The Equations of Motion

4/06/2003

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Abstract

Using the space/time geometry developed in the previous paper ("Linear Motion in Space-Time, the Dirac Matrices, and Relativistic Quantum Mechanics"), I shall show clearly why the Lagrangian equation has been a successful tool in both classical, relativistic and quantum physics.

A general form of the energy equation.

We start with Figure 1 below in which the energy triangle previously discussed, has been drawn and each segment labeled.

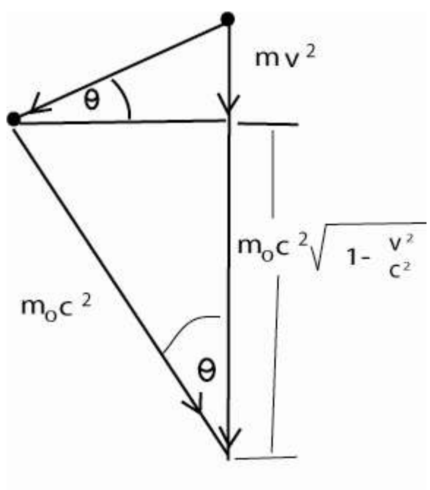

Figure 1

Substituting $m_0 = m\sqrt{1-V_P^2/c^2}$ for the m_0 term, yields Figure 2 below.

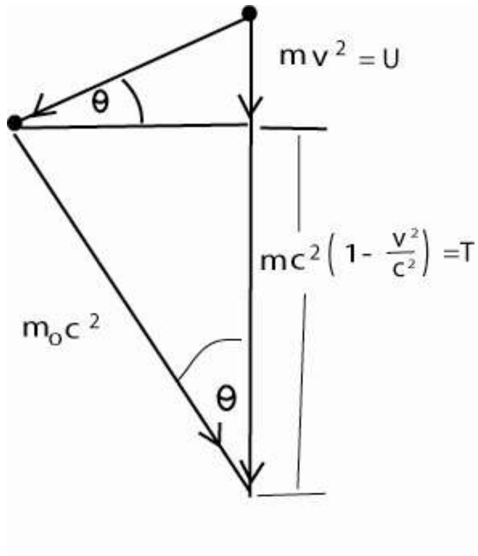

Figure 2

In the remainder of this paper the space/time velocity V_P shall simply be designated as v.

We define U and T as follows.

$$U = mv^2$$
$$T = mc^2\left(1 - v^2/c^2\right)$$

It is clear that the total energy is simply given by $E_T = T + U$.
If the total energy is constant ($E_T = mc^2$) then, m is a constant.

Each component of the total energy as well as the total energy can be written in the general functional form:

$$a + bv^2 \quad (1.10)$$

Furthermore, we know the classical energy equation $E_T = m_0c^2 + \frac{1}{2}m_0v^2$ is also in the form of 1.10.

Consider a point object which has a spatial position variable q. Clearly there is an associated velocity in the q direction given by $\frac{dq}{dt} = \dot{q}$.

Both q and $\dot{q}$ can be written as functions of time t.

$q = f(t)$ (1.20)

$\dot{q} = g(t)$ (1.30)

Taking the inverse function of 1.20 and substituting it into 1.30 yeilds $\dot{q}$ as a function of q.

Define a function $L = a + b\left(\dot{q}\right)^2$ (1.40)

Clearly 1.40 is in the form of 1.10.

It is easy to see that $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 2b\frac{d\dot{q}}{dt} = 2b\ddot{q}$ (1.50)

Also, taking $\dot{q}$ to be a function of q, we can see that $\frac{\partial L}{\partial q} = 2b\dot{q}\frac{\partial \dot{q}}{\partial q}$ (1.60)

Now, $\dot{q}\frac{\partial \dot{q}}{\partial q} = \frac{\partial \dot{q}}{\partial q}\frac{dq}{dt} = \frac{d\dot{q}}{dt} = \ddot{q}$

We can therefore conclude that $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$ (1.70)

1.70 is the LaGrangian equation of motion. There is a discrepancy between the above derivation and the LaGrangian methods used in physics today. In the above formulization, $\dot{q}$ was not independent of q. In fact, it was necessary to make $\dot{q}$ a function of q in order to arrive at the LaGrangian equation. Next we shall overcome this problem.

Consider the function: $L = a + b\left(\dot{q}\right)^2 + b\left(\dot{q}\right)^2$ (1.80)

Clearly 1.80 is in the form of the general energy equation 1.10.

We consider the second term to be a function of q by using the substitution $f(q)=\dot{q}(q)$ to arrive at: $L=a+b\left(\dot{q}\right)^2+b(f(q))^2$ (1.90)

Considering $\dot{q}$ and q to be independent variables, we see that:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)=2b\frac{d\dot{q}}{dt}=2b\ddot{q}$$

$$\frac{\partial L}{\partial q}=2bf(q)\frac{\partial f(q)}{\partial q}$$

But $f(q)=\dot{q}(q)$ and hence $f(q)\frac{\partial f(q)}{\partial q}=\dot{q}\frac{\partial \dot{q}}{\partial q}=\frac{\partial \dot{q}}{\partial q}\frac{dq}{dt}=\frac{d\dot{q}}{dt}=\ddot{q}$

We can therefore conclude that $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)=\frac{\partial L}{\partial q}$ for a function of the form of 1.90 where q and $\dot{q}$ are treated as independent variables.

We now have the necessary background to formulate and actual physical Lagrangian.
We know that $E_T=T+U$.
Consider the following equation:

$L=E_T-U-U$ (2.00)

Clearly 2.00 is in the form of 1.90 and the Lagrangian differential equation can be applied. We need a term to be solely the function of $\dot{q}$ and a term that is a function of q.
E_T-U is simply the measurable or kinetic energy T and is in the form $b\left(\dot{q}\right)^2$. We simply write the second U as a function of q, and the equation of motion for the object can be determined.

The function $L = E_t - U - U = T - U$ is exactly the equation Lagrange proposed over 200 years ago.

It is not a surprise that the Lagrangian method has proved successful in classical, relativistic and relativistic quantum physics since the energy can be written in the form of 1.10 in all cases.