Velocity Addition in Space/Time

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Abstract

Using the space/time geometry developed in the previous paper ("Linear Motion in Space-Time, the Dirac Matrices, and Relativistic Quantum Mechanics"), I shall develop a velocity addition formula. The formula differs from the one originally proposed by Einstein.

Relativistic Velocity Addition

The present relativistic velocity addition or composition formula, as it is more appropriately called, has been derived in numerous ways. The most direct is simple substitution of one transformation into another. Assume that there are two objects each traveling at a different velocity relative to an observer. We label the observer A, and the moving objects B and C respectively. Assume C's velocity is the larger of the two. We can write the following transformation equations (c is taken to be 1):

$$x' = (x - v_B t)\Big/\sqrt{1-v_B^2} \qquad 1.10$$

$$t' = (t - x v_B)\Big/\sqrt{1-v_B^2} \qquad 1.20$$

$$x'' = (x' - ut')\Big/\sqrt{1-u^2} \qquad 1.30$$

v_B is the velocity of B relative to A and u is the velocity of C relative to B. Substituting 1.10 and 1.20 into 1.30, one arrives at:

$$x'' = (x - v_C t)\Big/\sqrt{1-v_C^2}$$ where;

$$v_C = \frac{u + v_B}{1 + u v_B}$$ v_C is the velocity of C relative to A.

As long as both u and v_B are less than c (1 in the above example), v_C will always be less than c. This fact, and the existence of no other formula has lead virtually everyone to accept this formula as correct.

Space/Time geometry and velocity addition

In my paper ("Linear Motion in Space-Time, the Dirac Matrices, and Relativistic Quantum Mechanics"), I developed two different ways to represent linear motion in space/time. In one (P-T), the time vector was held fixed in direction and the axis of motion rotated towards it with the angle of rotation determining the space/time velocity. In the other (T-S) the axis of motion was fixed in direction and the time vector rotated towards it.

Lorentz transforms or Loerentz like transforms can be developed from either. Since energy is derived from the same vectors, it was seen that the P-T representation was more compatible with our notions of energy as a scalar and in fact the Dirac matrices could be developed from this representation.

In the S-T representation all space time motion lies on the same axis and the same reasoning that produces the Einstein velocity addition formula could be applied.

I shall show in the P-T space/time representation that the Einstein formula makes no sense and I shall derive one that does.

We start by considering the space/time representation of the two objects B and C discussed in the above example. Refer to Figure 1 below.

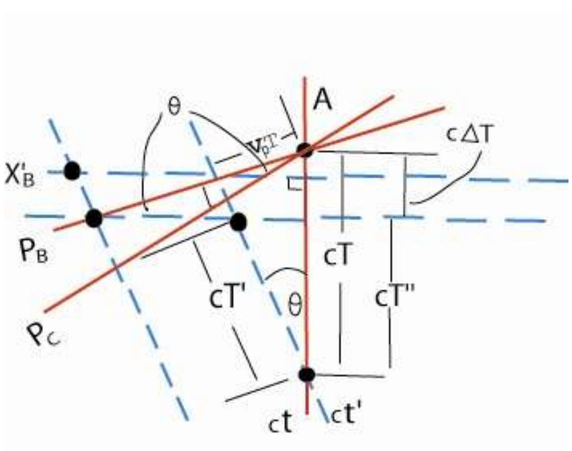

Figure 1

In Figure 1, the axes of motion for each moving object are shown. P_B is the motion axis for object B and P_C is the motion axis for object C. Both are moving relative to A. The dashed lines are reference lines for the purpose of determining the transformation equations for B. To avoid confusion the reference lines for C are not shown. The lines for C would simply be orientated relative to P_C . We know from the previous paper and /or the above diagram that:

$$X_B^{'} = (P_B - V_B T) \Big/ \sqrt{1 - V_B^2 / c^2} \qquad 1.40$$

$$X_C^{'} = (P_C - V_C T) \Big/ \sqrt{1 - V_C^2 / c^2} \qquad 1.50$$

Now if we analysis the motion of C relative to B we get the diagram of Figure 2 below:

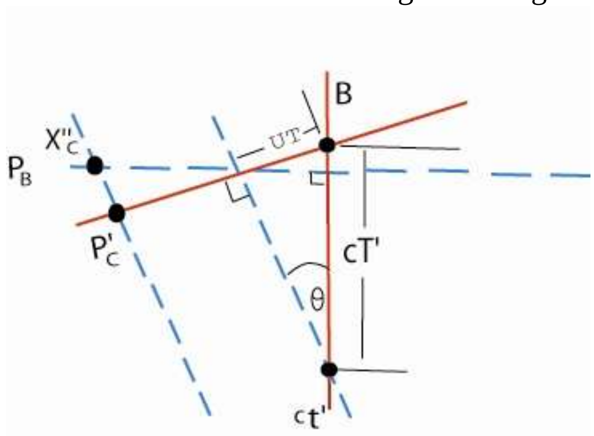

Figure 2

In Figure 2, the details were left out for clarity. The velocity of C relative to B is u and the axis of motion is labeled $P_C^{'}$ since the motion is the result of the projection of cT'. The transformation equation for the point $X_C^{''}$ is:

$$X_C^{''} = (P_C^{'} - uT') \Big/ \sqrt{1 - u^2 / c^2} \qquad 1.60$$

If we follow the previous reasoning, we would substitute for $P_C^{'}$ and T'. Paralleling the previous argument, we would substitute $X_C^{'}$ from 1.50 but this would not be valid since the points do not even lie on the same axis. We therefore conclude that the substitution of $X_C^{'}$ for $P_C^{'}$ would not give an exact equation. It is clear that another method is required.

A vector velocity Addition formula

All the axes of motion in space/time have direction and therefore any velocity addition formula must be developed from vector considerations. We shall proceed with the simplest which is shown below in Figure 3.

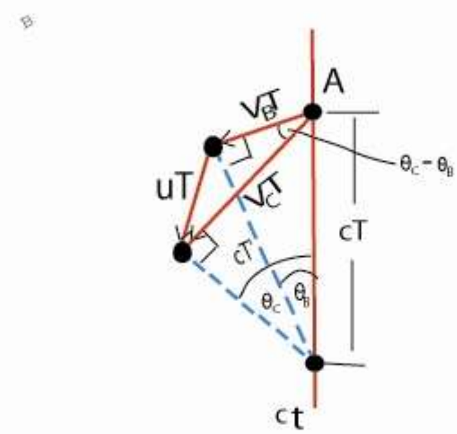

Figure 3

In Figure 3, the motion of both B and C relative to A is shown. A vector labeled U connects $V_B T$ and $V_C T$. When V_C equals V_B, U is zero and when V_C equals c (V_C lies on the cT axis) UT equals cT'. From B's point of view the velocity of C in units of c, is UT/cT'. This follows from the definition of velocity in the previous paper. It clearly ranges from zero to one as viewed by B.

It is easy to develop an expression for UT/cT'. We start by considering the diagram in Figure 4 below.

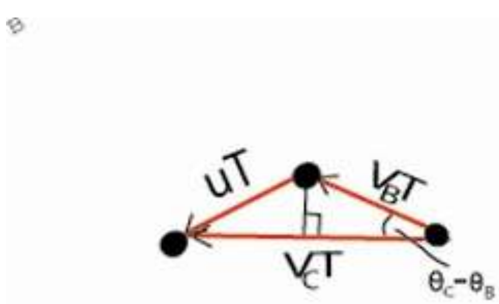

Figure 4

In Figure 4 a perpendicular is dropped from the apex of the triangle to the $V_C T$ line. From basic geometry we quickly arrive at the following equation.

$$\frac{UT}{cT'} = T/_{cT'}\sqrt{\left(V_C - V_B\cos(\theta_C - \theta_B)\right)^2 + \left(V_B\sin(\theta_C - \theta_B)\right)^2} \qquad 1.70$$

From the previous paper we know that $T/_{T'} = 1/_{\cos(\theta_B)}$. Letting c equal 1 and defining $U_C = UT/_{cT'}$ (the velocity of C relative to B) we finally arrive at:

$$U_C = \frac{1}{\cos(\theta_B)}\sqrt{\left(V_C - V_B\cos(\theta_C - \theta_B)\right)^2 + \left(V_B\sin(\theta_C - \theta_B)\right)^2} \qquad 1.80$$

At this point it is instructive to calculate some actual velocities and see how they compare to the Einstein formula. The chart below show several combinations of velocity. Vb is the velocity of B relative to A. Uc is the velocity of C relative to B

Vb	Uc	Vc (eq 1.80)	Vc Einstein
0.001	0.001000002	0.002	0.002
0.001	0.00900005	0.01	0.00999996
0.001	0.099005012	0.1	0.099995112
0.001	0.29904606	0.3	0.29995636
0.0001	0.949968775	0.95	0.94997853
0.01	0.290460131	0.3	0.299589942
0.1	0.204125503	0.3	0.298041711
0.1	0.307886763	0.4	0.395703573
0.1	0.628225944	0.7	0.68518109
0.4	0.813722971	0.95	0.915679269
0.8	0.5336668	0.95	0.934638409
0.95	0.230369115	0.97	0.968428007

A program for calculating values is available at
http://members.triton.net/daveb/velocty_addition.htm

From the chart a number of things are clear.

For very small velocities both eq 1.80 and Einstein's formula give identical results.
The largest variations between 1.80 and Einstein occur when one or both velocities are in the mid range.

The chart clearly indicates that experimental results might favor one formula over another. Most experiments done to date (that the author is aware of) have one very high velocity (light in a medium) and one very low velocity (the medium). The chart clearly indicate (row5) that under these conditions there is a very small difference (on the order of .002 %) between eq 1.80 and the Einstein formula.

Conclusion

The space/time geometry developed in my previous paper lends itself to an simple velocity addition formula. The similarity between the results of the formula and the Einstein formula would explain why there is no substantial experimental evidence contrary to the Einstein formula results.

One should be able to test my theory with a properly devised experiment.