Linear Motion in Space-Time, the Dirac Matrices, and Relativistic Quantum Mechanics

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Time is generally assumed to be non-directional. By allowing time to be directional, a space-time geometry can be developed that explains the Lorentz transforms as well the Dirac matrices.
 Keywords: Space-time geometry; Dirac matrice; Special relativity; Relativistic quantum mechanics; Quantum field theory; Quantum electrodynamics

The nature of Space-Time

About a century ago Poincaré arrived at the conclusion that, "No matter what observational facts are found, the physicist is free to ascribe to physical space any one of the mathematically possible geometrical structures…."[3, p. v]. In this paper, I shall take the liberty afforded by Poincaré and propose a geometrical structure for space-time that is not consistent with present theory.

In present relativity theory, space and time are combined into what is called a 4 vector [1]. A presumed and/or agreed upon direction for x, y and z is essential to coordinate this mathematical representation to any real physical phenomenon. The distinction of an opposite sign in a metric or the multiplication by the square root of negative one, are taken as adequate reasons for considering time to be non-directional [3].

In this paper, time will be treated as a vector. The direction of the time vector relative to any spatial axis will be related to the velocity of the object or system under consideration. Only in the case of stationary objects will the time vector be perpendicular to all spatial axes.

I will use the new term 'space/time' to represent space and time treated in the manner described. A vector space representation of space/time will be developed in this paper. The older term 'space-time' will refer to the conventional four-dimensional or four-vector representation.

Time and the Geometric Representation of Motion

The physical definition of time varies substantially from reference source to reference source, and the particular definition in a scientific context is often expressed in terms of relativity theory. When referring to time in this paper, its definition will not be limited to or associated with the common physical definition. Prior to making epistemological assumptions about the nature of Time a brief discussion is in order.

For every change in a measurable property of an observable there is an associated change in time. For example, take the two statements: I am in Los Angeles. I am in Chicago. The two cannot be true unless the times associated with the events are different.

In his landmark book, Reichenbach states: "Every lapse of time is connected with some process, for otherwise it could not be perceived at all." [3, p. 116]. A process by its very definition implies a change. Therefore we can conclude: *Any change in an observable is associated with a time change and any time change is associated with a change in some observable.*

If time stopped, all clocks would certainly stop. If time were to speed up or slow down, one would expect all clocks to speed up or slow down so as to track the rate of time. It is reasonable then to call the observed change on a clock's display the effect of time. *Time is the cause and the observed change on the display of the clock is the effect.*

Assume that a clock is put into linear motion. It does not seem as reasonable to say that time is the cause of the linear motion. However, moving the clock linearly involves some sort of process not unlike the process that moves the hands or increments the display. It is the designer of the clock who decided just how to move the hands. The same designer could use a very similar process to propel the clock in linear motion. One could infer that the linear motion is just as reasonably the effect of time. *Time is the cause and linear motion is the effect.*

If time is the cause of both the change in the display of the clock and the clock's linear motion, then time might best be treated as having components. This is the first motivation for the vector representation of time introduced here. We also believe from experiment that the rate at which the clock increments time is not independent of the linear motion. As the clock moves faster, its hands rotate slower. This effect is not in the design of clocks, but fundamental to the nature of time. There is a maximum time rate that the clock can achieve and this occurs when the clock has zero linear velocity. There is also a maximum speed (the speed of light) that the clock can achieve, and as this speed is approached, the time rate is reduced to zero. Proper choice of units/conversion factors can make these maximum values of clock rate and light speed numerically identical. It would appear then that time also should be treated as having a magnitude. This is the second motivation for the vector representation of time introduced here. In mathematics, something having both a magnitude and components is called a vector; so in this paper, time will be treated as a vector.

I now make the following epistemological assumptions regarding the nature of time:

- There exists a cause and effect relationship between time and change.
- Time is the cause, and change is the effect.
- Any and all observable change is the effect of time.

- The quantity of time available to affect any and all changes in any and all properties of an observable is fixed.

Motion is a perceivable change and therefore is the effect of time.

Linear space/time motion:

To translate the above assumptions into a mathematical representation of linear space/time motion, I make the following assumptions:

1) Space and time are related as proposed, by axiom, by N. Vivian Pope and Anthony Osborne, specifically "(A2) Observational distance and time have a constant ratio of units, c ...." [2, p193]. The value c is simply a conversion factor.

2) Space/time is a vector space with each dimension having units of distance.

3) The path that an object experiencing linear motion follows in space/time is a line in space/time. The line shall be referred to as "the axis of motion".

4) Any perceived change in time by an observer, multiplied by the speed of light, is a vector in space/time and is designated by $c\mathbf{T}$.

5) The vector representing the objects motion in space/time is the vector projection of $c\mathbf{T}$ onto the axis of motion.

6) The vector projection of an observer's $c\mathbf{T}$ vector onto a line perpendicular to the axis of motion, herein referred to as "the orthogonal component", is the observed progression of time for the observable multiplied by c.

7) All objects moving at the same space/time velocity will move along the same axis of motion.

The above assumptions do not fix $c\mathbf{T}$ in any particular direction. However from 5, it is clear that as the space/time velocity approached zero the angle between $c\mathbf{T}$ and the axis of motion approaches 90 degrees. When the space/time velocity is zero (i.e. a stationary object) we will call the axis of motion x and consider it to have the properties of all space. The axis on which the $c\mathbf{T}$ vector lies for zero space/time velocity shall be called the ct axis. From 7, it is clear that these axes are the same for all observables and hence can be used as coordinate axes. Any vector in space/time can be described in terms of its x and ct components.

For now, I shall limit the discussion to two dimensions using the plane of x and ct .

The following definitions and conventions will be used in the discussion of linear space/time motion:

- The symbol designating a vector in space/time will be in bold type (*i.e.* $\mathbf{T}$) and its magnitude in italic (*i.e.* T).
- The vector representing the component of $c\mathbf{T}$ projected onto the axis of motion shall be designated $c\mathbf{T}_p$.
- The axis of motion shall be called the p axis.
- The vector representing the space/time velocity shall be designated $\mathbf{v}_p$, and it is defined by the formula

$\mathbf{V}_P = c\mathbf{T}_P / T$.

- The orthogonal component shall be designated $c\mathbf{T}'$.

Figure 1 is a geometric representation of linear motion in the vector space. There the $c\mathbf{T}$ vector of observer A has been projected onto the path of change to represent the motion of the observable B. The orientation with respect to x, ct is not important at this point in the discussion.

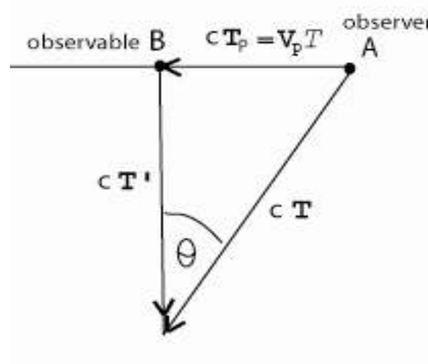

Figure 1

From Figure 1 and simple geometry it is clear that $V_P^2T^2 + c^2T'^2 = c^2T^2$. Multiplying each term by $1/c^2T^2$ and simplifying yields:

$$\cos\theta = T'/T = \sqrt{1 - V_P^2/c^2} \qquad 1.1$$

It is also apparent that:

$$\sin\theta = V_P / c \qquad 1.2$$

To understand the significance of T', we will evaluate an object at rest and then in motion. We wish to have a device that has moving parts but does not move on the p axis. We will use a clock. When the clock is stationary, $c\mathbf{T}$ lies on the ct axis and is perpendicular to the x axis. The clock, however, is changing and hence requires a component of time. This is represented in Fig. 2.

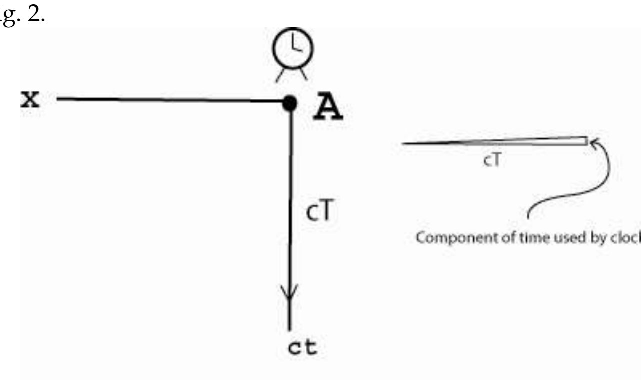

Figure 2

The time that increments the clocks display is represented as a small component of time perpendicular to the plane of x, ct . If it weren't perpendicular the clock would have translational motion. The component of time the clock requires is a function of its internal construction and is proportional to T.

Next the clock is put into motion. This is illustrated in Fig. 3. There, the clock is moving relative to A. The axis is in general, no longer the x axis but the axis of motion p. The amount of time available to run the clock from the point of view of A is now T'. The stationary observer A would then see the clock run slower. Its time would be dilated. The time on the clock relative to the stationary observer's time would be:

$$T' = T\sqrt{1 - V_p^2 / c^2} \qquad 1.3$$

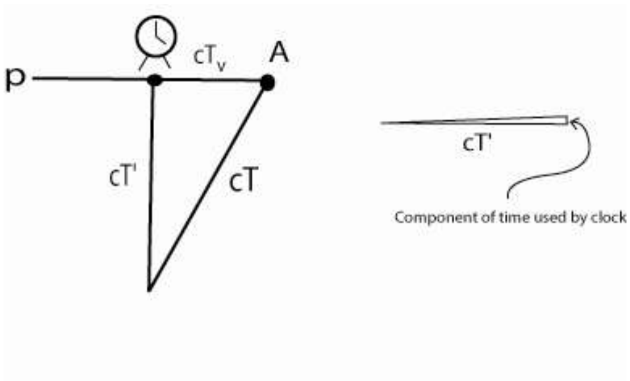

Figure 3

Next, I shall develop space/time transform equations using two separate assumptions. First I will assume that the rest observer's time direction is fixed on the ct axis. I will then assume that the rest observer's direction of motion is fixed on the x axis.

Fixed Time Direction Space/Time Transforms

In the following discussion, assume that the direction of the observer's $c\mathbf{T}$ vector is fixed for all possible values of motion and lies on the ct axis. The linear motion of an observable will be represented by rotating the axis of motion toward the fixed ct axis.

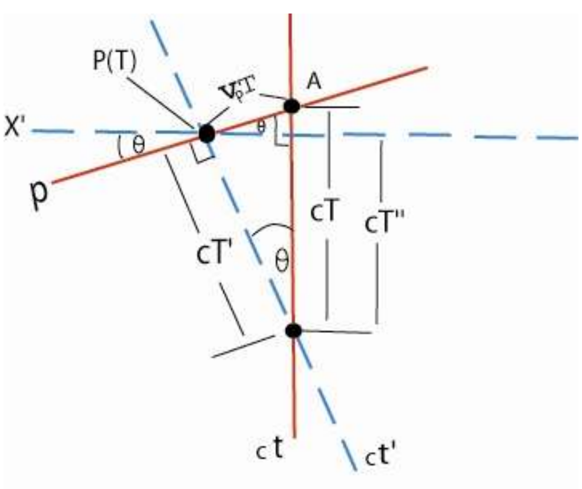

Figure 4

It is common practice to use a primed coordinate system to refer to the moving object or system and an unprimed coordinate system to refer to the rest object or system. In Fig. 4, the unprimed system is represented by the solid lines and the primed by the dashed lines. The center of the unprimed system is labeled A. The point $P(T)$ represents the moving object or the center of the moving system. For an observer in the moving system, the point $P(T)$ is stationary and hence the p axis is also the p' axis. The x' axis represents the motion of A relative to $P(T)$. The intersection of the x' axis with the ct axis yields another time T'' and it is clear that $T'' / T' = T' / T$.

It is important to note that in this representation, the two coordinate systems are symmetrical. If one were to flip the primed coordinate system around, its motion and time axis could be aligned to lie directly on the motion and time axis of the unprimed system. The only difference would be the direction of positive displacement on the respective motion axes.

We now develop the space/time transform for the positions on the motion lines. Refer to Fig. 5. As discussed above, the moving object's zero motion axis is just the p axis and hence the transformation equation for any point P' moving with $P(T)$ is simply:

$$P' = P - V_P T \qquad 1.4$$

Eq. 1.4 is a Galilean transformation

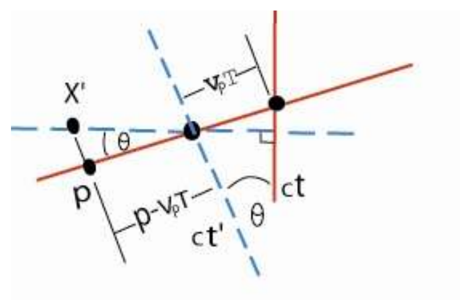

Figure 5

Now, if the observer at rest (A), believes that his/her spatial direction (zero motion line) must be the same as an observer traveling with $P(T)$, he/she would use the x' axis as the spatial axis for $P(T)$.

If asked to develop a transformation equation, he/she would take a random point P on the p axis and draw a line parallel to $P(T)$'s time line and find the intersection with the x' axis. This would be the point the stationary observer would assign (as the spatial point) to P based on the observers zero motion space line. It is clear from Fig. 5 and Eq. (0.1) that:

$$\frac{P - V_P T}{X'} = \cos\theta = \sqrt{1 - V_P^2 / c^2} \qquad 1.5$$

The above equation can be written in the more recognizable form:

$$X' = (P - V_p T) \Big/ \sqrt{1 - V_p^2 / c^2}$$

The above transform is identical in form to the Lorentz transform for spatial position with P substituted for x and V_P substituted for v.

To get the complete form of the space/time time transformation, refer to Fig. 6. T' is now the time associated with a random point P on the p axis. It is clear that $T''/T' = \cos\theta$. But T'' is just $T - \Delta T$. Using Eq. 1.1 we conclude that:

$$\frac{T - \Delta T}{T} = \sqrt{1 - V_P^2 / c^2}$$

Now $c\Delta T = P\sin\theta$, and it is obvious that $\sin\theta = V_P T / cT$. Substituting yields:

$$c\Delta T = PV_P / c \text{ or } \Delta T = PV_P / c^2$$

Substituting and rearranging yields the space/time time transform

$$T' = (T - V_p P / c^2) \Big/ \sqrt{1 - V_p^2 / c^2} \qquad 1.6$$

The above transform is identical to the Lorentz time transform with P replacing x and V_P replacing v.

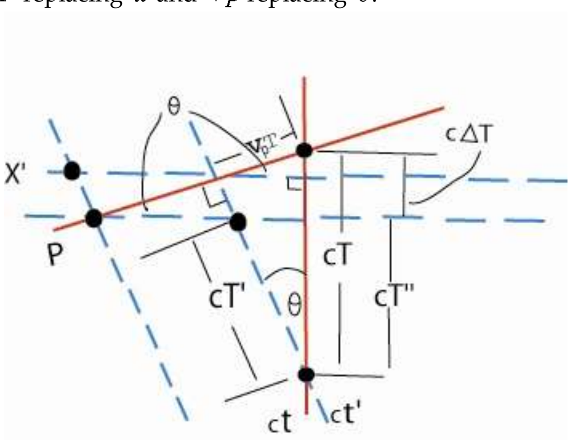

Figure 6

In the above discussion it was assumed that the observer's time was fixed in direction. The axis of motion was rotated towards time. This representation shall be referred to as $P \rightarrow T$ space/time representation.

Fixed spatial direction Space/time transforms

In this Section the p axis shall be fixed and assumed to lie along the x axis. Motion shall be represented by rotating $c\mathbf{T}$ toward it. The representation shall be referred to as $T \rightarrow S$ space/time representation. Figure 7 represents two observers labeled A and B. We let A be at rest and B is moving away from A. X_1 represents a random point on the x axis. T_1 is the time that B passes or will pass the point X_1. Since the motion is along a spatial x axis, it will be represented by the conventional symbol v. B's motion must be represented by projecting a component of $c\mathbf{T}$, $c\mathbf{T}_p$ on the x axis. The position of B is then given in general by $x + cT_p$. From our previous definition of velocity, B's position can also be written as $x + vT$.

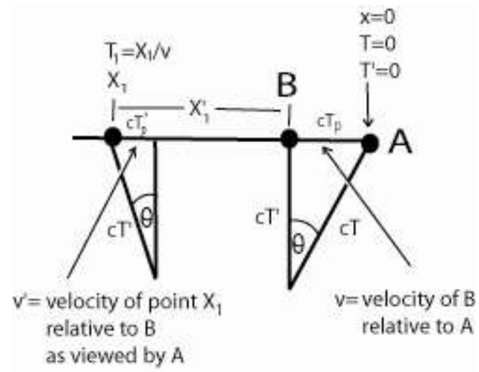

Figure 7

The time available to run B's clock is $T' = T\sqrt{1 - v^2 / c^2}$. As before, the rest observer A will have a different space/time representation (different time directions) for objects moving at different velocities.

To get the transformation equations for a specific point, refer again to Fig. 7. Assume that both clocks read zero at the starting point and call this the $x = 0$ point. We are interested in finding the time and position transformation equations for the point X_1. Clearly:

$$T_1' = T_1\sqrt{1 - v^2 / c^2} \qquad 1.7$$

Multiplying the numerator and denominator by $\sqrt{1 - v^2 / c^2}$ yields:

$$T_1' = (T_1 - v^2 T_1 / c^2) \Big/ \sqrt{1 - v^2 / c^2}$$

Substituting $T_1 = X_1 / v$ for the second T_1 in the above, and eliminating the subscript, yields the popular form of the Lorentz transform:

$$T' = (T - vX / c^2) \Big/ \sqrt{1 - v^2 / c^2} \qquad 1.8$$

We see that 1.7 is identical to 1.8.

To get the position transform we again refer to Fig. 7. From A's point of view the distance between B and X_1, at any time T, is simply $X_1 - vT$. It is not so obvious what the distance X' is for B from A's point of view. What is clear is that B must see the point moving toward it at speed v. In order to represent this (again from A's perspective), B's time, T' must be projected onto the x axis at the same angle θ and its speed is $v = cT_p' / T'$, which is identical to cT_p / T. This speed is obviously dilated as viewed form A, although B would measure it as v. A would measure it as $v' = cT_p' / T$, and the following speed ratio is apparent:

$$v' / v = \sqrt{1 - v^2 / c^2} \qquad 1.9$$

Let ΔT be the time on A's clock that it takes X_1 to reach B. The time on B's clock is then $\Delta T'$. B's distance from X_1 from A's perspective is $v'\Delta T$.

The distance that B would report is simply B's perceived speed v times the lapsed time on B's clock $\Delta T'$. B's distance then is $X' = v\Delta T' = v\Delta T\sqrt{1 - v^2/c^2} = v'\Delta T$. But $v'\Delta T$ is just the distance from A's perspective, $X - vT$. Hence we conclude that:

$$X' = X - vT \qquad 1.10$$

As before, we arrive at a **Galilean transformation** for spatial distance.

In the $P \to T$ representation we found that a Lorentz-like transform resulted from the rest observer assuming that his/her spatial direction was fixed for all moving objects.

In the above scenario, we would then expect to get a Lorentz-like transformation if the rest observer uses his/her time in both calculations. If we fix ΔT for both measurements, then A's assumed distance is given by the same formula $X - vT = v'\Delta T$. B's distance would now be given by $X' = v\Delta T$. From this we get $X'/(X - vT) = v'/v$. Using 1.9 we get

$$\frac{X'}{X - vT} = 1\Big/\sqrt{1 - v^2/c^2} \qquad 1.11$$

1.11 is exactly the Lorentz transform for spatial position.

In the $P \to T$ representation X cannot equal $V_P\mathbf{T}$

It should be clear that, in the $P \to T$ representation, no one single spatial dimension (x, y or z) can equal $V_P\mathbf{T}$. Assume that a straight ruler is placed in a direction and an object is sent moving down the ruler at constant velocity. See Fig. 8. The ruler is not moving and lies exactly perpendicular to the observer's time line. To find the position one would measure on the ruler, a line perpendicular to the ruler intersecting the point P is drawn (The point of intersection with the ruler is labeled X_c). From the previous discussion, it is clear that this yields the Fitzgerald length contraction formula.

$$X_c = P\sqrt{1 - v^2/c^2}$$

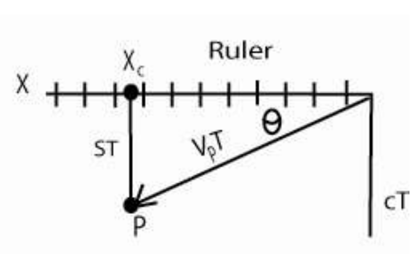

Figure 8

In Fig. 8, a line labeled *ST* connects a point X on the *x* axis with a point P on the *p* axis. The line is labeled *ST* because it is in general a combination of space and time. In the case above, where *ST* is perpendicular to the *x* axis we can write

$$(V_P T)^2 = X_c^2 + ST^2$$

Energy and Momentum

Both representations of motion in space/time result in the same space/time triangle. To develop an energy/momentum representation, we redraw the triangle using vector notation as shown below in Fig. 11. $\mathbf{p}$ represents a unit vector in the p direction, $\mathbf{t}$ a unit vector in the $\mathbf{T}$ direction and $\mathbf{t}'$ a unit vector in the $\mathbf{T}'$ direction.

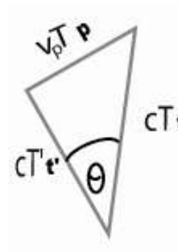

Figure 11

Next we multiply each side by m_0/T' where m_0 is the rest mass of the moving object. This yields the diagram in Fig. 12. Clearly $T'/T = \sqrt{1 - V_P^2/c^2}$. Letting $m = m_0\Big/\sqrt{1 - V_P^2/c^2}$ and substituting yields Fig. 13. All three sides have been transformed into momentum.

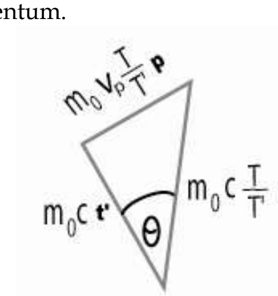

Figure 12

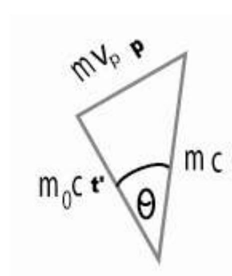

Figure 13

Next we define the following variables:

$$P_0 = m_0 c,\ \ P_p = mV_p,\ \ P_T = mc$$

$$E_0 = m_0c^2,\ \ E_p = mcV_p,\ \ E_T = mc^2$$

In terms of these variables, the momentum and energy triangles are depicted below in Fig. 14. It is obvious from Fig. 14, that:

$$\cos\theta = T'/T = P_0/P_T = E_0/E_T = \sqrt{1-V_P^2/c^2} \qquad 2.1$$

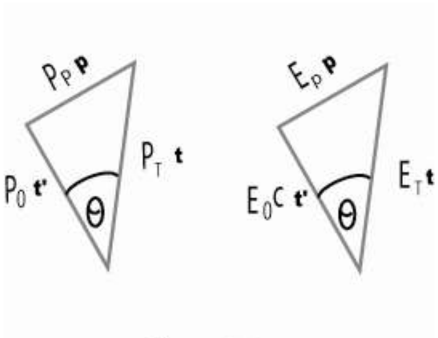

Figure 14

We have developed a vector representation of energy. To understand exactly what this means we must first look at exactly what making a measurement entails. As we discussed previously, any spatial measuring device (ruler for example) will lie on a line perpendicular to the observer's time line.

Any device for measuring energy will be stationary relative to the observer and must lie on a line perpendicular to the observer's time line as well. It is the measuring device, means or method that communicates the measurement to the observer. The situation is illustrated in Fig. 15. There the moving object encounters the measuring device on what is called the actual line of measurement. The time of the encounter with the measuring device differs from the observed time. The energy the observer would associate with this measurement is E_0'. The significance of $E_q^{'}$ will become clear shortly.

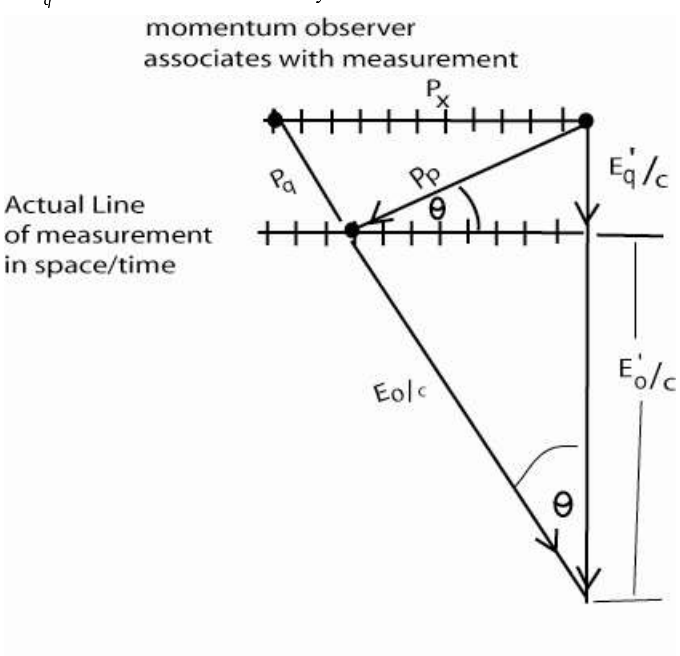

Figure 15

If the observer's time direction is fixed, energy has both a scalar and a vector representation. The scalar representation is just the vector projections onto the observer's time line. The vector magnitude is given by:

$$E_T^2 = E_0^2 + E_P^2 \qquad 2.2$$

Eq. 2.2 can be written in the more familiar form:

$$E_T^2 = m_0^2c^4 + P_P^2c^2 \qquad 2.3$$

The scalar equation can be written as:

$$E_T = E_0' + E_q \qquad 2.4$$

$E_0' = E_0\cos\theta$ and $E_q = cmV_P\sin\theta$. But $\sin\theta = V_P/c$ and hence:

$$E_T = E_0\cos\theta + mV_P^2 \qquad 2.5$$

In terms of the measurable energy we can write:

$$E_0' = E_T - mV_P^2 \qquad 2.6$$

In Equation 2.6, the second term is the amount of energy that cannot be directly measured. It ranges from minus infinity to 0. It has the exact same form as the potential energy of an object in a circular orbit ($-mv^2$) with a $1/r$ potential well.

We can write (1.4) in terms of E_0 by noting that $E_P = E_0\tan\theta$ and $E_q^{'} = E_P\sin\theta$ to yield:

$$E_T = E_0(\cos\theta + \tan\theta\sin\theta) \qquad 2.7$$

For small angles we can approximate 2.7. Using the first order expansion terms $\cos\theta \approx 1-\theta^2/2$, $\sin\theta \approx \theta = V_P/c$, and $\tan\theta \approx \theta$. Substituting $E_0 = m_0c^2$ for the second term and noting that $V_P \to v$ yields:

$$E_T = E_0 + m_0v^2/2 \qquad 2.8$$

Equation 2.8 is the classic Newtonian energy.

In present theory, the vector components and scalar components are often used interchangeably leading to much confusion. It has led some to speculate that the energy formulas and/or the theory cannot be correct [5].

Which representation to use?

We have developed two different representations of motion in space/time, which we call $P \to T$ and $T \to S$. In the $P \to T$ representation the direction of time is the same for all objects. If energy is to be treated as a scalar, the $P \to T$ representation must be used. We shall therefore use it in the following discussion to explain the Dirac matrices.

The Dirac Matrices

Before showing how the geometry developed previously can explain the Dirac matrices, we have to determine just how to add the y and z dimensions.

It is commonly accepted that we live in, or at least perceive, three spatial dimensions. Mathematically these three dimensions are represented as orthogonal or perpendicular. They are considered independent. That is to say, one can often calculate an event along one of the three without considering the others. Motion or velocity is one property that is considered independent. x, y and z (or some transformation of them) are treated as independent variables. It can be shown that the three dimensions cannot be truly independent.

In present theory the Lorentz transforms are assumed to be independently applicable to v_x, v_y and v_z. Each individual component can be transformed to any velocity less than c. The vector sum of these transforms $v=\sqrt{v_x^2+v_y^2+v_z^2}$ could exceed c.

In classical relativity, this problem is generally handled by choosing a coordinate system such that one axis, say x, lies exactly along the direction of motion.

A quantum object does not move in just one direction and therefore using the concept of independence of spatial dimensions is not compatible with QM. One would calculate states that have velocity components in each direction that are near the speed of light and hence the total velocity could exceed c.

We wish to find a three-component representation that meets the following two criteria.

A) $vT \le cT$ for all possible values of v_x, v_y and v_z.

B) The representation is compatible with standard mathematical transformations, specifically $v^2 = v_x^2 + v_y^2 + v_z^2$.

Criterion A is obvious if one hopes to achieve any sensible results. Criterion B simply makes the task easier. First we must note that there is not a one to one correspondence between the perpendicularity of v_x, v_y and v_z and the common transformation equation $v^2 = v_x^2 + v_y^2 + v_z^2$.

Clearly if v_x, v_y and v_z are real and mutually perpendicular, then $v^2 = v_x^2 + v_y^2 + v_z^2$. But given that $v^2 = v_x^2 + v_y^2 + v_z^2$, one cannot assume that the vectors are mutually perpendicular. Assume v_y equals $i\mathbf{a}$ where $\mathbf{a}$ is a real vector and i is the square root of negative one. Now $\mathrm{v}^2=\mathrm{v}_x^2-\mathrm{a}^2+\mathrm{v}_z^2$, and $\mathbf{a}$ cannot be perpendicular to both $\mathbf{v}_x$ and $\mathbf{v}_y$.

Criterion A can be satisfied if the three spatial components add as vectors to $\mathbf{V}_P$, and $\mathbf{V}_P$ represents the space-time velocity in the $P \to T$ representation. $(V_P T)^2 = (cT)^2 - (cT')^2$ and as V_P approaches c, T' approaches zero, and hence $v \le c$.

Consider the diagram in Fig. 11 with each vector given a direction as shown in Fig. 12. $\mathbf{P}_q$ now represents the amount of momentum that must be added to $\mathbf{P}_x$ to equal $\mathbf{P}_v$.

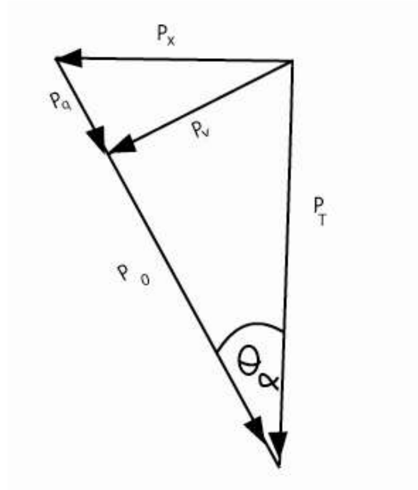

Figure 12

The following vector equations are apparent from the diagram in Fig. 12.

$$\mathbf{P}_T = \mathbf{P}_0 + \mathbf{P}_v \quad 3.1$$

$$\mathbf{P}_v = \mathbf{P}_x + \mathbf{P}_q \quad 3.2$$

Next a two vector representation of $\mathbf{P}_q$ is created. This is depicted in Fig. 13. Two vectors labeled $\mathbf{P}_z$ and $i\mathbf{P}_y$ are created. The two meet at a point above the $\mathbf{P}_x,\mathbf{P}_T$ plane as illustrated below in Fig. 14. $i\mathbf{P}_y$ comes directly out of the plane and is perpendicular to $\mathbf{P}_q$. $\mathbf{P}_z$ on the other hand is not perpendicular to $\mathbf{P}_q$. Obviously $P_q^2 = P_z^2 - P_y^2$ and:

$$\mathbf{P}_q = i\mathbf{P}_y - \mathbf{P}_z \quad 3.3$$

The choice of the two vectors may seem arbitrary. What is important is that they sum to $\mathbf{P}_q$ and that the magnitude of $\mathbf{P}_v$ is equal to the square root of the sum of the squares of the magnitudes of three components.

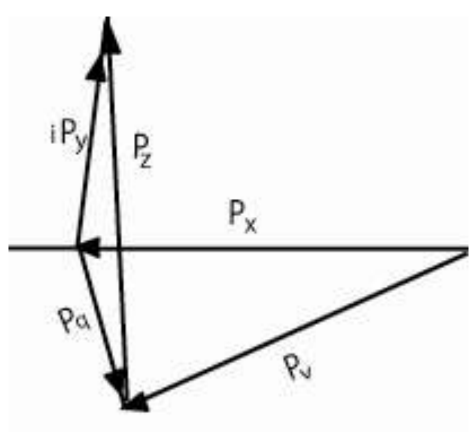

Figure 13

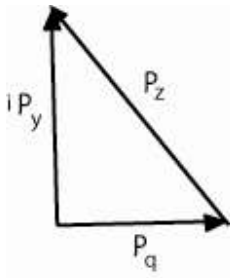

Figure 14

The magnitude of $\mathbf{P}_q$ is clearly $\sqrt{P_z^2 - P_y^2}$ and hence $\mathbf{P}_q$ can be considered to have components $(-P_z, i_s P_y)_m$. The subscript m is used to indicate that these 'vector components' are valid in so far as they yield the proper magnitude of their vector sum (using the square root of the sum of the squares).

From Fig. 15, one can see that $\mathbf{P}_x$ and $i\mathbf{P}_y$ are in fact orthogonal, and can thus be treated as a conventional complex number.

If $\mathbf{P}_z$ is combined with $i\mathbf{P}_y$ then the magnitude of the resultant is determined simply by the Pythagorean Theorem. On the other hand, if $\mathbf{P}_x$ is combined with $i\mathbf{P}_y$ the magnitude of the resultant is determined by multiplying $\mathbf{P}_x + i\mathbf{P}_y$ by its complex conjugate. Not surprisingly, $\mathbf{P}_x$ and $i\mathbf{P}_y$ do always appear together as a single variable in Dirac's equations, and in solving the equations, conventional complex methods are employed. Also note that $\mathbf{P}_z$ will always have the opposite sign of $i\mathbf{P}_y$ which again is consistent with the Dirac formulas.

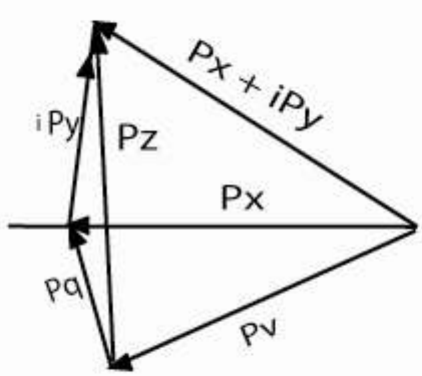

Figure 15

Combining Eqs. 3.1, 3.2, and 3.3 yields the following vector equation:

$$\mathbf{P}_T = \mathbf{P}_0 + \mathbf{P}_x + i\mathbf{P}_y - \mathbf{P}_z \qquad 3.4$$

It is clear that Equation 3.4 is valid irregardless of the plane that $\mathbf{P}_T$ and $\mathbf{P}_0$ lie in. $\mathbf{P}_0$ is orthogonal to $\mathbf{P}_v$ and hence it can be considered orthogonal to all other components by choice of dimensions.

If $\mathbf{P}_q$ is chosen to be perpendicular to $\mathbf{P}_x$, all components can be treated as orthogonal (for the purpose of determining the total magnitude). In this representation $\mathbf{E}$ is always in the same direction and hence we can write:

$$E_T / c = P_0 + P_x + iP_y - P_z \qquad 3.5$$

E is taken to be a scalar and all the components on the right are independent vector components.

From the above discussion there are two obvious facts, alluded to earlier.

1) P_x and iP_y should always appear as a single complex variable. Treating them as such will maintain their orthogonality as well as their compatibility with quantum mechanics.

2) P_z will always have the opposite sign of iP_y.

All four of Dirac's equations meet the requirements of 1 and 2 above as shown below.

$$E / c = m_0 c + P_x + iP_y - P_z$$

$$E / c = -m_0 c + P_x - iP_y + P_z$$

$$E / c = m_0 c + P_x - iP_y + P_z$$

$$E / c = -m_0 c + P_x + iP_y - P_z$$

In terms of Dirac matrices, $\mathbf{E} / \boldsymbol{c} = \boldsymbol{\alpha} \cdot \mathbf{P} + \beta \mathrm{mc}$, where the Dirac Matrices [7] are listed below.

$$\alpha_x = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \ \alpha_y = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \ \alpha_z = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$\beta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

So far it is not clear as to why there are four coupled equations in Diracs set. One explanation is the result of the direction of $\mathbf{P}_z$. This is illustrated in Fig. 16. It is clear that just negating $\mathbf{P}_z$ does not give the correct vector equation. It is necessary to define an up direction $\mathbf{P}_{Zu}$ and a down direction $\mathbf{P}_{Zd}$. The net energy and total momentum are unaffected by weather one uses the up or down direction, but the momentum and hence angular momentum in the case of orbiting electrons, for example, should be distinguishable. At any rate, a total quantum mechanical solution must include both distinguishable states and we can now write two equations:

$$E / c = m_0 c + P_x + iP_y - P_{Zu}, \quad E / c = m_0 c + P_x - iP_y + P_{Zd}$$

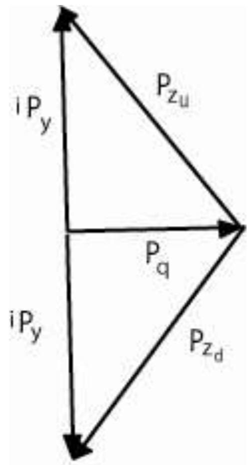

Figure 16

To get the remaining two equations refer to Fig. 17. It is apparent that negating $\mathbf{P}_0$ results in two distinguishable states for $\mathbf{P}_T$ which are labeled $\mathbf{P}_{TU}$ and $\mathbf{P}_{Td}$, and hence, the two equations must be further split each into two distinguishable states. The final four equations are:

$$(E/c)_u = m_0c + P_x + iP_y - P_{Zu},\ (E/c)_d = -m_0c + P_x + iP_y - P_{Zu}$$

$$(E/c)_u = m_0c + P_x - iP_y + P_{Zd},\ (E/c)_d = -m_0c + P_x - iP_y + P_{Zd}$$

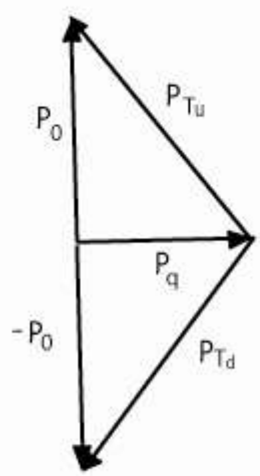

Figure 17

Quantum Mechanics in the $P \to T$ space-time representation

In quantum mechanics, one replaces the components of momentum with differential operators [4]. What about $\mathbf{P}_{Zu}$ and $\mathbf{P}_{Zd}$? One might attempt to find a separate differential operator but using symmetry considerations, it makes more sense to replace them with the same operator and define separate wave functions. The same argument applies to $(E/c)_u$ and $(E/c)_d$.

It is apparent that four separate wave equations will be required. The following eigenvector equations can be written. The p's now refer to operators and $p_x \pm ip_y$ is treated as a single complex variable.

$$(p_t - m_0c)\psi_1 - p_z\psi_1 - (p_x - ip_y)\psi_1 = 0 \qquad 3.6$$

$$(p_t - m_0c)\psi_2 + p_z\psi_2 - (p_x + ip_y)\psi_2 = 0 \qquad 3.7$$

$$(p_t + m_0c)\psi_3 - p_z\psi_3 - (p_x - ip_y)\psi_3 = 0 \qquad 3.8$$

$$(p_t + m_0c)\psi_4 + p_z\psi_4 - (p_x + ip_y)\psi_4 = 0 \qquad 3.9$$

Clearly for the same states of energy and momentum there will be relationships between the wave functions.

The geometric relationship between the spatial components of momentum is not affected by $p_0(m_0c)$. The equivalent states for the positive energy and negative energy solutions should have the same magnitude of P_z and the same sign. As was clear from the previous discussion, it is not possible to simply negate P_z. (you can't replace a $+P_z$ with a $-(-P_z)$)

The $-p_z\psi$ terms in Eqs. (2.6) and (2.8) should be identical for identical states. Similarly, the $+p_z\psi$ terms in Eqs 3.7 and 3.9 should be identical for identical states. Substituting yields:

$$(p_t - m_0c)\psi_1 - p_z\psi_3 - (p_x - ip_y)\psi_1 = 0 \qquad 3.10$$

$$(p_t - m_0c)\psi_2 + p_z\psi_4 - (p_x + ip_y)\psi_2 = 0 \qquad 3.11$$

$$(p_t + m_0c)\psi_3 - p_z\psi_1 - (p_x - ip_y)\psi_3 = 0 \qquad 3.12$$

$$(p_t + m_0c)\psi_4 + p_z\psi_2 - (p_x + ip_y)\psi_4 = 0 \qquad 2.13$$

There is insufficient coupling in the above equations to achieve a unique solution for each wave function for any given state. Additional substitutions are required.

A similar substitution based on $p_x + ip_y$ would result in no additional coupling. Referring to Fig. 15, it is clear that reversing the sign of $+ip_y$, which results in reversing the sign of p_z creates an exact mirror image using the plane of p_v and p_x as the mirror. The solutions will have identical values of energy and momentum.

Limiting the solutions to those that are symmetrical with regards to a reversal of $+ip_y$ we can conclude that we can exchange wave functions terms in any two equations, provided that we replace one operated on by $p_x + ip_y$ with one operated on by $p_x - ip_y$, if p_z has the opposite sign in the two equations.

Considering the above restriction, a substitution with regards to the wave function operated on by $p_x \pm ip_y$ in Eqs. 3.13 and

3.10 is valid as well as a substitution of the same terms in Eqs. 3.11 and 3.12. Making the substitutions yields:

$$(p_t - m_0 c)\psi_1 - p_z\psi_3 - (p_x - ip_y)\psi_4 = 0$$

$$(p_t - m_0 c)\psi_2 + p_z\psi_4 - (p_x + ip_y)\psi_3 = 0$$

$$(p_t + m_0 c)\psi_3 - p_z\psi_1 - (p_x - ip_y)\psi_2 = 0$$

$$(p_t + m_0 c)\psi_4 + p_z\psi_2 - (p_x + ip_y)\psi_1 = 0$$

The above equations are exactly the four coupled differential equations that result from the Dirac matrices formulization of relativistic quantum mechanics.

Some consequences of this theory

1) The spatial dimensions in this representation are not homogeneous. The variables do not all range from minus infinity to plus infinity. It is not difficult to show that in the $P \to T$ representation, $P_x \le P_0$ when P_q is perpendicular to P_T. Also, $|P_y| \le |P_z|$. This may explain the cut off points [7] that are commonly required in present quantum field theory to achieve only valid results.

2) The direction of P_T for anti matter is not the opposite direction as it is for matter. One obvious consequence of this is the fact that mater and anti matter do not have perfect symmetry with regards to energy. One would expect that using devices and material made purely of matter, and used to measure anti matter, would not give results that are a 'mirror image' of the results for matter. Experiments seem to bear this out [6].
Measuring anti matter with devices and material made entirely of anti matter should give the exact 'mirror image' results.

3) The i (square root of negative one) is required only for the purpose of making the vector magnitude equation work. It should be possible to formulate this space-time geometry with all real vectors. The author is working on formulating this representation in a real homogeneous space/time.

4) If momentum in space/time is conserved, certain phenomenon can be explained. For example: beta decay. A neutron decaying into a proton/ electron pair would not conserve momentum in space/time. The proton's motion line would be at a much smaller angle to a fixed spatial axis than the electrons. A neutrino may be a requirement of conservation of momentum in beta decay.

Summary

When Dirac developed his matrices he did so for mathematical reasons. There was no tangible physical rationality behind the development. A similar statement could be made in regard to Lorentz. Although the Lorentz transforms were given a degree of physical meaning by Einstein, the Diracs matrices remain a mystery.

The space/time geometry that results by treating time as a vector and motion as a projection of the time vector, results in a simple geometric explanation of both the Lorentz transforms and the Dirac matrices. This theory unifies the two concepts.

There is obviously much yet to explore with this theory. Questions and comments are welcome.

Acknowledgment

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