\mathbf{r}_i(t_i;t_{i0})}{t_i - t_{i0}} =$$

$$= \begin{cases} \mathbf{v}_P^i(t_{i0};t_{i0}) = \mathbf{v}_{P0}^i,\ t_i = t_{i0} \\ \dfrac{1}{t_i - t_{i0}}\displaystyle\int_{t_{i0}}^{t_i}\mathbf{v}_P^i(t_i;t_{i0})dt_i,\ t_i > t_{i0} \end{cases} =$$

$$= \bar{\mathbf{v}}_P^i(t_i) \text{ for known and fixed } t_{i0},$$

$\mathbf{v}_{O_k}^m \equiv v_{O_k}^m\mathbf{u}$ is the constant velocity of O_k relative to O measured in terms of t_m, k, m $\in\{1, 2, …, s\}$, $v_{O_i}^r \leq v_{O_j}^r$ is accepted, $r\in\{i, j\}$,

$\boldsymbol{v}_{ji}^k \equiv v_{ji}^k\mathbf{u} \equiv (v_{O_j}^k - v_{O_i}^k)\mathbf{u}$ is the constant relative speed vector of O_j with respect to O_j measured all in terms of t_κ, k $\in$ {i, j}; notice that $\boldsymbol{v}_{ji}^i \equiv -\boldsymbol{v}_{ij}^i$, that $\boldsymbol{v}_{ji}^i \neq \boldsymbol{v}_{ji}^j \equiv (v_{Oj}^j - v_{Oi}^j)\mathbf{u} \equiv -\boldsymbol{v}_{ij}^j$ is possible if $T_i \neq T_j$, that $\boldsymbol{v}_{ji}^{(.)} = v_{ji}^{(.)}\mathbf{u}$ is called *the transfer velocity* and that $v_{ji}^{(.)}$ is called *the transfer speed,*

$\boldsymbol{v}_{ji} \equiv v_{ji}\mathbf{u}$ denotes both $\boldsymbol{v}_{ji}^i$ and $\boldsymbol{v}_{ji}^j$ if and only if they are equal: $\boldsymbol{v}_{ji}^i = \boldsymbol{v}_{ji}^j = \boldsymbol{v}_{ji} = -\boldsymbol{v}_{ij}$. In the Lorentz transformations there is v_{ji} denoted by *v* because it is considered invariant. Evidently, $v_{ji}^i = v_{ij}^i = v_{ji}^j = v_{ij}^j = v_{ij} = \nu = 0$ for *i = j*.

If and only if the point P represents a light signal L, then:

$\mathbf{r}_P^O = \mathbf{r}_L = \mathbf{r} \in R^n$, $\mathbf{r}_L = \rho_L\mathbf{u} = \rho\mathbf{u}$, $\mathbf{r}_P^{O_{(.)}} = \mathbf{r}_{L(.)} \in R_{(.)}^n$, and $\mathbf{v}_P^{O_{(.)}} = \mathbf{v}_L^{O_{(.)}} = \mathbf{c}_{(.)}^{(.)} = c_{(.)}^{(.)}\mathbf{u}$, (.) = *i, j*; *i,j*$\in\{1, 2,…, s\}$, $i \leq j$.

3. *TIME* FEATURES

The physical reality and the experience led to the following axiomatic form of the characterisation of *time*, which expresses its crucial properties, agrees with Newton's explanation of *time* [34], and which has permitted new fundamental results in the relativity theory [10], [11], [14] – [20]:

Axiom 1.

a) ***Time***

Time **(*temporal variable*)** denoted by t (or by τ) *is* ***an independent physical variable.*** *Its value occupies (covers, encloses, impregnates, is over and in, and penetrates) equally beings, energy, matter, objects, and space. Its value has been and will be smoothly and strictly monotonously continuously increasing equally in all directions in space and independently of beings, energy, matter, objects and space, independently of all other (physical and mathematical) variables, independently of all movements, of all processes and all events.*

b) **Physical dimensionality of *time***

Time is its own unique component. It is a **basic physical variable**. Its dimensionality is a basic physical dimensionality denoted by "T", t[T], where "T" means "*time*".

c) ***Time*** **value and its flow**

The ***value of time*** is called **moment** or **instant.** It is the shortest possible duration that is the duration of a single *time* value. It can happen exactly once and then it is the same for all beings, energy, matter, for all objects, in the whole space, for all events, movements and processes. It is denoted by t and a subscript, e.g. t_2. An arbitrary instant will be denoted as *time* itself by t. The *time* value is determined accurately up to an unknown additive constant. *A* ***sequence of time values*** *determines uniquely the order of events happening.*

A flow *(i. e.* ***an oriented variation)*** ***of time values*** (for short: ***a temporal flow***) from one *time* value to another one is the ***duration between them*** (for short: ***duration***). Its orientation is the *temporal* orientation (the *temporal* sense, the *temporal* order), which is from a smaller (from a past) *time* value to a bigger (to a future) *time* value: dt > 0.

There can be assigned exactly one real number to every moment, and vice versa. The set *T* of all moments is in one-to-one correspondence with the set *R* of all real numbers. An accepted rule of the correspondence determines ***a relative zero numerical value of time***, **a** ***time*** **scale** and ***a time unit***, and vice versa.

A total zero moment $t_{ZeroTotal}$ has not existed and will not occur. Any instant may be accepted for ***a relative zero time value*** t_{zero}.

d) ***Time interval***

The temporally ordered connected set of all instants between two different *time* values is a ***time interval***. The *time* interval $[t_0, t_{trm}[$, $t_{trm} > t_0$, reflects the duration from the initial instant t_0 to the terminal instant t_{trm} either of the existence or of the non-existence of the related being, or of the related form of energy, of the related kind of matter, of the related object, of the related event, of the related movement, of the related process or of the related rest.

e) **Age**

Moment (***instant***) reflects *an instantaneous internal physical situation* of a material object called its **age**. ***Time*** **value difference** $t - t_0$, $t > t_0$, (or equivalently, ***time*** **interval** $[t_0, t]$) expresses, and is used to measure ***the duration*** of the (non-)existence of a being, of a form of energy, of a kind of matter, of an object, of a movement, of a process, of a rest, to measure duration of the (non-) existence of somebody or of something, relative to an accepted ***initial moment*** t_0. If it is the instant of the beginning of the (non-)existence of a being, of a form of energy, of a kind of matter, of an object, of a movement, of a process, of a rest, of somebody or of something, then the *time* value difference $t - t_0$, $t > t_0$, (or equivalently, *time* interval $[t_0, t]$) represents ***the age*** of the being, of the form of energy, of the kind of matter, of the object, of the movement, of the process, of the rest, of the somebody or of the something.

A co-ordinate axis used for a *time* axis is in the sequel immovable relative to the environment. It is denoted by *T*. It is a geometrical representation of the ***time set*** denoted also by *T*, which is the temporally ordered set of all instants:

$T = \{t: t \in R,\ t \in C^{(1)}(R),\ dt > 0\}$.

Evidently, $T \neq R$ due to dt > 0.

Once a *time* axis has been accepted with a fixed *time* scale including a fixed *time* unit, then the (relative) zero moment $t_{zero} = 0$ should have been also accepted. Afterwards, we can select any instant $t_0 \in T$ for an initial instant. We accept that it has been chosen, known and fixed. It can be $t_0 = 0$, but need not.

Different processes can propagate with different speeds in different beings or objects, which can hold also in the same being or object at its different points or parts. Therefore, different *time* scales, different initial moments and/or different *time* units can be assigned to different material objects and/or to different parts of the same material object giving a relative meaning to *time* (in this sense that is Newtonian).

We should recognise the fact, which has been often ignored, that **Newton** himself [34: pp. 8 – 10] introduced and explained both the absolute and relative sense of *time*. Moreover, we should appreciate the truth that Newton's explanation of the relativity of *time* incorporates that of Einstein [2, p. 20], [4, pp. 26 – 27], [5, pp. 23 – 40], which was explained and proved in [10], [11], [14] – [19].

4. DYNAMICAL SYSTEMS WITH MULTIPLE *TIME* SCALES

A large class of dynamical systems is adequately mathematically modelled by (S1) and (S2), [7] – [9], [12],

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}_1(t, \mathbf{x}, \mathbf{y}, M), \mathbf{x} \in R^p, \mathbf{y} \in R^s, \quad \text{(S1)}$$

$$\mathbf{f}_1(.): R \times R^p \times R^s \times R^{s \times s} \to R^p, M \in R^{s \times s},$$

$$M\frac{d\mathbf{y}}{dt} = \mathbf{f}_2(t, \mathbf{x}, \mathbf{y}, M), \quad \text{(S2)}$$

$$\mathbf{f}_2(.): R \times R^p \times R^s \times R^{s \times s} \to R^s.$$

The matrix M = diag$\{\mu_1\ \mu_2\ \ldots\ \mu_s\}$ contains different (possibly small) parameters μ_k that enable the introduction of s different *time* scales and/or units, $\mu_k \in R^+$, k = 1, 2, ... , s, $R^+ =]0, \infty[$:

$$t_k - t_{k0} = \mu_k (t - t_0),\ t_{k0} = \mu_k t_0, \mu_k \in R^+,\ k = 1, 2, \ldots, s. \quad (1)$$

5. COMPATIBILITY OF TRANSFORMATIONS

The *time* scaling coefficients α^i_j and α^j_i, and the space scaling coefficients λ^i_j and λ^j_i should be positive real valued. They intervene in the following generic co-ordinate transformations (2) through (5) :

$$t_i - t_{i0} = \alpha^i_j [(t_j - t_{j0}) + \frac{v^j_{ji}}{\upsilon^j \omega^j} \rho_j (t_j; t_{j0})],$$

$$i,j \in \{-, 1, 2, \ldots, s\},\ i \leq j, \quad (2)$$

$$t_j - t_{j0} = \alpha_i^j[(t_i - t_{i0}) - \frac{v_{ji}^i}{\upsilon^i\omega^i}\rho_i(t_i;t_{i0})],$$

$$i,j \in \{-, 1, 2, \ldots, s\},\ i \le j, \quad (3)$$

$$\mathbf{r}_i(t_i;t_{i0}) = \lambda_j^i[\mathbf{r}_j(t_j;t_{j0}) + \boldsymbol{v}_{ji}^j(t_j - t_{j0})],$$

$$i,j \in \{-, 1, 2, \ldots, s\},\ i \le j, \quad (4)$$

$$\mathbf{r}_j(t_j;t_{j0}) = \lambda_i^j[\mathbf{r}_i(t_i;t_{i0}) - \boldsymbol{v}_{ji}^i(t_i - t_{i0})],$$

$$i,j \in \{-, 1, 2, \ldots, s\},\ i \le j, \quad (5)$$

where

$\upsilon^{(.)}, \omega^{(.)}$ are the reference speeds, the values of which are measured relative to $T_{(.)}$, and which obey the following:

$\upsilon^{(.)}, \omega^{(.)} \in R^+$,

$(.) = i, j;\ i, j \in \{1, 2, \ldots, s\},\ i \le j$,

and

$\upsilon^{(.)}\omega^{(.)} \in \left\{\left[c_{(.)}^{(.)}\right]^2, \left[v_P^{(.)}\right]^2\right\}$ is permitted,

$(.) = i, j;\ i, j \in \{1, 2, \ldots, s\},\ i \le j$,

$c_{(.)}^{(.)}$ is the light speed relative to the integral space $I_{(.)} = T_{(.)} \mathrm{x} R_{(.)}^n$, $(.) = i, j;\ i, j \in \{1, 2, \ldots, s\},\ i \le j$.

The *time* scaling coefficients α_j^i and α_i^j, and the space scaling coefficients λ_j^i and λ_i^j are permitted a priori to be different in order to express that *time* is independent of space.

The co-ordinate transformations (2) through (4) do not contain the speed of light. In a special case $\upsilon^{(.)}\omega^{(.)} = \left[c_{(.)}^{(.)}\right]^2$. Then the co-ordinate transformations contain the speed of light.

The condition for the preservation of the generalised length in integral spaces will be used in its general form (6),

$$\left[\mathbf{r}_i^T \quad (t_i - t_{i0})\left[\mathbf{v}_P^{O_i}(t_i)\right]^T\right] G \left[\mathbf{r}_i^T \quad (t_i - t_{i0})\left[\mathbf{v}_P^{O_i}(t_i)\right]^T\right]^T \equiv$$

$$\left[\mathbf{r}_j^T \quad (t_j - t_{j0})\left[\mathbf{v}_P^{O_j}(t_j)\right]^T\right] G \left[\mathbf{r}_j^T \quad (t_j - t_{j0})\left[\mathbf{v}_P^{O_j}(t_j)\right]^T\right]^T . \quad (6)$$

The matrix G = blockdiag{A -B} in (6), A and B are positive definite matrices with A = B possible but not required, A and $B \in R^{n x n}$. The block diagonal form of the matrix G reflects *time* independence of space (Axiom 1).

The temporal co-ordinate equations (1) are inherent for multiple *time* scale dynamical systems [10], [11], [14], [17]. The condition (6) expresses the Gaussian transformation of the *time*-space length. It is a crucial condition of Einstein's general relativity theory for validity of the co-ordinate transformations defined by (2) through (5), [2], [3].

Definition 1.

a) The temporal co-ordinate transformations (2) and (3) are ***compatible*** if, and only if they yield an identity as soon as one temporal co-ordinate and the corresponding (with the same subscript) spatial co-ordinate are eliminated from them [without using the spatial co-ordinate transformations (4) and (5)]. Otherwise, they are incompatible.

Likewise, the spatial co-ordinate transformations (4) and (5) are compatible if, and only if they yield an identity as soon as one spatial co-ordinate and the corresponding (with the same subscript) temporal co-ordinate are eliminated from them [without using the temporal co-ordinate transformations (2) and (3)]. Otherwise, they are incompatible.

b) The transformations (2) through (5) are ***pairwise compatible*** if, and only if both the temporal co-ordinate transformations (2) and (3) are compatible and the spatial co-ordinate transformations (4) and (5) are compatible. Otherwise, they are pairwise incompatible.

c) The transformations (2) through (5) are ***entirely compatible*** if, and only if both 1) and 2) hold:

1) The temporal co-ordinate transformations (2) and (3) yield, by means of the spatial co-ordinate transformations (4) and (5), an identity as soon as temporal and spatial co-ordinates with the same subscripts are eliminated from them.

2) The spatial co-ordinate transformations (4) and (5) yield, by means of the temporal co-ordinate transformations (2) and or (3), an identity as soon as temporal and spatial co-ordinates with the same subscripts are eliminated from them.

Otherwise they are ***entirely incompatible***.

d) The transformations (2) through (5) are ***partially*** (i.e. ***restrictively***) [***pairwise***, ***entirely***] ***compatible*** if, and only if they are, respectively, [pairwise, entirely] compatible exclusively when the arbitrary point P moves with the speed restricted by certain conditions (which do not allow its arbitrary non-zero value), e. g. with the light speed.

e) The transformations (2) through (5) are ***completely*** (***pairwise***, ***entirely***) ***compatible*** if, and only if they are, respectively, (pairwise, entirely) compatible regardless of the speed of the arbitrary point P different from the light speed.

Definition 2. The co-ordinate transformations (2) through (5) are *consistent* if, and only if the same *time* scale and the same *time* unit are applied to all variables related to the same integral space.

6. CONSTRAINTS OF THE EINSTEINIAN RELATIVITY THEORY

The original Lorentz transformations were determined under the following a priori accepted conditions, which were also a priori adopted by Einstein, hence in the Einsteinian relativity theory [1] - [6], [22] - [27]:

Constraint 1. All *time* scaling coefficients α^i_j and α^j_i are equal: $\alpha^i_j \equiv \alpha^j_i \equiv \alpha$.

Constraint 2. All space scaling coefficients λ^i_j and λ^j_i are equal: $\lambda^i_j \equiv \lambda^j_i \equiv \lambda$.

Constraint 3. The arbitrary point P moves with the velocity of light: $\mathbf{v}^{(.)}_P(t_{(.)}) \equiv \mathbf{c}^{(.)}_{(.)} \equiv c^{(.)}_{(.)}\mathbf{u}$, (.) = i,j; i,j∈{-,1,2,…, s},

Constraint 4. The numerical value of every speed, including the value $c^{(.)}_{(.)}$ of the light speed, is invariant relative to *time* axes: $v^{(.)}_{(..)}(t_{(.)}) \equiv v_{(..)}(t_{(.)})$, $c^{(.)}_{(.)} \equiv c_{(.)}$, (.) = i,j; i,j∈{-,1,2,…, s}, i ≤ j,
and the value of the light speed, hence its numerical value as well, is also invariant with respect to inertial spatial co-ordinate systems:
$c_{(.)} \equiv c$, (.) = i,j; i,j∈{-,1,2,…, s}, i ≤ j.

Constraint 5. The position of the arbitrary point P is the position of the corresponding light signal: $\mathbf{r}_{(.)}(t_{(.)}; t_{(.)0}) \equiv \rho_L(t_{(.)}; t_{(.)0})\mathbf{u} \equiv c(t_{(.)} - t_{(.)0})\mathbf{u}$, $\rho_L(t_{(.)}; t_{(.)0}) \in R_+$, $\forall t_{(.)} \in T_{(.)}$,(.) = i,j; i,j∈{-,1,2,…, s}, i ≤ j.

Constraint 6. The matrices A and B in G, (6), are equal, A=B → G = blockdiag{A -A}.

In Einstein's special relativity they are the identity matrix I, A = B = I.

These constraints will be avoided in the fundamentals of the new relativity theory.

7. FUNDAMENTALS OF THE COMPATIBLE AND CONSISTENT RELATIVITY THEORY

7.1. Problem statement

There are two essentially different cases relative to the mutual relationship among the scaling coefficients, which are constant in either case:

General case: $\alpha^j_i \neq \alpha^i_j$ and/or $\lambda^j_i \neq \lambda^i_j$.

Special case: $\alpha^j_i = \alpha^i_j$ and $\lambda^j_i = \lambda^i_j$.

A particular sub-case of the special case concerns the invariance of the light speed and of the transfer speed relative to a choice of a *time* axis. It is the ***singular case***.

Problem statement. *Determine the values of the scaling coefficients* α^i_j, α^j_i, λ^i_j, λ^j_i, *and* μ_i, *(1) – (5), for the movement of the arbitrary point P with an arbitrary velocity, so that they are positive real numbers, that the equations (2) through (5) hold, and that they together with (1) imply the identity (6) in the general case. Verify compatibility of the resulting co-ordinate transformations.*

7.2 Problem solutions for the general case

We will allow for the (numerical) value of any speed, hence of the light speed, to depend on an integral space with respect to which it is determined.

Theorem 1. Let the *time* scaling coefficients μ_i be defined by (1). If the speed of the arbitrary point P is arbitrary, then, in order for the scaling coefficients $\alpha^j_i \in R^+$, $\alpha^i_j \in R^+$, $\alpha^j_i \neq \alpha^i_j$, $\lambda^j_i \in R^+$ and $\lambda^i_j \in R^+$, $\lambda^j_i \neq \lambda^i_j$, to be constant and to obey the equations (2) through (5), and for (1) through (5) to imply (6), it is necessary and sufficient that the speed of the point P is constant and that the following equations hold for any choice of the *time* scaling coefficients $\mu_i \in R^+$ and $\mu_j \in R^+$:

$$\alpha^i_j = \frac{\mu_i}{\mu_j}\frac{1}{1+\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}} = \frac{v^j_P}{v^i_P}\frac{1}{1+\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}} = \frac{c^j_j}{c^i_i}\frac{1}{1+\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}},$$

$i,j \in \{-, 1, 2, \ldots, s\}$, $i \leq j$, (7)

$$\alpha^j_i = \frac{\mu_j}{\mu_i}\frac{1}{1-\frac{v^i_{ji}v^i_P}{\upsilon^i\omega^i}} = \frac{v^i_P}{v^j_P}\frac{1}{1-\frac{v^i_{ji}v^i_P}{\upsilon^i\omega^i}} = \frac{c^i_i}{c^j_j}\frac{1}{1-\frac{v^i_{ji}v^i_P}{\upsilon^i\omega^i}},$$

$i,j \in \{-, 1, 2, \ldots, s\}$, $i \leq j$, (8)

$$\lambda^i_j = \frac{1}{1+\frac{v^j_{ji}}{v^j_P}},\ i,j \in \{-,1,2,\ldots,s\},\ i \le j, \tag{9}$$

$$\lambda^j_i = \frac{1}{1-\frac{v^i_{ji}}{v^i_P}},\ i,j \in \{-,1,2,\ldots,s\},\ i \le j, \tag{10}$$

$$0 \le v^{(.)}_{ji} < v^{(.)}_P,\ (.) \in \{i,j\},\ i,j \in \{-,1,2,\ldots,s\},\ i \le j, \tag{11}$$

and

$$\frac{\mu_j}{\mu_i} = \frac{v^i_P}{v^j_P} = \frac{c^i_i}{c^j_j},\ i,j \in \{-,1,2,\ldots,s\},\ i \le j. \tag{12}$$

The equations (7) – (10) give the next form to the equations (2) – (5):

$$t_i - t_{i0} = \frac{\mu_i}{\mu_j}\,\frac{(t_j - t_{j0}) + \frac{v^j_{ji}}{\upsilon^j\omega^j}\rho_j(t_j;t_{j0})}{1+\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}}, \tag{13}$$

$$t_j - t_{j0} = \frac{\mu_j}{\mu_i}\,\frac{(t_i - t_{i0}) - \frac{v^j_{ji}}{\upsilon^j\omega^j}\rho_i(t_i;t_{i0})}{1-\frac{v^i_{ji}v^i_P}{\upsilon^i\omega^i}}, \tag{14}$$

$$\mathbf{r}_i(t_i;t_{i0}) = \frac{\mathbf{r}_j(t_j;t_{j0}) + \mathbf{v}^j_{ji}(t_j - t_{j0})}{1+\frac{v^j_{ji}}{v^j_P}}, \tag{15}$$

$$\mathbf{r}_j(t_j;t_{j0}) = \frac{\mathbf{r}_i(t_i;t_{i0}) - \mathbf{v}^i_{ji}(t_i - t_{i0})}{1-\frac{v^i_{ji}}{v^i_P}}, \tag{16}$$

The transformations (13) through (16) are completely both entirely and pairwise compatible.

Proof. *Necessity*. Let the *time* scaling coefficients $\mu_i \in R^+$, $i = 1, 2, \ldots, s$, be defined by (1). Let the coefficients $\alpha^j_i \in R^+$, $\alpha^i_j \in R^+$, $\alpha^j_i \neq \alpha^i_j$, $\lambda^j_i \in R^+$ and $\lambda^i_j \in R^+$, $\lambda^j_i \neq \lambda^i_j$, be constant and obey (2) through (5), and let (1) through (5) imply (6). Since $\rho_j(t_j;t_{j0})$ represents the position of the arbitrary point P measured relative to the integral space $I_j = T_j \times R^n_j$, then $\rho_j(t_j;t_{j0}) = \bar{v}^{O_j}_P(t_j;t_{j0})(t_j - t_{j0})$, j= -, 1, 2, … , s. This, (1) and (2) yield:

$$t_i - t_{i0} = \mu_i(t - t_0) = \alpha^i_j\left[1+\frac{v^j_{ji}\bar{v}^{O_j}_P(t_j;t_{j0})}{\upsilon^j\omega^j}\right](t_j - t_{j0}) =$$

$$= \alpha^i_j\left[1+\frac{v^j_{ji}\bar{v}^{O_j}_P(t_j;t_{j0})}{\upsilon^j\omega^j}\right]\mu_j(t - t_0),$$

so that:

$$\alpha^i_j = \frac{\mu_i}{\mu_j}\left[1+\frac{v^j_{ji}\bar{v}^{O_j}_P(t_j;t_{j0})}{\upsilon^j\omega^j}\right]^{-1}. \tag{17}$$

Since the scaling coefficients are constant, it follows that $\bar{v}^{O_j}_P(t_j;t_{j0}) \equiv const. \Leftrightarrow v^{O_j}_P(t_j;t_{j0}) \equiv v^{O_j}_P = const. \Leftrightarrow \bar{v}^{O_i}_P(t_i;t_{i0}) \equiv const. \Leftrightarrow v^{O_i}_P(t_i;t_{i0}) \equiv v^{O_i}_P = const.$ This proves the constancy of the speed of the arbitrary point P. This and the result (17) imply the first equation in (7). The first equation in (8) is analogously proved by combining $\rho_i(t_i;t_{i0}) = v^i_P(t_i - t_{i0})$ with (1) and (3). From (1) and from (4) we deduce the following:

$$\mathbf{r}_i(t_i;t_{i0}) = v^{O_i}_P\mu_i(t - t_0)\mathbf{u} = \lambda^i_j(v^{O_j}_P + v^j_{ji})\mu_j(t - t_0)\mathbf{u},$$

$$\lambda^i_j = \frac{\mu_i v^{O_i}_P}{\mu_j v^{O_j}_P}\left(1+\frac{v^j_{ji}}{v^{O_j}_P}\right)^{-1}. \tag{18}$$

We transform the right hand side of (6) as follows by using (1) and $\rho_k(t_k;t_{k0}) = v^{O_k}_P(t - t_k)$, k= 1, 2, …, s:

$$\begin{bmatrix}\mathbf{r}_i(t_i;t_{i0})\\ (t_i - t_{i0})\mathbf{v}^{O_i}_P\end{bmatrix}^T G \begin{bmatrix}\mathbf{r}_i(t_i;t_{i0})\\ (t_i - t_{i0})\mathbf{v}^{O_i}_P\end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix} v^{O_j}_P(v^{O_j}_P)^{-1}\mu^{-1}_j\mu_j\mu_i(t - t_0)v^{O_i}_P\mathbf{u}\\ v^{O_j}_P(v^{O_j}_P)^{-1}\mu^{-1}_j\mu_j\mu_i(t - t_0)v^{O_i}_P\mathbf{u}\end{bmatrix}^T G \bullet$$

$$\bullet \begin{bmatrix} v^{O_j}_P(v^{O_j}_P)^{-1}\mu^{-1}_j\mu_j\mu_i(t - t_0)v^{O_i}_P\mathbf{u}\\ v^{O_j}_P(v^{O_j}_P)^{-1}\mu^{-1}_j\mu_j\mu_i(t - t_0)v^{O_i}_P\mathbf{u}\end{bmatrix} \equiv$$

$$\begin{bmatrix} v^{O_i}_P(v^{O_j}_P)^{-1}\mu_i\mu^{-1}_j\mathbf{r}_j(t_j;t_{j0})\\ v^{O_i}_P(v^{O_j}_P)^{-1}\mu_i\mu^{-1}_j(t_j - t_{j0})\mathbf{v}^{O_j}_P\end{bmatrix}^T G \bullet$$

$$\bullet \begin{bmatrix} v^{O_i}_P(v^{O_j}_P)^{-1}\mu_i\mu^{-1}_j\mathbf{r}_j(t_j;t_{j0})\\ v^{O_i}_P(v^{O_j}_P)^{-1}\mu_i\mu^{-1}_j(t_j - t_{j0})\mathbf{v}^{O_j}_P\end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix}\mathbf{r}_j(t_j;t_{j0})\\ (t_j - t_{j0})\mathbf{v}^{O_j}_P\end{bmatrix}^T G \begin{bmatrix}\mathbf{r}_j(t_j;t_{j0})\\ (t_j - t_{j0})\mathbf{v}^{O_j}_P\end{bmatrix}.$$

The last identity implies:

$$\left(\mu_i v^{O_i}_P\right)\left(\mu_j v^{O_j}_P\right)^{-1} = 1. \tag{19}$$

This implies the first equality in (12). Since the point P is arbitrary, then it can represent the light signal so that then $v_P^{O_{(.)}} = c_{(.)}^{(.)}$, (.) = i, j, which implies the second equation in (12) in view of the preceding result. The equations (18) and (19) yield:

$$\lambda_j^i = \left(1 + \frac{v_{ji}^j}{v_P^{O_j}}\right)^{-1}.$$

This proves the equation (9). The equation (10) is proved in the same manner. It and the definition of $v_{ji}^{(.)}$ imply the inequalities in (11). The first equations in (7) and (8) together with (12) result, respectively, in the second and the third equations in (7) and (8).

Sufficiency. Let *time* scaling coefficients $\mu_k \in R^+$, k = 1, 2, ... , s, be defined by (1). Let (7) – (12) and all the conditions of the theorem statement hold. We start with (1), (7) and (12):

$$t_i - t_{i0} = \mu_i (t - t_0)\alpha_j^i (\alpha_j^i)^{-1} =$$

$$= \mu_i (t - t_0)\alpha_j^i \frac{\mu_j}{\mu_i}\left(1 + \frac{v_{ji}^j v_P^j}{\upsilon^j \omega^j}\right),$$

or, by using (1) for k = j, and $\rho_j(t;t_j) = v_P^j (t - t_j)$:

$$t_i - t_{i0} = \alpha_j^i \left(1 + \frac{v_{ji}^j v_P^j}{\upsilon^j \omega^j}\right)(t_j - t_{j0}) =$$

$$= \alpha_j^i \left[(t_j - t_{j0}) + \frac{v_{ji}^j}{\upsilon^j \omega^j}\rho_j(t_j;t_{j0})\right].$$

This proves validity of (2). The equation (3) is proved analogously by starting with $t_j - t_{j0}$. We transform $\mathbf{r}_i(t_i;t_{i0}) = v_P^{O_i}(t_i - t_{i0})\mathbf{u}$ by using $\rho_i(t_i;t_{i0}) = v_P^{O_i}(t_i - t_{i0})$, (1), (9) and (12):

$$\mathbf{r}_i(t_i;t_{i0}) = v_P^{O_i}(t_i - t_{i0})\mathbf{u} = v_P^{O_i}\lambda_j^i(\lambda_j^i)^{-1}(t_i - t_{i0})\mathbf{u} =$$

$$= v_P^{O_i}\lambda_j^i\left(1 + \frac{v_{ji}^j}{v_P^{O_j}}\right)(t_i - t_{i0})\mathbf{u} =$$

$$= v_P^{O_i}\lambda_j^i\left(1 + \frac{v_{ji}^j}{v_P^{O_j}}\right)\frac{\mu_i}{\mu_j}(t_j - t_{j0})\mathbf{u} =$$

$$= v_P^{O_j}\lambda_j^i\left((t_j - t_{j0}) + \frac{v_{ji}^j}{v_P^{O_j}}(t_j - t_{j0})\right)\mathbf{u} =$$

$$= \lambda_j^i\left(\rho_j(t_j;t_{j0}) + v_{ji}^j(t_j - t_{j0})\right)\mathbf{u} =$$

$$= \lambda_j^i\left(\mathbf{r}_j(t_j;t_{j0}) + v_{ji}^j(t_j - t_{j0})\mathbf{u}\right).$$

This proves (4). The equation (5) is analogously proved by starting with $\mathbf{r}_j(t_j;t_{j0}) = v_P^{O_j}(t_j - t_{j0})\mathbf{u}$. Let us now transform the left-hand side of (6) as follows by using $\mathbf{r}_i(t_i;t_{i0}) = v_P^{O_i}(t_i - t_{i0})\mathbf{u}$, $\mathbf{v}_P^{O_i} = v_P^{O_i}\mathbf{u}$, in view of constancy of the velocity $\mathbf{v}_P^{O_i}$, and (1):

$$\begin{bmatrix} \mathbf{r}_i(t_i;t_{i0}) \\ (t_i - t_{i0})\mathbf{v}_P^{O_i} \end{bmatrix}^T G \begin{bmatrix} \mathbf{r}_i(t_i;t_{i0}) \\ (t_i - t_{i0})\mathbf{v}_P^{O_i} \end{bmatrix} \equiv$$

$$\begin{bmatrix} v_P^{O_j}(v_P^{O_j})^{-1}\mu_j(\mu_j)^{-1}\mu_i(t - t_0)v_P^{O_i}\mathbf{u} \\ v_P^{O_j}(v_P^{O_j})^{-1}\mu_j(\mu_j)^{-1}\mu_i(t - t_0)v_P^{O_i}\mathbf{u} \end{bmatrix}^T G \bullet$$

$$\bullet \begin{bmatrix} v_P^{O_j}(v_P^{O_j})^{-1}\mu_j(\mu_j)^{-1}\mu_i(t - t_0)v_P^{O_i}\mathbf{u} \\ v_P^{O_j}(v_P^{O_j})^{-1}\mu_j(\mu_j)^{-1}\mu_i(t - t_0)v_P^{O_i}\mathbf{u} \end{bmatrix}.$$

This, $\mathbf{r}_j(t_j;t_{j0}) = v_P^{O_j}(t_j - t_{j0})\mathbf{u}$, $\mathbf{v}_P^{O_j} = v_P^{O_j}\mathbf{u}$, (1) and (12) imply:

$$\begin{bmatrix} \mathbf{r}_i(t_i;t_{i0}) \\ (t_i - t_{i0})\mathbf{v}_P^{O_i} \end{bmatrix}^T G \begin{bmatrix} \mathbf{r}_i(t_i;t_{i0}) \\ (t_i - t_{i0})\mathbf{v}_P^{O_i} \end{bmatrix} \equiv$$

$$\left[(v_P^{O_j}\mu_j)^{-1}v_P^{O_i}\mu_i\right]^2 \begin{bmatrix} \mathbf{r}_j(t_j - t_{j0})\mathbf{u} \\ (t_j - t_{j0})\mathbf{v}_P^{O_j} \end{bmatrix}^T G \begin{bmatrix} \mathbf{r}_j(t_j - t_{j0})\mathbf{u} \\ (t_j - t_{j0})\mathbf{v}_P^{O_j} \end{bmatrix} \equiv$$

$$\equiv \begin{bmatrix} \mathbf{r}_j(t_j - t_{j0})\mathbf{r}_0 \\ (t_j - t_{j0})\mathbf{v}_P^{O_j} \end{bmatrix}^T G \begin{bmatrix} \mathbf{r}_j(t_j - t_{j0})\mathbf{r}_0 \\ (t_j - t_{j0})\mathbf{v}_P^{O_j} \end{bmatrix}.$$

This proves (6) and completes the proof of sufficiency.

Compatibility. We verify compatibility of the transformations (13) through (16) as follows. We replace $(t_j - t_{j0})$ from (14) into (13):

$$t_i - t_{i0} =$$

$$= \frac{\mu_i}{\mu_j}\left[\frac{\mu_j}{\mu_i}\frac{(t_i - t_{i0}) - \frac{v_{ji}^i}{\upsilon^i\omega^i}\rho_i(t_i;t_{i0})}{1 - \frac{v_{ji}^i v_P^i}{\upsilon^i\omega^i}} + \frac{v_{ji}^j}{\upsilon^j\omega^j}\rho_j(t_j;t_{j0})\right] \bullet$$

$$\bullet \left(1 + \frac{v_{ji}^j v_P^j}{\upsilon^j\omega^j}\right)^{-1},$$

and use $\rho_{(.)}(t_{(.)};t_{(.)0}) = v_P^{(.)}(t_{(.)} - t_{(.)0})$ for (.) = i, j,

$$t_i - t_{i0} =$$

$$= \frac{\mu_i}{\mu_j}\left[\begin{array}{l} \frac{\mu_j}{\mu_i}\left(1-\frac{v^i_{ji}v^i_P}{\upsilon^i\omega^i}\right)(t_i - t_{i0})\left(1-\frac{v^i_{ji}v^i_P}{\upsilon^i\omega^i}\right)^{-1} + \\ +\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}(t_j - t_{j0}) \end{array}\right]\bullet$$

$$\bullet\left(1+\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}\right)^{-1} =$$

$$= \left[(t_i - t_{i0}) + \frac{\mu_i}{\mu_j}\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}(t_j - t_{j0})\right]\left(1+\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}\right)^{-1}.$$

(1) permits us to replace $(t_j - t_{j0})$ by $(\mu_j/\mu_i)(t_i - t_{i0})$:

$$(t_i - t_{i0}) = \left[\begin{array}{l}(t_i - t_{i0}) + \\ +\frac{\mu_i}{\mu_j}\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}\frac{\mu_j}{\mu_i}(t_i - t_{i0})\end{array}\right]\left(1+\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}\right)^{-1} =$$

$$= \left(1+\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}\right)(t_i - t_{i0})\left(1+\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}\right)^{-1} = (t_i - t_{i0}).$$

This proves compatibility between (13) and (14) independently of the position and the speed of the arbitrarily accepted point P in space. Hence, their compatibility is complete. We verify their compatibility with their relationships with velocities expressed by:

$(t_{(.)} - t_{(.)0}) = (v_P^{(.)})^{-1}\rho_{(.)}(t_{(.)};t_{(.)0})$, (.) = i, j,

as follows by utilising (1) applied to (13):

$$t_i - t_{i0} = \frac{\mu_i}{\mu_j}\frac{(t_j - t_{j0}) + \frac{v^j_{ji}}{\upsilon^j\omega^j}\rho_j(t_j;t_{j0})}{1+\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}} =$$

$$= \frac{\mu_i}{\mu_j}\frac{\frac{\mu_j}{\mu_i}(t_i - t_{i0}) + \frac{v^j_{ji}}{\upsilon^j\omega^j}v^j_P\frac{\mu_j}{\mu_i}(t_i - t_{i0})}{1+\frac{v^j_{ji}v^j_P}{\upsilon^j\omega^j}} = t_i - t_{i0}.$$

This proves their compatibility. In the same way we prove compatibility of (14) with

$(t_{(.)} - t_{(.)0}) = (v_P^{(.)})^{-1}\rho_{(.)}(t_{(.)};t_{(.)0})$, (.) = i, j.

In order to verify compatibility of the spatial co-ordinate transformations we replace at first $\mathbf{r}_j(t_j;t_{j0})$ by the right-hand side of (16) into (15), we use $\mathbf{r}_i(t_i;t_{i0}) = \mathbf{v}_P^{O_i}(t_i - t_{i0})$ and afterwards we apply (1) and (12):

$$\mathbf{r}_i(t_i;t_{i0}) = \frac{\frac{\mathbf{r}_i(t_i;t_{i0}) - v^i_{ji}(t_i - t_{i0})\mathbf{u}}{1-\frac{v^i_{ji}}{v_P^{O_i}}} + v^j_{ji}(t_j - t_{j0})\mathbf{u}}{1+\frac{v^j_{ji}}{v_P^{O_j}}} =$$

$$= \frac{\frac{\left(1-\frac{v^i_{ji}}{v_P^{O_i}}\right)\mathbf{r}_i(t_i;t_{i0})}{1-\frac{v^i_{ji}}{v_P^{O_i}}} + v^j_{ji}\frac{\mu_j}{\mu_i}\mu_i(t - t_0)\mathbf{u}}{1+\frac{v^j_{ji}}{v_P^{O_j}}} =$$

$$= \frac{\mathbf{r}_i(t_i;t_{i0}) + v^j_{ji}\frac{\mu_j}{\mu_i}(t_i - t_{i0})\mathbf{u}}{1+\frac{v^j_{ji}}{v_P^{O_j}}} =$$

$$= \frac{\mathbf{r}_i(t_i;t_{i0}) + v^j_{ji}\frac{v_P^{O_i}}{v_P^{O_j}}(t_i - t_{i0})\mathbf{u}}{1+\frac{v^j_{ji}}{v_P^{O_j}}} =$$

$$= \frac{\left(1+\frac{v^j_{ji}}{v_P^{O_j}}\right)\mathbf{r}_i(t_i;t_{i0})}{1+\frac{v^j_{ji}}{v_P^{O_j}}} = \mathbf{r}_i(t_i;t_{i0}).$$

This proves compatibility between (15) and (16) for the arbitrary point P rather than only for the light signal. Let us check their compatibility with their relationships with the velocities: $\mathbf{r}_{(.)}(t_{(.)};t_{(.)0}) = \overset{-}{v}_P^{O_{(.)},(.)}(t_{(.)} - t_{(.)0})\mathbf{u}$ by using (1), (12), (15) and (16),

$$\mathbf{r}_i(t_i;t_{i0}) = \frac{v_P^{O_j}(t_j - t_{j0})\mathbf{u} + v^j_{ji}(t_j - t_{j0})\mathbf{u}}{1+\frac{v^j_{ji}}{v_P^{O_j}}} =$$

$$= v_P^{O_j}\frac{\mu_j}{\mu_i}(t_i - t_{i0})\mathbf{u} = v_P^{O_i}(t_i - t_{i0})\mathbf{u} = \mathbf{r}_i(t_i;t_{i0}).$$

They are compatible also with their relationships with the velocities. The equations (15) and (16) are completely compatible. Altogether, the transformations (13) through (16) are completely pairwise compatible.

We will verify their complete entire compatibility as follows. We replace, respectively, $(t_j - t_{j0})$ and $\mathbf{r}_j(t_j;t_{j0})$ from (13) and (16) into (15):

$$\mathbf{r}_i(t_i;t_{i0}) = \left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right)^{-1} \bullet$$

$$\bullet\left[\mathbf{r}_j(t_j;t_{j0}) + v_{ji}^j \frac{\mu_j}{\mu_i}\frac{(t_i - t_{i0}) - \frac{v_{ji}^i}{\upsilon^i\omega^i}\rho_i(t_i;t_{i0})}{1-\frac{v_{ji}^i v_P^i}{\upsilon^i\omega^i}}\mathbf{u}\right] =$$

$$= \left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right)^{-1} \bullet$$

$$\bullet\left[\mathbf{r}_j(t_j;t_{j0}) + v_{ji}^j \frac{\mu_j}{\mu_i}\frac{1-\frac{v_{ji}^i v_P^i}{\upsilon^i\omega^i}}{1-\frac{v_{ji}^i v_P^i}{\upsilon^i\omega^i}}(t_i - t_{i0})\mathbf{u}\right] =$$

$$= \left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right)^{-1} \bullet$$

$$\bullet\left[\frac{\mathbf{r}_i(t_i;t_{i0}) - v_{ji}^i(t_i - t_{i0})\mathbf{u}}{1-\frac{v_{ji}^i}{v_P^{O_i}}} + v_{ji}^j\frac{\mu_j}{\mu_i}(t_i - t_{i0})\mathbf{u}\right] =$$

$$= \left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right)^{-1}\left(\frac{1-\frac{v_{ji}^i}{v_P^{O_i}}}{1-\frac{v_{ji}^i}{v_P^{O_i}}} + \frac{\mu_j}{\mu_i}\frac{v_{ji}^j}{v_P^{O_i}}\right)\mathbf{r}_i(t_i;t_{i0}) =$$

$$= \left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right)^{-1}\left(1+\frac{v_{ji}^j}{v_P^{O_j}}\right)\mathbf{r}_i(t_i;t_{i0}) = \mathbf{r}_i(t_i;t_{i0}).$$

This proves complete entire compatibility among the transformations (13), (15) and (16). We prove in the same way complete entire compatibility among (14), through (16). Altogether, the transformations (13) through (16) are completely entirely compatible. Q. E. D.

The equations (13) through (16) are beyond the Lorentz - Einstein relativity theory. The former are rational functions of $v_{ji}^i/\upsilon^i\omega^i$, $v_{ji}^j/\upsilon^j\omega^j$, $v_{ji}^i/v_P^{O_i}$ or $v_{ji}^j/v_P^{O_j}$, while the latter are not. The former do not contain square roots or squared speeds, while the latter do both. The former contain the relative values of all speeds, but not the latter. The former are proved completely pairwise and entirely compatible, while the latter are not completely pairwise compatible. The former do not contain the light speed in general, while the latter do necessarily. Therefore, the former show that the light speed is not a necessary reference speed, while the latter demands that exclusively.

The preceding theorem results from the characterisation of *time* based on the physical reality and our experience (Axiom 1). The *time* independence is expressed by different scaling factors in equations (2) through (5). They essentially differ from those by Lorentz. The above result is general. The special case will be examined in the sequel.

The preceding theorem takes the following form in the case the reference frames R_i^n and R_j^n move in parallel with the same velocity: $\mathbf{v}_{ji}^i = \mathbf{v}_{ji}^j = \mathbf{0}$. This is important for dynamical systems (S1), (S2).

Corollary 1. If the reference frames R_i^n and R_j^n move with the same velocity in the same direction and sense, and if the speed of the arbitrary point P is arbitrary, then, in order for the scaling coefficients α_i^j, α_j^i, $\alpha_i^j \neq \alpha_j^i$, λ_i^j and λ_j^i, to be constant and to obey the equations (2) through (5), and for (1) through (5) to imply (6), it is necessary and sufficient that the following equations hold for any choice of $\mu_i \in R^+$ and $\mu_j \in R^+$**:**

$$\alpha_j^i = \frac{\mu_i}{\mu_j} = \frac{c_j^j}{c_i^i}, \quad \alpha_i^j = \frac{\mu_j}{\mu_i} = \frac{c_i^i}{c_j^j}, \quad \lambda_j^i = \lambda_i^j = 1,$$

and

$$\frac{\mu_j}{\mu_i} = \frac{v_P^{O_i}}{v_P^{O_j}} = \frac{c_i^i}{c_j^j},\ i, j = 1,2,\ldots,\text{n}.$$

The (numerical) value of the light speed need not be the same with respect to $I_i = T_i \times R_i^n$ and $I_j = T_j \times R_j^n$ as soon as the *time* scales of T_i and T_j are different: $\mu_i \neq \mu_j$.

The space scaling factors $\lambda_i^j = \lambda_j^i = 1$ because R_i^n and R_j^n move with the same speed.

The preceding corollary is important for dynamical systems (S1), (S2) with multiple time scales. It links the relativity theory with the theory of these systems. It establishes the conditions under which the dynamical system (S1), (S2) can have different time scales.

7.3 *Problem solution for the special case*

In the case the relative values c_i^i and c_j^j of the light speed are mutually equal, $c_i^i = c_j^j$, then they are denoted by c_{ij} or by c_{ji}, $c_i^i = c_j^j = c_{ij} = c_{ji}$. This designates the same relative value of the light speed with respect to I_i and I_j. Then, v_{ji} denotes that v_{ji}^i and v_{ji}^j are mutually equal: $v_{ji}^i = v_{ji}^j = v_{ji} = -v_{ij} = -v_{ij}^i = -v_{ij}^j$.

Theorem 2. Let the *time* scaling coefficients $\mu_i \in R^+$, $i = 1, 2, ..., s$, be defined by (1). In order for the coefficients $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j = \alpha_j^i = \alpha_{ij} = \alpha_{ji}$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j = \lambda_j^i = \lambda_{ij} = \lambda_{ji}$, to be positive real numbers and to obey (2) through (5), and for (1) through (5) to imply (6) it is necessary (but not sufficient) that the following equations hold for any choice of the *time* scaling coefficient $\mu_i \in R^+$:

$$\overset{-}{v}_P^{O_{(.)}}(t_{(.)}) \equiv v_P^{O_{(.)}}(t_{(.)}) \equiv v_P^{O_{(.)}} = \sqrt{\upsilon^{(.)}\omega^{(.)}} = \text{const.},$$

$(.) \in \{i,j\}$, $i,j \in \{-, 1, 2, \ldots, s\}$, $i \le j$, (20)

$$\frac{v_{ji}^i}{v_{ji}^j} \equiv \frac{v_P^i}{v_P^j} \equiv \frac{c_i^i}{c_j^j}, \quad i,j \in \{-, 1, 2, \ldots, s\}, i \le j, \qquad (21)$$

$$\alpha_{ij} = \frac{1}{\sqrt{1-\left(\frac{v_{ji}^i}{\sqrt{\upsilon^i\omega^i}}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{v_{ji}^j}{\sqrt{\upsilon^j\omega^j}}\right)^2}},$$

$i,j \in \{-, 1, 2, \ldots, s\}$, $i \le j$, (22)

$$\lambda_{ij} = \frac{1}{\sqrt{1-\left(\frac{v_{ji}^i}{v_P^i}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{v_{ji}^j}{v_P^j}\right)^2}},$$

$i,j \in \{-, 1, 2, \ldots, s\}$, $i \le j$, (23)

$$\frac{\mathbf{u}^T\mathbf{A}\mathbf{u}}{\mathbf{u}^T\mathbf{B}\mathbf{u}} = \left(\frac{v_{ji}^i}{v_{ji}^j}\right)^2 = \left(\frac{v_P^i}{v_P^j}\right)^2 = \left(\frac{c_i^i}{c_j^j}\right)^2,$$

i,j ∈ {-, 1, 2, …, s}, i ≤ j, (24)

$v_{ji}^{(.)} < v_P^{(.)}$, $(.) \in \{i,j\}$, $i,j \in \{-, 1, 2, \ldots, s\}$, $i \le j$, (25)

$$\mu_j = \mu_i\sqrt{\frac{1-\frac{v_{ji}^j}{v_P^j}}{1+\frac{v_{ji}^j}{v_P^j}}} = \mu_i\sqrt{\frac{1-\frac{v_{ji}^i}{v_P^i}}{1+\frac{v_{ji}^i}{v_P^i}}},$$

$i,j \in \{-, 1, 2, \ldots, s\}$, $i \le j$. (26)

The equations (2) through (5) become the equations (27) through (30),

$$t_i - t_{i0} = \frac{t_j + \frac{v_{ji}^j}{\upsilon^j\omega^j}\rho_j(t_j;t_{j0})}{\sqrt{1-\left(\frac{v_{ji}^j}{\sqrt{\upsilon^j\omega^j}}\right)^2}},$$

$i,j \in \{-, 1, 2, \ldots, s\}$, $i \le j$, (27)

$$t_j - t_{j0} = \frac{t_i - t_{i0} + \frac{v_{ji}^i}{\upsilon^i\omega^i}\rho_i(t_i;t_{i0})}{\sqrt{1-\left(\frac{v_{ji}^i}{\sqrt{\upsilon^i\omega^i}}\right)^2}},$$

$i,j \in \{-, 1, 2, \ldots, s\}$, $i \le j$, (28)

$$\mathbf{r}_i(t_i;t_{i0}) = \frac{\mathbf{r}_j(t_j;t_{j0}) + v_{ji}^j(t_j - t_{j0})\mathbf{u}}{\sqrt{1-\left(\frac{v_{ji}^j}{v_P^j}\right)^2}},$$

$i,j \in \{-, 1, 2, \ldots, s\}$, $i \le j$, (29)

$$\mathbf{r}_j(t_j;t_{j0}) = \frac{\mathbf{r}_i(t_i;t_{i0}) + v_{ji}^i(t_i - t_{i0})\mathbf{u}}{\sqrt{1-\left(\frac{v_{ji}^i}{v_P^i}\right)^2}},$$

$i,j \in \{-, 1, 2, \ldots, s\}$, $i \le j$. (30)

If additionally A = B in G, (6), then for the coefficients $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j = \alpha_j^i = \alpha_{ij} = \alpha_{ji}$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j = \lambda_j^i = \lambda_{ij} = \lambda_{ji}$, to be positive real numbers and to obey (2) through (5), and for (1) through (5) to imply (6) it is necessary and sufficient that the equations (20) – (26) and the following equations hold for any choice of the *time* scaling coefficient $\mu_i \in R^+$:

$v_{ji}^i = v_{ji}^j = v_{ji}$, $v_P^i = v_P^j = v_{Pij}$, (31)

$i,j \in \{-, 1, 2, \ldots, s\}$, $i \le j$,

$c_i^i = c_j^j = c_{ij} = c_{ji}$, $i,j \in \{-, 1, 2, \ldots, s\}$, $i \le j$, (32)

The transformations (21) through (23) are partially (but not completely) both pairwise and entirely compatible.

Proof. The proof of this theorem is very long. It is therefore omitted due to the space limitation. It is along the same lines as the proof of Theorem 2 in [18].

The equations (22), (23) generalise the Lorentz formulas for α and λ, which have been basic in the Einsteinian relativity theory.

7.4 Problem solution for the singular case

In the singular case Constraints 1 through Constraint 6 are valid. They give the next form to Theorem 2:

Corollary 2. Let Constraint 1 through Constraint 6 be valid. Let the *time* scaling coefficients $\mu_i \in R^+$, i = 1, 2, ... , s, be defined by (1). In order for the coefficients $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j = \alpha_j^i = \alpha_{ij} = \alpha_{ji}$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j = \lambda_j^i = \lambda_{ij} = \lambda_{ji}$, to be positive real numbers and to obey (2) through (5), and for (1) through (5) to imply (6) it is both necessary and sufficient that the equations (20) through (30) hold for any choice of the *time* scaling coefficient $\mu_i \in R^+$ and for $c_{ij} = c_{ji} = c$ and $v_{ij} = -v_{ji} = -v$. The equations (2) through (5) become the Lorentz transformations (L1) through (L4):

$$\mathrm{t_i - t_{i0}} = \frac{(\mathrm{t_j - t_{j0}}) + \frac{v}{c^2}\rho_j(\mathrm{t_j;t_{j0}})}{\sqrt{1-\left(\frac{v}{c}\right)^2}}, \quad \text{(L1)}$$

$$\mathrm{t_j - t_{j0}} = \frac{(\mathrm{t_i - t_{i0}}) - \frac{v}{c^2}\rho_i(\mathrm{t_i;t_{i0}})}{\sqrt{1-\left(\frac{v}{c}\right)^2}}, \quad \text{(L2)}$$

$$\mathbf{r}_i(\mathrm{t_i;t_{i0}}) = \frac{\mathbf{r}_j(\mathrm{t_j;t_{j0}}) + \boldsymbol{v}(\mathrm{t_j - t_{j0}})}{\sqrt{1-\left(\frac{v}{c}\right)^2}}, \quad \text{(L3)}$$

$$\mathbf{r}_j(\mathrm{t_j;t_{j0}}) = \frac{\mathbf{r}_i(\mathrm{t_i;t_{i0}}) - \boldsymbol{v}(\mathrm{t_i - t_{i0}})}{\sqrt{1-\left(\frac{v}{c}\right)^2}}. \quad \text{(L4)}$$

Corollary 2, hence the fact that the Lorentz transformations (L1) through (L4) are obtained by starting with Axiom 1 on *time*, show that there is not any collision among them. The Lorentz transformations (L1) through (L4) may not be used, and cannot be used, to state any of the following essentially and fundamentally wrong claims:

Wrong claim 1. *Time* is a dependent variable.

Wrong claim 2. *Time* depends on space.

Wrong claim 3. The speed of *time* value evolution changes for a clock that changes its speed.

Comment 1. It is not *time* that forces a clock (a watch) to operate. Any variation of the speed of a clock (of a watch) hand corresponds only to an adequate variation of the *time* unit, but not to any variation of the speed of the *time* value evolution. It is energy that forces the clock (the watch) to operate. Marmet [28] - [33] explained why a variation of the speed of a clock (of watch) itself causes a variation of the speeds of the clock (of the watch) hands. However, let us repeat once more, a change of the speed of the clock (of the watch) hand means nothing else than the corresponding change of the *time* unit. Once we determine the scaling coefficients among the *time* scales corresponding to different speeds of the clock (of the watch) hands, then we can use the equation (1) to verify that all clocks (watches) show always the same moment, provided they are identical, operate identically in the same conditions, their hands were set at the same initial position and they started to operate at the same instant.

Comment 2 . It is not *time* that causes a variation of the mass of a body when its velocity varies. It is a variation of the energy exchange between the body and its environment, which causes the variation of the mass of the body. For the explanation see the papers by Marmet [28] - [33].

8. SPEED TRANSFORMATIONS

The transformations (2) – (5) together with (7) – (10), i. e. (13) – (16), are completely both entirely and pairwise compatible only in the general case. The speed transformations will be presented for that case.

Theorem 3. Let the *time* scaling coefficients $\mu_i \in R^+$, *i* = 1, 2, ... , *s*, be defined by (1). Let the scaling coefficients μ_i, $\alpha_i^j \in R^+$, $\alpha_j^i \in R^+$, $\alpha_i^j \neq \alpha_j^i$, $\lambda_i^j \in R^+$ and $\lambda_j^i \in R^+$, $\lambda_i^j \neq \lambda_j^i$, be positive real numbers and obey (7) through (10), and let (1) through (5) imply (6). Then, a constant non-zero speed $\mathbf{v}_{\mathrm{P}}^{\mathrm{O_i}}$ of the arbitrary point P with respect to the origin $\mathrm{O_i}$ of R_i^n and relative to $\mathrm{t_i}$ and the speed $\mathbf{v}_{\mathrm{P}}^{\mathrm{O_j}}$ of the same point P with respect to the origin $\mathrm{O_j}$ of R_{j}^n and relative to $\mathrm{t_j}$ are interrelated as follows:

$$\mathbf{v}_P^{O_i} = \frac{\mu_j}{\mu_i}\frac{\mathbf{v}_P^{O_j} + \mathbf{v}_{ji}^j}{1 + \frac{v_{ji}^j}{v_{\mathrm{P}}^{\mathrm{O_j}}}} = \frac{\mu_j}{\mu_i}\mathbf{v}_P^{O_j} = \frac{c_i^i}{c_j^j}\mathbf{v}_P^{O_j}, \quad (31)$$

$$\mathbf{v}_P^{O_j} = \frac{\mu_i}{\mu_j}\frac{\mathbf{v}_P^{O_i} - \mathbf{v}_{ji}^i}{1-\frac{v_{ji}^i}{v_P^{O_i}}} = \frac{\mu_i}{\mu_j}\mathbf{v}_P^{O_i} = \frac{c_j^j}{c_i^i}\mathbf{v}_P^{O_i}. \qquad (32)$$

The transformations are completely compatible.

Proof. Let all the conditions hold. Hence, Theorem 1 holds. The velocity is defined as:

$$\mathbf{v}_P^{O_{(.)}}(t_{(.)}) = \frac{d\mathbf{r}_{(.)}(t_{(.)};t_{(.)0})}{dt_{(.)}}, (.) = i, j. \qquad (33)$$

By applying (13) and (15) to the right hand side of the preceding equation and by using (1), $d\rho_P(t_j;t_{j0})/dt_j = v_P^{O_j}$, $v_{ji}^j\mathbf{r}_0 = \boldsymbol{v}_{ji}^j$ and (12) we find:

$$\mathbf{v}_P^{O_i} = \frac{d\left[\dfrac{\mathbf{r}_j(t_j;t_{j0}) + v_{ji}^j(t_j - t_{j0})\mathbf{u}}{1+\dfrac{v_{ji}^j}{v_P^{O_j}}}\right] : dt_j}{d\left[\dfrac{\mu_i}{\mu_j}\dfrac{(t_j - t_{j0}) + \dfrac{v_{ji}^j}{\upsilon^j\omega^j}\rho_j(t_j;t_{j0})}{1+\dfrac{v_{ji}^j v_P^{O_j}}{\upsilon^j\omega^j}}\right] : dt_j} =$$

$$= \frac{\dfrac{\mathbf{v}_P^{O_j} + v_{ji}^j\mathbf{u}}{1+\dfrac{v_{ji}^j}{v_P^{O_j}}}}{\dfrac{\mu_i}{\mu_j}\dfrac{1+\dfrac{v_{ji}^j v_P^{O_j}}{\upsilon^j\omega^j}}{1+\dfrac{v_{ji}^j v_P^{O_j}}{\upsilon^j\omega^j}}} =$$

$$= \frac{\mu_j}{\mu_i}\frac{\mathbf{v}_P^{O_j} + \mathbf{v}_{ji}^j}{1+\dfrac{v_{ji}^j}{v_P^{O_j}}} = \frac{c_i^i}{c_j^j}\frac{\mathbf{v}_P^{O_j} + \mathbf{v}_{ji}^j}{1+\dfrac{v_{ji}^j}{v_P^{O_j}}} = \frac{\mu_j}{\mu_i}\mathbf{v}_P^{O_j} = \frac{c_i^i}{c_j^j}\mathbf{v}_P^{O_j}.$$

This proves (31). The equation (32) is analogously proved by using (1), (12), (14), (16), (33), $d\rho_i(t_i;t_{i0})/dt_i = v_P^i$ and $v_{ji}^i\,\mathbf{u} = \mathbf{v}_{ji}^i$. Complete compatibility of (31) and (32) is obvious. Q. E. D.

The equations (31) and (32) confirm the equations (12).

The velocity transformation equations (31) and (32) are beyond the Lorentz – Poincaré - Einstein relativity theory. The speed value ratios in the denominators of the right hand side quotients in (31) and (32) do not contain the light speed value, and the speed value $v_P^{O_{(.)}}$ appears in the denominators of the ratios "$v_{ji}^{(.)}/v_P^{(.)}$", (.) = i, j, rather than in the numerators of the ratio "$vv_P^{O_{(.)}}/c$" that is the ratio in the speed transformations resulting from the Lorentz transformations, [21] - [24]. Formulas (31) and (32) contain the light speed values c_i^i and c_j^j relative to I_i and I_j, respectively. However, the light speed value is independent of *time* scale and of *time* unit in the Lorentz – Poincaré - Einstein relativity theory [1] – [5], [21] - [24], [35].

9. Conclusion

The physical reality and our experience permitted us to present and to accept the axiomatic characterisation of the properties of *time*. They are expressed in Axiom 1. They are in Newton's sense. They show that *time* is a unique physical variable. It is a basic physical variable. The numerical value of the speed of *time* value evolution equals one (with respect to all *time* axes). It is invariant relative to a choice of a *time* axis. *Time* is the unique physical variable with such property of its speed. Even the numerical value of the speed of light is not invariant with respect to a choice of a *time* axis. The features of *time* enabled us to avoid all constraints a priori accepted in the Einsteinian relativity theory after Lorentz and Einstein, and to start establishing fundamentals of a new relativity theory.

The co-ordinate transformations are *compatible* if, and only if the application of the inverse transformation to the transformation itself results in an identity. If, and only if this holds for both pairs, the pair of the temporal co-ordinate transformations and the pair of the spatial co-ordinate transformations, then the transformations are *pairwise compatible*. If, and only if this is true for the temporal and spatial transformations altogether, then they are *entirely compatible*. If, and only if this compatibility of the transformations is valid only for the light speed of an arbitrary point, then the compatibility of the transformations is *partial (restrictive) compatibility*. Their compatibility is *complete* if, and only if it holds for arbitrary value of the speed of an arbitrary point.

Uniformity of the transformations means that the temporal co-ordinate transformations are independent of a choice of an arbitrary point (of its position and of its speed), i. e. that they hold uniformly over space. Uniform co-ordinate transformations were established in [17] through [20]. Otherwise, the transformations are *non-uniform*, which holds for the transformations (13) through (16) established herein.

The *time* scaling factors defined by the equation (1), which are used in the theory of dynamical systems with multiple *time* scales, are introduced in the relativity theory in Theorem 1 in this framework.

The new relativity theory is compatible and consistent. It is called *compatible and consistent* (for short: *CC*) *relativity theory*. The *time* scaling coefficients and the space scaling coefficients are a priori all permitted to be mutually different. This led to their new forms and to the new forms of the corresponding co-ordinate transformations. They are crucially different from the Lorentz transformations. They show that the relative light speed value should be used in computations, i. e. that the value of the light speed depends on the integral space relative to which it is measured. They have implied new formulas for speed. They are proved for the general case and their complete entire and pairwise compatibility is also verified.

The new scaling factors determined by (7) through (10), or by (22), (23), and the related new co-ordinate transformations (13) through (16), respectively, (27 through (30), do not contain the speed of light in general.

It is shown that starting with Axiom 1 and by accepting Constraint 1 through Constraint 6, we get the Lorentz co-ordinate transformations. This clarifies that they may not, and cannot be used to claim either *time* dependence on space or a variation of the speed of *time* value evolution. Such claims are wrong. *Time* does not forces any clock (any watch) to work. A variation of the speed of clock (watch) hands means only the corresponding variation of the *time* units, and vice versa. It does not mean either a variation of the speed of *time* value evolution or the existence of different *time*s for clocks (for the watches) moving with different speeds. Energy, not *time*, forces the clock (the watch) to operate. *Time* is unique. There are not two or more different *times*. There are many different *time* scales and *time* units. Newton himself explained this in [34].

Bibliographical references

[1] R. D'Inverno, *Introducing Einstein's Relativity*, Clarendon Press, 0xford (1995).

[2] A. Einstein, *La théorie de la relativité*, Gauthier-Villars et Cie, Paris (1921).

[3] A. Einstein, *L'éther et la théorie de la relativité.* Gauthier-Villars et Cie, Paris (1921).

[4] A. Einstein, *The meaning of relativity.* Methuen & Co. Ltd., London (1950).

[5] A. Einstein, *Relativity.* University paperbacks, Methuen & Co. Ltd., London (1960).

[6] A. Einstein and L. Infeld, *L'évolution des idées en physique*, Flammarion, Paris (1938).

[7] Lj. T. Grujić, Singular perturbations and large-scale systems, *International Journal of Control*, **29**, 159-169, 1979.

[8] Lj. T. Grujić, Singular perturbations, large-scale systems and asymptotic stability of invariant sets, *International Journal of Systems Science*, **10**, 1323 - 1341, 1979.

[9] Lj. T. Grujić, Sets and singularly perturbed systems, *Systems Science*, **5**, 327 - 338, 1979.

[10] Lj. T. Grujić, *Time and Modeling*, Proc. 1996 IEEE International Conference on Systems, Man and Cybernetics, Beijing, **3** (1996), 2438-2443.

[11] Lj. T. Grujić, *Multiple Time Scale Systems, Time and Modeling*, Pre-prints of the IFAC-IMACS-IFIP Conference on Control of Industrial Systems, Belfort, France, **1**, (1997), 293-298; also in Proceedings of the IFAC Conference Control of Industrial Systems, Pergamon, Elsevier, **1** (1997), 189-194.

[12] Lj. T. Grujić, A. A. Martynyuk and M. Ribbens-Pavella, *Large Scale Systems Stability under Structural and Singular Perturbations*, Springer Verlag, Berlin (1987).

[13] Ly. T. Gruyitch, *Physical continuity and uniqueness principle. Exponential natural tracking control*, Journal: Neural, Parallel and Scientific Computing, Dynamic Publishers, Atlanta, USA, Vol. 6, No. 2, (1998), 143 - 170.

[14] Ly. T. Gruyitch, "*Time*, Relativity and Physical Principle: Generalisations and Applications", *Proc. VI International Conference Physical Interpretations of Relativity Theory*, London, (1998), 134-170.

[15] Ly. T. Gruyitch, "*Time*, Relativity and Physical Principle: Generalizations and Applications", Nelinijni Koluvanna, Kiev, Ukraine, **2**, No. 4, (1999), 465-489.

[16] Ly. T. Gruyitch, "Gaussian Generalisations of the Relativity Theory Fundaments with Applications", *Preprints of the VII International Conference: Physical Interpretations of Relativity Theory*, pp. 125 - 136, London, 15 - 18 September, 2000.

[17] Ly. T. Gruyitch, "Systems Approach to the Relativity Theory Fundaments", *Nonlinear Analysis*, **47**, (2001), 37-48.

[18] Ly. T. Gruyitch, "*Time* and Uniform Relativity Theory Fundaments", *Problems of nonlinear analysis in engineering systems*, No. 2(14), **7**, (2001), 1-28.

[19] Ly. T. Gruyitch, "Compatibility and Uniformity in the Relativity Theory: Problems and Solutions", *VIII International Conference: Physical Interpretations of Relativity Theory*, London, 6 - 9 September, 2002, pp. 176-190

[20] Ly. T. Gruyitch, "ON *TIME*: Einsteinian and Non-Einsteinian CC relativity theory", Problems of Nonlinear Analysis in Engineering Systems, 9, No. 3 (19), pp. 18 - 54, Kazan, Russia, 2003.

[21] R. E. Kalman, P. L. Falb and M. A. Arbib, *Topics in Mathematical System Theory.* Mc Graw Hill, New York (1969).

[22] H. A. Lorentz, *Versuch einer theorie der electricschen und optischen Erscheinungen in bewegten Körpern*, E. K. Brill, Leiden, (1895).
[23] H. A. Lorentz, *Das Relativitätsprinzip*, B. G. Teubner, Leipzig (1914).
[24] H. A. Lorentz, *The theory of Electrons*, B. G. Teubner, Leipzig (1916).
[25] H. A. Lorentz, *Deux mémoires de Henri Poincaré sur la physique mathématique*, Acta mathematica, Zeitscrift, **38** (1921), 294-308.
[26] H. A. Lorentz, A. Einstein and H. Minkowski, *Das Relativitätsprinzip*, B. G. Teubner, Leipzig (1923).
[27] H. A. Lorentz, A. Einstein, H. Minkowski and H. Weyl, *The Principle of Relativity*, Dover Publications, Inc., New York (1952).
[28] P. Marmet, *Einstein's Theory of Relativity versus Classical Mechanics*, Newton Physics Books, 2401 Ogilvie Road, Gloucester, ON, Canada, K1J 7N4, 1997.
[29] P. Marmet, "Relationship between $E = mc^2$ and $m = \gamma m_o$", the Internet site by Paul Marmet: *http://www.newtonphysics.on.ca/faq/gamma-mass-13.html*, updated May 2002 (the text cited was copied for this book June 7, 2003).
[30] P. Marmet, "Natural Length Contraction Mechanism Due to Kinetic Energy", the Internet site by Paul Marmet: *http://www.newtonphysics.on.ca/kinetic/length.html*, June 10, 2003.
[31] P. Marmet, "Natural Physical Length Contraction Due to Gravity", the Internet site by Paul Marmet: *http://www.newtonphysics.on.ca/Gravity/gravity.html*, June 10, 2003.
[32] P. Marmet, "The GPS and the Constant Velocity of Light", the Internet site by Paul Marmet: *http://www.newtonphysics.on.ca/Illusion/index.html*, June 10, 2003.
[33] P. Marmet, "Fundamental Nature of Relativistic Mass and Magnetic Fields", the Internet site by Paul Marmet: *http://www.newtonphysics.on.ca/magnetic/mass.html*, September 23, 2003.
[34] I. Newton, *Mathematical Principles of Natural Philosophy - BOOK I. The Motion of Bodies*, (The first edition 1687), William Benton, Publisher, Encyclopaedia Britannica, Inc., Chicago (1952).
[35] I. Prigogine, Etude Thermodynamique des Phénomènes Irréversibles, Dunod, Paris (1947).