From wave-digital principles to relativity theory

Alfred Fettweis

1 Origin of the challenge

There are essentially three different simple expressions available by means of which an inductance can be described, i. e.,

$$u = \mathrm{D}(L'i), \quad u = L''\mathrm{D}i, \quad \mathrm{D} = \mathrm{d}/\mathrm{d}t \tag{1}$$

$$u = \sqrt{L}\mathrm{D}(\sqrt{L}i) = \frac{1}{2}(\mathrm{D}(Li) + L\mathrm{D}i), \tag{2}$$

u being the voltage, i the current. In general, these expressions are not equivalent, but the requirement for $L \geq 0$ to always hold is necessary and sufficient for guaranteeing passivity (in fact, losslessness).

In the simplest nonlinear case, i.e. if the inductance depends only on its own current i, the following can be shown to hold

$$L' = \frac{1}{2}L + \frac{1}{2i}\int_0^i L\mathrm{d}i, \quad L = \frac{2}{i^2}\int_0^i iL''\mathrm{d}i, \tag{3}$$

$$L'' = L + \frac{1}{2}i\frac{\mathrm{d}L}{\mathrm{d}i}, \quad L = 2L' - \frac{2}{i^2}\int_0^i iL'\mathrm{d}i. \tag{4}$$

From (3) it follows that

$$L'' \geq 0 \;\;\Rightarrow\;\; L \geq 0 \;\;\Rightarrow\;\; L' \geq 0. \tag{5}$$

None of the implication arrows in (5) may be reversed, as follows from (4) and as can be shown rigorously by means of the counterexamples $L = L_0/(1 + x^2)$ and $L' = L_0/(1 + x^2)$, respectively, where $x = i/i_0$, L_0 and i_0 being constants. Hence, in order to have $L \geq 0$, $L' \geq 0$ is not sufficient and $L'' \geq 0$ not necessary (always for all i).

In the most general case, i.e. if L depends on any of the depending variables in the entire circuit and/or on t, the situation is far more complicated, but the simplicity of (2) and the requirement $L \geq 0$ remain untouched. This confirms the fundamental role of (2) versus either one of the expressions (1), and this is even further strengthened in the multidimensional case, where D can be any of the partial differential operators then involved.

Despite all this, the relation between force, mass, and velocity is, in classical relativity, not introduced as

$$\mathbf{f} = \sqrt{m}\mathrm{D}\left(\sqrt{m}\mathbf{v}\right) = \frac{1}{2}\left(\mathrm{D}\left(m\mathbf{v}\right) + \mathbf{v}\mathrm{D}m\right) \tag{6}$$

but in the form

$$\mathbf{f} = \mathrm{D}\left(m'\mathbf{v}\right), \tag{7}$$

where we have written m' instead of m in order to conform with the notation in (1), m and m' being related in the same way as L an L' in (3) and (4). In fact, (7) is imposed as a postulate to be confirmed by experiment and is thus not derived by an immediately cogent logical deduction. This corresponds to placing prime emphasis on $\mathrm{m}'\mathbf{v}$, thus on momentum, while (6), like (2), places prime emphasis on passivity, thus on energy. The classical expression obtained for m' is

$$m' = m_0/\alpha, \quad \alpha = \sqrt{1-\beta^2}, \quad \beta = v/c, \quad v^2 = \mathbf{v}^{\mathrm{T}}\mathbf{v} \tag{8}$$

where m_0 is a constant also called rest mass and c is the velocity of light. The corresponding value obtained for m is rather unwieldy.

The question thus arises whether some different kind of postulate couldn't reverse that situation. Such a postulate must be of an immediately comprehensible physical nature and must be of relevance also to aspects of energy and thus of work done. It must not affect the validity of the Einsteinian kinematics, which is based on the Lorentz transformation and is the result of an admirable logic deduction. The results must of course be in agreement with unquestionable experimental facts.

A very brief outline of this challenging topic will be presented hereafter. Some details can be found in [1] to [4], with [3] being the most complete presentation so far, although certain aspects will be mentioned that have not yet been published.

2 The principle of Newtonian limit

As needed in the context of relativistic kinematics we consider two reference frames S and S', with S' moving with constant velocity $\mathbf{v}_0$ with respect to S. We assume $\mathbf{v}_0 = (v_0, 0, 0)^{\mathrm{T}}$ so that the Lorentz transformation is given by

$$x' = \frac{1}{\alpha_0}\left(x - v_0 t\right), \quad y' = y, \quad z' = z, \quad t' = \frac{1}{\alpha_0}\left(t - \beta_0\frac{x}{c}\right), \tag{9}$$

$$\alpha_0 = \sqrt{1-\beta_0^2}, \quad \beta_0 = v_0/c, \tag{10}$$

unprimed and primed coordinates referring to S and S', respectively. The corresponding relations for the velocities of a particle are

$$v'_x = \frac{v_x - v_0}{1-\beta_0\beta_x}, \quad v'_y = \frac{\alpha'}{\alpha}v_y, \quad v'_z = \frac{\alpha'}{\alpha}v_z, \tag{11}$$

$$\beta_x = v_x/c, \quad 1-\beta_0\beta_x = \alpha_0\alpha/\alpha', \tag{12}$$

$$\mathbf{v} = \left(v_x, v_y, v_z\right)^{\mathrm{T}}, \quad v^2 = \mathbf{v}^T\mathbf{v}, \quad \beta = v/c, \quad \alpha = \sqrt{1-\beta^2}, \tag{13}$$

$$\mathbf{v}' = \left(v'_x, v'_y, v'_z\right)^{\mathrm{T}}, \quad v'^2 = \mathbf{v}'^T\mathbf{v}', \quad \beta' = v'/c, \quad \alpha' = \sqrt{1-\beta'^2}. \tag{14}$$

We call a particle instantaneously motionless if in the reference frame and at the time instant under consideration its velocity is zero. We then define the principle of Newtonian limit as requiring that for a relevant quantity that can be defined in a given reference frame, say in S', under Newtonian and under relativistic conditions the ratio of the two corresponding quantities must go to unity if one approaches a time instant where the particle is instantaneously motionless.

3 Newton's second law

Let us then add a subscript 1 to quantities evaluated at some arbitrary fixed time instant t_1, thus correspondingly at t'_1. We consider a particle P of rest mass m_0 and assume S' to be such that $\mathbf{v}_0 = \mathbf{v}_1$, i.e., that P is instantaneously motionless at t'_1. The principle of Newtonian limit then immediately yields

$$\mathbf{f}'_1 = m_0 \left(\mathrm{D}'\mathbf{v}'\right)_1, \quad \mathrm{D}' = \mathrm{d}/\mathrm{d}t', \tag{15}$$

which is also valid in classical relativity. Consider however the work done in S' between t'_1 and $t'_2 = t'_1 + \Delta t'$, i.e.,

$$\Delta W' = \int_{t'_1}^{t'_2} \mathbf{f}'^{\mathrm{T}} \mathbf{v}' \mathrm{d}t'. \tag{16}$$

For the corresponding quantity under Newtonian conditions, which imply $t' = t$, $\mathbf{f}' = \mathbf{f}$, and $\mathbf{v}' = \mathbf{v} - \mathbf{v_0}$, we have

$$\left(\Delta W'\right)_N = \int_{t_1}^{t_2} \mathbf{f}^{\mathrm{T}} \left(\mathbf{v} - \mathbf{v}_0\right) \mathrm{d}t. \tag{17}$$

Both $\Delta W'$ and $\left(\Delta W'\right)_N$ become zero for $\Delta t' \to 0$, but according to the principle of Newtonian limit we require

$$\lim_{\Delta t' \to 0} \left(\Delta W' / \left(\Delta W'\right)_N\right) = 1, \tag{18}$$

and this obviously for any value of $\mathbf{f}'_1$.

From (9) to (18) one obtains, using some lengthy calculations, an expression for $\mathbf{f}$ that has precisely the form of (6) with

$$m = m_0/\alpha^2, \quad \alpha^2 = 1 - \beta^2 \tag{19}$$

thus with α as in (8). The alternative relativistic extension of Newton's 2nd law is thus indeed given by (6) and (19). For the power absorbed one finds

$$\mathbf{v}^{\mathrm{T}}\mathbf{f} = \mathrm{D}W_k, \quad W_k = \frac{1}{2} m v^2 = m_0 c^2 \beta^2 / 2\alpha^2, \tag{20}$$

W_k being the kinetic energy. It is different but simpler than the expression $m_0 c^2 \left(1/\alpha - 1\right)$ of classical relativity. The total energy is then

$$W = W_0 + W_k, \quad W_0 = \mathrm{const.},$$

the rest (internal) energy W_0 being in fact an integration constant; it is unspecified by (20). Noticing the valid decomposition $W_k = \frac{1}{2} mc^2 - \frac{1}{2} m_0 c^2$ one is tempted to set $W = \frac{1}{2} mc^2$ and $W_0 = \frac{1}{2} m_0 c^2$, but the classical $W_0 = m_0 c^2$ is also compatible with (20).

Observe that the quadruple

$$\begin{pmatrix} \mathbf{f} \\ \frac{1}{c}\mathbf{v}^{\mathrm{T}}\mathbf{f} \end{pmatrix} = \begin{pmatrix} \mathbf{f} \\ \frac{1}{c}\mathrm{D}W_k \end{pmatrix} \tag{21}$$

is identical to the four-vector originally introduced by Minkowski and thus has Lorentz invariant form.

The result given by (6) and (19) can be extended to the case of nonnegative definite matrices $\mathbf{m}_0$ and $\mathbf{m} = \mathbf{m}_0/\alpha^2$. However, even if the properties of a corresponding object are constant (like for a rigid body) these matrices may not be constant because the object may change its orientation while moving. Hence, a proper extension for the rigid case should be of the form

$$\mathbf{f} = \frac{1}{\alpha}\mathbf{m}_0^{\mathrm{T}/2}\mathrm{D}\left(\frac{1}{\alpha}\mathbf{m}_0^{1/2}\mathbf{v}\right) \tag{22}$$

where $\mathbf{m}_0^{\mathrm{T}/2} = \left(\mathbf{m}_0^{1/2}\right)^{\mathrm{T}}$ and $\mathbf{m}_0 = \mathbf{m}_0^{\mathrm{T}/2}\mathbf{m}_0^{1/2}$. The kinetic energy is now given by

$$W_k = \frac{1}{2\alpha^2}\mathbf{v}^{\mathrm{T}}\mathbf{m}_0\mathbf{v} = \frac{1}{2}\mathbf{v}^{\mathrm{T}}\mathbf{m}\mathbf{v}, \tag{23}$$

but the Lorentz invariance mentioned in relation with (21) is usually no longer valid.

The case of a nonrigid object requires more care.

4 Newton's third law and some consequences

Newton's first law is purely qualitative and thus always valid. Modifying his 2nd law as given by (6) and (19), however, can be shown to require, in the alternative theory, his 3rd law ($\mathbf{f}_1 = -\mathbf{f}_2$), which remains untouched in classical relativity, to be modified according to

$$\alpha_1\mathbf{f}_1 = -\alpha_2\mathbf{f}_2, \quad \alpha_i = \sqrt{1-\beta_i^2}, \quad \beta_i = v_i/c, \qquad i = 1,\ 2, \tag{24}$$

the subscripts 1 and 2 referring to the two particles P_1 and P_2 now involved, and v_1 and v_2 thus being their respective velocities.

If there are n particles P_ν, with rest masses $m_{\nu 0}$, velocities $\mathbf{v}_\nu$, and momenta (defined as in classical relativity)

$$\mathbf{p}_\nu = m_{\nu 0}\mathbf{v}_\nu/\alpha_\nu, \quad \alpha_\nu = \sqrt{1-\beta_\nu^2}, \quad \beta_\nu = v_\nu/c, \quad v_\nu^2 = \mathbf{v}_\nu^{\mathrm{T}}\mathbf{v}_\nu,$$

$\nu = 1 \ \text{to} \ n$, one obtains, using (24),

$$\mathrm{D}\left(\mathbf{p}_1 + \mathbf{p}_2 + \cdots + \mathbf{p}_n\right) = \mathbf{0}.$$

Hence conservation of momentum holds exactly as in classical relativity.

Consider still a particle P moving with velocity $\mathbf{v}$ in a field that exerts an actual force $\mathbf{f}$ (in the sense of the alternative theory) upon P. Let $\mathbf{f}_0$ be the force that would act upon P according to classical expressions. One finds, using again (24),

$$\mathbf{f} = \mathbf{f}_0/\alpha, \quad \alpha = \sqrt{1-\beta^2}, \quad \beta = v/c, \quad v^2 = \mathbf{v}^{\mathrm{T}}\mathbf{v}, \tag{25}$$

This can be confirmed, independently of (24), for an electromagnetic field. Using the known rules for transforming such a field from S to S' and furthermore the Lorentz invariance mentioned for (21) one finds that $\mathbf{f}$ is indeed given by (25) with

$$\mathbf{f}_0 = q\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right), \tag{26}$$

q being the charge of the particle, $\mathbf{E}$ the electric field, and $\mathbf{B}$ the magnetic induction.

Obviously, (22) and (25) combined imply that classical relativity and the alternative approach lead to exactly the same behavior of particles in fields. As a further consequence of (25), assuming that at a given location (position and time instant) one can assign to an electromagnetic field a velocity $\mathbf{v}$, one must replace the classical expressions for the energy density w and the Poynting vector $\mathbf{S}$ by

$$w = \frac{1}{2\alpha}\left(\varepsilon\,|\mathbf{E}|^2 + \mu\,|\mathbf{H}|^2\right), \quad \mathbf{S} = \frac{1}{\alpha}\mathbf{E}\times\mathbf{H}. \tag{27}$$

5 Moving electromagnetic field

We consider an electromagnetic field (in vacuum) using standard notation, apply the rules recalled subsequently to (25), and adopt primed and unprimed notation as in Section 3 but write $\mathbf{v}$, α, and β instead of $\mathbf{v}_0$, α_0, and β_0. Assuming that at the location under consideration we have $\mathbf{S}' = \mathbf{0}$ one finds for the field energy in an elementary volume $\mathrm{d}V$ of S,

$$w\mathrm{d}V = w'\mathrm{d}V' + \frac{1}{2\alpha^2}\mathbf{v}^{\mathrm{T}}\mathbf{m}_0\mathbf{v}\mathrm{d}V' \tag{28}$$

where, using (27),

$$\mathbf{S}' = \mathbf{E}'\times\mathbf{H}', \quad w' = \frac{1}{2}\left(\varepsilon\,|\mathbf{E}'|^2 + \mu\,|\mathbf{H}'|^2\right), \quad \mathrm{d}V = \mathrm{d}x\cdot\mathrm{d}y\cdot\mathrm{d}z,$$

$$\mathbf{m}_0 = \frac{2}{c^2}\left(2w'\mathbf{1} - \varepsilon\mathbf{E}'\mathbf{E}'^{\mathrm{T}} - \mu\mathbf{H}'\mathbf{H}'^{\mathrm{T}}\right), \quad \mathbf{1} = \text{unit matrix}$$

etc. For a given location and a given field in S one can always achieve $\mathbf{S}' = \mathbf{0}$ by choosing $\mathbf{v}$ according to

$$2\mathbf{v}\big/\left(1 + v^2/c^2\right) = \mathbf{S}/w, \quad v = |\mathbf{v}| < c. \tag{29}$$

This suggests interpreting (as would not be possible in the context of classical relativity) the field at the given location to be instantaneously motionless in S' (thus $\mathbf{v}' = \mathbf{0}, \quad \alpha' = 1$), moving in S with $\mathbf{v}$ given by (29), and having in $\mathrm{d}V$ a rest energy and a kinetic energy given by the first and the second term in the right-hand side of (28). There are infinitely many choices other than (29) that lead to $\mathbf{S}' = \mathbf{0}$ but yield the same lateral velocity and this way also the same kinetic energy.

One is thus further tempted to associate with that kinetic energy a force density $\mathbf{f}$ related to $\mathbf{m}_0$ and $\mathbf{v}$ in a way corresponding to (22). We restrict ourselves here to the simplest possible case and interpretation. It then turns out that due to $\mathbf{S}' = \mathbf{0}$ one may first decompose $\mathbf{f}$ into a sum, and the simplest expressions thus obtainable that are compatible with (22) and (28) are, assuming Cartesian coordinates,

$$\mathbf{f} = \mathbf{f}_E + \mathbf{f}_H, \qquad \mathbf{f}_E = \frac{2\varepsilon}{\alpha^2c^2}\mathbf{E}'\times\mathrm{D}\left(\frac{1}{\alpha}\mathbf{v}\times\mathbf{E}'\right),$$

$$\mathbf{f}_H = \frac{2\mu}{\alpha^2c^2}\mathbf{H}'\times\mathrm{D}\left(\frac{1}{\alpha}\mathbf{v}\times\mathbf{H}'\right),$$

while the expressions relating the fields in S and S' yield, due to (29),

$$\mathbf{E}' = \frac{1}{\alpha}\left(\mathbf{E} + \mu\mathbf{v}\times\mathbf{H}\right), \quad \mathbf{H}' = \frac{1}{\alpha}\left(\mathbf{H} - \varepsilon\mathbf{v}\times\mathbf{E}\right).$$

To this should be added, in a pure electromagnetic field, $\mathbf{f} = (\rho\mathbf{E} + \mu\mathbf{i} \times \mathbf{H})/\alpha$, $\mathbf{i}$ being the current density (in general $\mathbf{i} \neq \rho\mathbf{v}$) in the relevant Maxwell equation, and $\mathbf{f}$ being also related to the velocity in a way as just discussed. If such results should be confirmed they open up new perspectives for examining the behavior of objects consisting of fields.

6 The Bertozzi experiment

In 1964, Bertozzi [5] had reported, among other things, results about calorimetric measurements of the kinetic energy W_k of fast electrons (charge e) accelerated by a field of voltage u (notation modified to better suit our purpose) and then impacting and thus heating a target. Let W_c be the estimate of W_k as determined by calorimetry and define $\delta = W_c/eu$. Since there are unavoidable losses (e.g. by X-ray generation) we have $W_c < W_k$, thus $\delta < 1$ if indeed $W_k = eu$ as is the case according to classical relativity. For the alternative theory we have, as can be shown to follow from (24) and the presence of unavoidable secondary effects, $W_c > eu$, thus $\delta > 1$. Both test results ($eu =$ 1.5 and 4.5 MeV) mentioned in [5] yield δ = 1.067, thus indeed $\delta > 1$. However, it is also stated that the experiment was only 10% accurate, which would make the measured values to be not incompatible with $\delta < 1$. Obviously, higher accuracy is needed.

References

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